Case Study: Bacterial Regrowth in Water Distribution Networks

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Abstract: - A real-world problem (bacterial regrowth in water distribution systems) is suggested in the present paper as a case study for a course in mathematical modeling or numerical methods. Case studies motivate engineering students to learn theory and to develop necessary skills for using mathematical methods in applications. A relatively simple mathematical model can be used to solve the case. In addition, classical numerical schemes for the solution of hyperbolic and parabolic equations can be discussed since wave equation and heat equation are particular cases of bacterial regrowth model.

Key-Words: - Case studies, mathematical model, bacterial regrowth, water distribution system

1 Introduction

Case study method is usually used as a complement to traditional teaching methods like lectures or tutorials. Researchers have also used the case study research method for many years across a variety of disciplines (examples include social sciences, medicine, marketing). It is recognized nowadays that case study method can also be effectively used to teach mathematical modeling for engineers at undergraduate and graduate level. Several case studies based on problems in some British companies are considered in [1]. Recent publications discuss case studies and projects from industry at upper undergraduate and graduate level [2], [3]. Case studies based on real-world problems are also extensively used in mathematical modeling courses offered by Finnish Virtual University Network [4]. The shift in teaching philosophy can be clearly seen from [1]-[4], that is, real-world problems are discussed first and mathematics is used as a necessary tool to solve these problems in contrast to standard teaching where mathematical methods are discussed first and then used to solve problems.

Experience has shown that case studies bring interesting, real-world situations into the classroom. Case studies motivate students to learn the theory and, therefore, provide a meaningful framework for the theoretical material. Case study approach can be really useful for engineering students if the following conditions are met [5]:

1. Case study should be based on a real-world problem.
2. A mathematical model used to describe the case should be relatively simple.
3. Several mathematical methods should be used for the solution.
4. Model results should be compared with experimental data.

In the present paper case study approach is used to analyze an important practical problem: bacterial regrowth in water distribution systems. The case study described in the paper can be used in courses on mathematical modeling or numerical methods.

One of the main objectives of drinking water treatment is to protect consumers against infectious diseases through efficient microbial inactivation [6]. On the other hand, proper management of disinfectant concentrations in water distribution system is essential to ensuring that a safe and acceptable product is supplied to the consumer [7]. Many studies have shown the complex structure of the processes that occur in water distribution networks [8], [9]. Recently a new bacterial regrowth model for water distribution systems was suggested in [10]. The model presented in [10] combines hydraulic network calculations with microbial and chemical processes that occur in the network. In particular, the following microbial processes are analyzed: the growth and mortality of free and attached bacteria, bacteria detachment and inactivation by chlorine.

All the four conditions specified above are satisfied for the case study based on the model suggested in [10]. In particular, special cases of the problem considered in [10] allow the instructor to discuss different numerical methods used to solve hyperbolic problems, parabolic problems or advection-diffusion problems. In addition, a simplified mathematical model for advection-diffusion equation is used to demonstrate the role of analytical solutions in mathematical modeling.

2 Problem Formulation

Case studies are used in practice (in particular, to teach courses in numerical methods or mathematical modeling) in order to provide students with necessary skills to apply different mathematical techniques to real-world problems. One of the objectives of case study method is to train students how to simplify a complicated real-world problem and formulate solvable mathematical models. Our experience shows that engineering students like practicality of problems discussed in class and they are more interested to learn new methods and techniques if they are convinced that the problem in hand is important in applications. Two real-world problems can be suggested for a mathematical modeling course (or for a course in numerical methods) to illustrate mathematical models used in fluid mechanics: leak detection in pipelines [5] and bacterial regrowth in drinking water distribution network. It is assumed that students have an experience in deriving basic equations of mathematical physics (like wave equation or heat equation). Some time in class should be dedicated to the discussion of the background of the problem. In particular, the role of important physical mechanisms such as diffusion, advection and nonlinearity should be briefly analyzed by the instructor before formulating a conceptual model for case study. This approach to teach mathematical modeling (or numerical methods) by introducing a real-world problem provides students with a reason to learn new methods and techniques and generates interest to the subject since these techniques are needed to solve a real problem. In addition, students gain practical experience and problem-solving skills so that they can apply what they learn in real applications.

