Automated Knowledge Assessment and Knowledge Spaces

SYLVIA ENCHEVA
Stord/Haugesund University College
Department Haugesund
Bjørnsonsg. 45, 5528 Haugesund
NORWAY

SHARIL TUMIN
University of Bergen
IT-Dept.
P. O. Box 7800, 5020 Bergen
NORWAY

Abstract: The goal of this paper is to develop a theoretical framework for efficient assessment of learners' understanding of terms and concepts that are of fundamental importance for the study of a subject. The model is based on the theory of knowledge spaces and meet-distributive lattices. The structure of the latter is used to select only knowledge states that imply understanding of key ideas and minimize the effect of lucky guesses while determining learner's knowledge.

Key–Words: Knowledge assessment, lattices

1 Introduction

Assessing the initial knowledge of a student and updating this assessment as the student progresses through a course is an important part of every intelligent tutoring system [1]. A framework for representing and measuring students' knowledge is developed in [6]. The key concepts in the theory of knowledge spaces are the *knowledge state* - a subset of problems that an individual is capable of solving correctly, and the *knowledge structure* - a distinguished collection of knowledge states [10].

Establishing the knowledge state of a student in a subject is crucial for providing him/her with individual help. However, in many occasions it may require a relatively long sequence of questions and thus disturb the flow of the learning process. It might also not be very effective for providing immediate help to the student while he/she is working on a particular part of the curriculum. In this paper we propose a model for automated assessment of learner's understanding of comparatively small units that are considered to be fundamental in a subject. The model is based on application of lattices of closure systems.

'Assessment and LEarning in Knowledge Spaces' (ALEKS) [14] and 'Relational Adaptive Tutoring Hypertext' (RATH) [15] are systems aiming at establishing the knowledge state of each student in a certain knowledge domain and then provide further guidance and personalized help. Our model differs from existing systems in the following: it is based on multiple choice tests, can assess high level thinking, does not apply coefficients for guessing correction, and knowledge states are arranged in meet-distributive lattices where student's understanding of an atom is found satisfactory if his/her response belongs to a sublattice of a lattice of the convex geometries on the set of related atoms.

The rest of the paper is organized as follows. Related work is listed in Section 2. The model description can be found in Section 3. The paper ends with a conclusion placed in Section 4.

2 Related Work

Let Q be a finite set. A family $\mathcal{K}$ of subsets of Q is a *knowledge space* on Q [6] if the empty set and the total set Q are members of the family $\mathcal{K}$, and the family $\mathcal{K}$ is closed under union. An atom at item q in knowledge space theory is a minimal knowledge state containing q. A state is called an atom if it is an atom at q for some item q. A finite closure space is a convex geometry if its dual $(Q, \mathcal{K})$ is a knowledge space in which all learning paths are gradations [6]. Subsets of relevent examination questions and certain skills from a branch of knowledge are listed as examples of knowledge states in [7]. They are followed by an important remark that not all possible subsets of such items turn out to be knowledge states. ALEKS [14] is based on mathematical cognitive science and involves computer algorithms while constructing specific knowledge structures. Markovian procedures are further employed for analyzing of a particular student's knowledge. RATH [15] combines mathematical hypertext model and knowledge space theory and is focused on teaching.

A model for student knowledge diagnosis through adaptive testing is presented in [12]. Permutational multiple choice question tests have been used for assessing high-level thinking [11]. Students' conceptual thinking can be assessed by presenting them with tests where all the correct answers should be chosen and/or answers require integration of several components or approaches [3]. The use of formula in a spreadsheet to convert the raw assessment marks into marks or grades corrected for guessing or additionally allowing for the maximum expected mark is demonstrated in [13]. An excellent introduction to ordered sets and lattices and to their contemporary applications can be found in [5].

3 The Model

For this scenario we consider a subject, given at several universities, members of a federated system. A subject is divided into units accepted by all course builders from these universities. They agree on terms, concepts and skills to be included in tests' questions following a unit, knowledge states going to be atoms, and knowledge states containing questions related to those atoms. The primary objective of this approach is to determine all knowledge states that indicate that a student possesses knowledge about an atom in a unit and is able to apply that knowledge solving problems related to other atoms in the same unit. This can be achieved by working with nodes of a meet-distributive lattice on the set of questions.