The model proposed in [10] for bacterial regwroth in water distribution systems can be described by the following system of equations

$$\frac{\partial X_a}{\partial t} = \mu_a X_a - k_{\det} X_a v - k_d X_a + k_{dep} X_b R_h \quad (1)$$

$$\frac{\partial X_b}{\partial t} = -v\frac{\partial X_b}{\partial x} + D\frac{\partial^2 X_b}{\partial x^2} + \mu_b X_b + k_{\det} X_a \frac{v}{R_h} - k_d X_b - k_{dep} X_b \quad (2)$$

$$\frac{\partial S}{\partial t} = -v\frac{\partial S}{\partial x} + D\frac{\partial^2 S}{\partial x^2} - \left(\frac{1}{Y_g \beta}\right)\left(\mu_a \frac{X_a}{R_h} + \mu_b X_b\right) \quad (3)$$

$$\frac{\partial Cl_2}{\partial t} = -v\frac{\partial Cl_2}{\partial x} + D\frac{\partial^2 Cl_2}{\partial x^2} - k_b Cl_2 - \frac{k_w}{R_h} \quad (4)$$

where x is the longitudinal coordinate, t is time, X_a is the concentration of attached bacteria on the surface, X_b is the concentration of free bacteria in bulk water, S is the concentration of the biodegradable carbonated substrates in water, Cl_2 is the chlorine concentration in water, v is water velocity, D is the diffusion coefficient, R_h is the hydraulic radius. The parameters μ_a and μ_b represent specific growth rate constants of attached bacteria and free bacteria in the bulk water, respectively. The rates μ_a and μ_b depend on the organic substrate concentration, chlorine concentration and temperature and are expressed as follows:

$$\mu_b = \mu_{\max,b}\left(\frac{S}{S+K_s}\right)\exp\left(-\frac{Cl_2 - Cl_{2t}}{Cl_{2c}}\right) \times \exp\left[-\left(\frac{T_{opt} - T}{T_{opt} - T_i}\right)^2\right] \quad (5)$$

if $Cl_2 > Cl_{2t}$ and

$$\mu_b = \mu_{\max,b}\left(\frac{S}{S+K_s}\right)\exp\left[-\left(\frac{T_{opt} - T}{T_{opt} - T_i}\right)^2\right] \quad (6)$$

if $Cl_2 \le Cl_{2t}$. Here $\mu_{\max,b}$ is the maximum growth rate of biomass in the bulk water, T_{opt} is the optimal temperature for bacterial activity, T_i is the temperature dependent shape parameter, T is the temperature, Cl_{2t} is the threshold above which chlorine affects bacterial activity and Cl_{2c} is the characteristic chlorine concentration. Equation similar to (5), (6) holds for μ_a.

The description of the other parameters of the model (1) – (4) is given in [10].

The instructor should discuss problem (1) – (4) with students in detail. In particular, attention should be given to the major physical mechanisms represented in the model. In addition, the major assumptions should be discussed as well. First, it is assumed that the model is one-dimensional (that is, advective-dispersive transport in the radial direction is neglected). Second, transformation is assumed between free and attached bacteria, that is, free bacteria can deposit as attached bacteria and attached bacteria can detach to form free bacteria. Third, the instructor should emphasize that advection and diffusion both play an important role in water distribution networks (that is why advection and diffusion terms are present in (1) – (4)). Fourth, the following dependent variables are used in the analysis: free bacteria in the bulk water, attached bacteria on the inner surface of the wall, biodegradable dissolved organic carbon in the bulk water, and chlorine in the bulk water. It is also recommended at this stage to pay students' attention to the fact that in order to obtain a unique solution of the system (1) – (4) one needs to specify the boundary and initial conditions.