3.1 A Test with Six Questions

After going through a section in a subject, a student is suggested to take a multiple choice test with six questions. Three of the questions consider understanding of new terms or applying new skills and are denoted by $\{a, b, c\} = \mathcal{B}$. The other three are denoted by $\{ab, ac, bc\} = \mathcal{R}$ where ab indicates student's ability to apply both a and b at the same time, ac indicates student's ability to apply both a and c at the same time and bc indicates student's ability to apply both b and c at the same time.

We assume that a student has sufficient knowledge and understanding of a question if he/she gives correct answers to:

a) one basic question from the set $\mathcal{B}$, say a and the two related to a questions from the set $\mathcal{R}$, i.e. ab and ac, or

b) two basic question from the set $\mathcal{B}$, say a, b and the related to a and b question from the set $\mathcal{R}$, i.e. ab.

Table 1: Correct answer combinations leading to a reduced test in a consecutive trial in the case of six questions

Correctly answered questions in a current test						Questions to be excluded
a	b	c	ab	ac	bc	
•			•	•		a
	•		•		•	b
		•		•	•	c
•	•		•			ab
•		•			•	ac
		•	•		•	bc
•	•		•	•		a, ab
•	•	•		•		b, ab
•		•	•		•	a, ac
	•	•	•		•	b, bc
	•	•		•	•	c, bc
•		•		•	•	c, ac
•	•	•	•	•		a, ab, ac
•	•	•	•		•	b, ab, bc
•	•	•		•	•	c, ac, bc

The idea is to filter out all answer combinations that do not imply that sufficient knowledge and understanding are obtained. The outcome is listed in Table 1 and has a graphical representation shown in Fig. 1.

In a case with six questions we consider the following five cases where if the student answers correctly to all of them the process of questioning is terminated.

`Case 1`:
If a student can answer correctly to less than three questions or to exactly three questions from either $\mathcal{B}$ or $\mathcal{R}$, the system will first present him/her with selected learning materials (theory and examples). Next time the student takes the same test he/she will be presented with six similar questions but developed by another course builder.

`Case 2`:
Suppose a student answers correctly to three questions, where one of them belongs to $\mathcal{B}$ and the other two are the related to it questions from $\mathcal{R}$, like c, ac, bc. This answer combination indicates mastering question c and makes no assumptions about other questions. The student will then be advised to work with selected learning materials (theory and examples) concerning questions a and b. In a consecutive test the student will be presented with five questions a, c, ab, ac, bc again developed by another course

builder. If the student answers correctly to all of them, the process of questioning is terminated. If the student fails to give correct answers to some of the questions a, b, ab, ac, bc then procedures similar to the following cases will be applied.

`Case 3`:
Suppose a student answers correctly to three questions, where two of them belong to $\mathcal{B}$ and the third one is the related to them question from $\mathcal{R}$, like b, c, bc. This answer combination indicates mastering question bc and makes no assumptions about other questions. The student will then be advised to work with selected learning materials (theory and examples) concerning questions a, b and c. In a consecutive test the student will be presented with five questions a, b, c, ab, ac again developed by another course builder. If the student fails to give correct answers to some of the questions a, b, c, ab, ac then procedures similar to the following cases will be applied.

`Case 4`:
Suppose a student answers correctly to four questions, where two of them belong to $\mathcal{B}$ and the other two are the related to them questions from $\mathcal{R}$, like f.ex. a, b, ab, ac. This answer combination indicates mastering questions a and ab and makes no assumptions about other questions. The student will then be advised to work with selected learning materials (theory and examples) concerning questions b and c. In this case we still repeat question b since by not answering correctly to question bc the student indicates possible difficulties applying knowledge from question b to other domains. Next time the student takes the same test he/she will be presented with four questions b, c, ac, bc again developed by another course builder. If the students fails to give correct answers to some of the questions b, c, ac, bc then procedures similar to the one in the following case will be applied.

`Case 5`:
Suppose a student answers correctly to five questions, where three of them belong to $\mathcal{B}$ and the other two are related to them questions from $\mathcal{R}$, like f.ex. a, b, c, ab, bc. This answer combination indicates mastering questions b, ab and bc and makes no assumptions about other questions. The student will then be advised to work with selected learning materials (theory and examples) concerning questions a and c. In this case we repeat questions a and c since by not answering correctly to question ac the student indicates possible difficulties applying knowledge from questions a and c to other areas. Next time the student takes the same test he/she will be presented with three questions a, c, ac again developed by another course builder. If the students fails to give correct answers to some of the questions a, c, ac then a similar procedure will be applied.

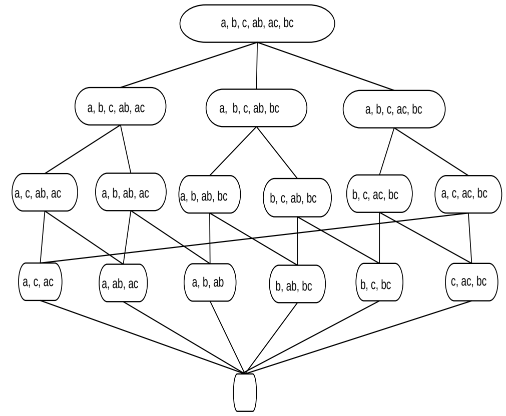

Figure 1: A lattice for the case with six questions

3.2 A Test with Eight Questions

After going through a even larger section in a subject, f.ex. a chapter, a student is suggested to take a multiple choice test with eight questions. Four of the questions consider understanding of new terms or applying new skills and are denoted by $\{a, b, c, d\} = \mathcal{B}$. The other four indicating student's ability to apply three terms/skills at the same time are denoted by $\{abc, abd, acd, bcd\} = \mathcal{R}$.

We assume that a student has sufficient knowledge and understanding of a question if he/she gives correct answers to:
a) one basic question from the set $\mathcal{B}$, say a and the three related to a questions from the set $\mathcal{R}$, i.e. abc, abd, and acd, or
b) three basic question from the set $\mathcal{B}$, say a, b, c and the related to a, b and c question from the set $\mathcal{R}$, i.e. abc.

In a case of incorrect answer the student is presented with theory and examples filling knowledge gaps, clarifying misunderstanding or misconception. Such recommendations are given by intelligent agents that base their decisions on using association rules. In case of a consecutive failure the procedure is repeated with another set of questions, developed by a different lecturer. If the student answers correctly to all questions the process of questioning is terminated.

Furthermore, a question is excluded from a consecutive trial if a student gives five correct answers as shown in Table 2 and Table 3 with a graphical representation shown in Fig. 2. The rest is similar to the case with six questions.

4 Conclusion

A theoretical framework for efficient assessment of learners' understanding of carefully chosen terms and concepts is presented. The model is based on the

Table 2: Correct answer combinations leading to a reduced test in a consecutive trial in the case of eight questions

Correctly answered questions in a current test								Questions to be excluded
a	b	c	d	abc	abd	acd	bcd	
•	•			•	•	•		a
•		•		•	•	•		a
•			•	•	•	•		a
•				•	•	•	•	a
•	•			•	•		•	b
	•	•		•	•		•	b
	•		•	•	•		•	b
	•			•	•	•	•	b
•		•		•		•	•	c
	•	•		•		•	•	c
		•	•	•		•	•	c
		•		•	•	•	•	c
•			•		•	•	•	d
	•		•		•	•	•	d
		•	•		•	•	•	d
			•	•	•	•	•	d
•	•	•	•	•				abc
•	•	•		•	•			abc
•	•	•		•		•		abc
•	•	•		•			•	abc
•	•	•	•		•			abd
•	•		•	•	•			abd
•	•		•		•	•		abd
•	•		•		•		•	abd
•	•	•	•			•		acd
•		•	•	•		•		acd
•		•	•		•	•		acd
•		•	•			•	•	acd
•	•	•	•				•	bcd
	•	•	•	•			•	bcd
	•	•	•		•		•	bcd
	•	•	•			•	•	bcd
•	•			•	•	•	•	a, b
•		•		•	•	•	•	a, c
•			•	•	•	•	•	a, d
•	•	•		•	•	•		a, abc
•	•		•	•	•	•		a, abd

Table 3: Correct answer combinations leading to a reduced test in a consecutive trial in the case of eight questions

Correctly answered questions in a current test								Questions to be excluded
a	b	c	d	abc	abd	acd	bcd	
•		•	•	•	•	•		a, acd
	•	•		•	•	•	•	b, c
	•		•	•	•	•	•	b, d
	•	•	•	•	•		•	b, bcd
•	•		•	•	•		•	b, abd
•	•	•		•	•		•	b, abc
•	•		•	•	•		•	b, abd
		•	•	•	•	•	•	c, d
•		•	•		•	•	•	c, acd
	•	•	•		•	•	•	c, bcd
•	•	•			•	•	•	c, abc
•	•		•		•	•	•	d, abd
•		•	•		•	•	•	d, acd
	•	•	•		•	•	•	d, bcd
•	•	•	•	•	•			abc, abd
•	•	•	•	•		•		abc, acd
•	•	•	•	•			•	abc, bcd
•	•	•	•		•	•		abd, acd
•	•	•	•		•		•	abd, bcd
•	•	•	•			•	•	acd, bcd
•	•	•		•	•	•	•	a, b, c, abc
•	•		•	•	•	•	•	a, b, d, abd
•		•	•	•	•	•	•	a, c, d, acd
	•	•	•	•	•	•	•	b, c, d, bcd
•	•	•	•	•	•	•		a, abc, abd, acd
•	•	•	•	•	•		•	b, abc, abd, acd
•	•	•	•	•		•	•	c, abc, abd, acd
•	•	•	•		•	•	•	d, abc, abd, acd