3 Special Cases of the Formulated Problem

The analysis presented in the previous section shows that 1) the problem under consideration is a real-world problem; 2) the solution of this problem is very important in practice and 3) the model is still rather complicated (at least for the first discussion in class). It is therefore recommended to the instructor at this stage to consider some special cases of (1) – (4). One option is to analyze scalar advection-diffusion equation of the form

$$\frac{\partial C}{\partial t} = -v\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2} \qquad (7)$$

Equation (5) is convenient for the analysis due to the following reasons. First, it represents a particular case of (1)-(4). Second, an analytical solution of (5) can be obtained in the form

$$C(x,t) = \frac{C_0}{2}\left\{ \begin{array}{l} erfc\left[\dfrac{x-vt}{2\sqrt{Dt}}\right] \\ +\exp\left(\dfrac{vx}{D}\right) erfc\left[\dfrac{x+vt}{2\sqrt{Dt}}\right] \end{array} \right\} \qquad (8)$$

for the following initial and boundary conditions:

$$C(x,0) = 0,\ C(0,t) = C_0,\ C(\infty,t) = 0 \qquad (9)$$

The instructor should stress the importance of analytical solutions in numerical analysis: analytical solutions in many cases are used to test the robustness of numerical algorithms. Third, equation (7) gives an opportunity for the instructor to discuss two classical problems which arise in a numerical approximation of partial differential equations: a hyperbolic problem (when $D = 0$ in (7)) and a parabolic problem (when $v = 0$ in (7)). It is well-known (see, for example, [11]) that different numerical schemes are used for approximation of hyperbolic and parabolic equations. The advantages and limitations of approximation methods can be effectively discussed by the instructor using equation (7) and its analytical solution (8). Fourth, equations (7) – (9) (for the case $v = 0$) can be used as a class project. The following steps are suggested for the class project:

1. Solve equation (7) under conditions (9) analytically using the method of Laplace transform.
2. Use explicit and implicit difference schemes for parabolic equations to solve equation (7), (9) numerically.
3. Analyze the limitations of numerical schemes used in Step 2.

In case where standard packages like IMSL or Matlab are available, students can perform numerical experiments and compare numerical solutions of (7), (9) with analytical solution (8) for different values of D.

3 Model Development and Simulations

Problem (1)-(4) is rather complicated and it is unlikely that students will be able to solve the case without the help from the instructor. The guidelines for the analysis should be provided by the instructor.

It is recommended to pay students' attention on major factors playing an important role in the model. Such physical phenomena as diffusion, advection and nonlinearity should be discussed by the instructor in detail. Also, analytical solution (8) needs a careful discussion in class. The instructor may review basic analytical methods such as separation of variables or methods of Laplace transform. The students may be asked to solve similar problems working in groups.

In addition, students should be advised what to do with the full model (1)-(4). First, it is recommended to spend some time on the analysis of split-operator method and show the main advantages of this method. In particular, it should be demonstrated that split-operator method provides a means to separate the transport operator from the reaction operator. This results in a very efficient and flexible approach to solve complex partial differential equations. Second, a model described by (1)-(4) should be applied to a pipe network. It should be made clear by the instructor that equations (1)-(4) are derived for a straight long pipe. In reality, however, one has to analyze a network of many pipes. In order to simplify discussion, the instructor is advised to use a hypothetical network containing just a few pipe segments of different lengths and diameters. Since at present it is difficult to obtain experimental data for such a problem, it is recommended to consider the network considered in [10] which consists of five pipe segments. Rather than develop their own code for numerical simulations, students should be advised to use available software like Matlab or IMSL for modeling purposes.

4 Conclusion

Case study approach is used in the present paper to discuss a real-world problem (bacteria regrowth in water distribution systems). The attractiveness of case studies for teaching courses in mathematical modeling or numerical methods lies in their ability to integrate theory and practice. Solving a case gives a student experience with how mathematics can be used to answer specific practical questions. The student is also given an opportunity to practice his or her communications skills and use available software for the solution. The importance of simple and reliable bacteria regrowth model for drinking water distribution networks is discussed.

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