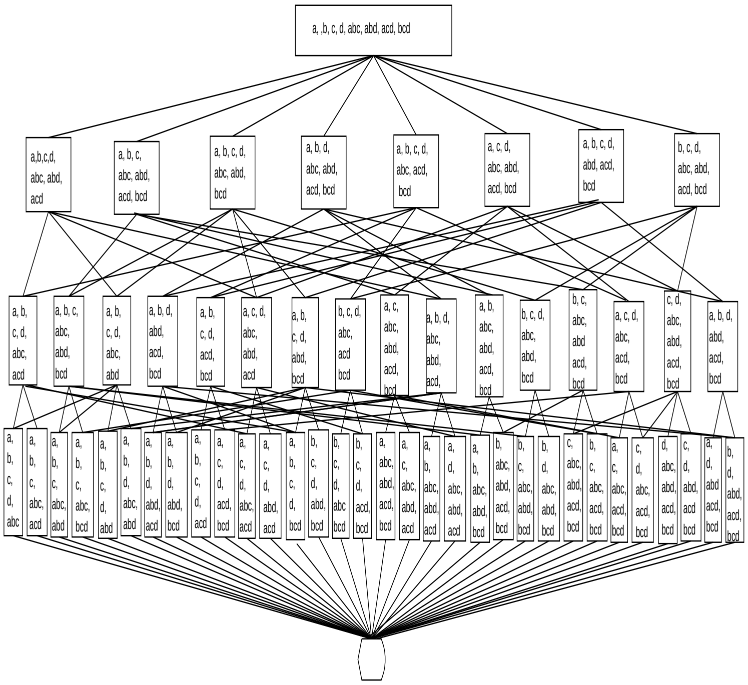

Figure 2: A lattice showing relations among eight questions

theory of knowledge spaces and meet-distributive lattices. In the future we plan to develop a prototype system based on the presented model. In our next phase of experiments we will evaluate the suitability of the current rules and the effectivity over time of the system. Association rules will be applied in the process of choosing appropriate learning materials.

References:

[1] Albert, D., Lukas, J. (eds.): Knowledge Spaces. Lawrence Erlbaum Associates, 1999

[2] Burton, R. F.: Guessing in selected-response tests. Medical Education **38**(1), 2004, pp. 112–112

[3] Bush, M. : A multiple choice test that rewards partial knowledge. Journal of Further and Higher Education, **25**2, 2001, pp. 157–163

[4] Carpineto, C. and Romano, G.: Concept Data Analysis: Theory and Applications, John Wiley and Sons, Ltd., 2004

[5] Davey, B. A. and Priestley, H. A.: Introduction to lattices and order. Cambridge University Press, Cambridge, 2005

[6] Doignon, J.-P. and Falmagne, J.-C.: Knowledge Spaces. Springer-Verlag, 1999

[7] Dowling C.E.: On the irredundant generation of knowledge spaces. Journal of Mathematical Psychology, **37**, 1993, pp. 49-62

[8] Downing, S.M.: Guessing on selected-response examinations. Medical Education **37**(8), 2003, pp. 670–671

[9] Edelman P.H. , R.E. Jamison, R.E.: The theory of convex geometries, Geom. Dedicata, **19**, 1985, pp. 247–270

[10] Falmagne, J.-C., Koppen, M., Villand, M., Doignon, J.-P., Johannesen, L.: Introduction to knowledge spaces: How to build test and search them. Psychological Review, **97**, 1990, pp. 201-224

[11] Farthing, D.W.: Permutational multiple choice questions: An objective and efficient alternative to essay-type examination questions. Proc. of 3rd Annual Conference on Integrating Technology into Computer Science Education (ITiCSE'98), 1998

[12] Guzmàn, E., Conejo, R.: A model for student knowledge diagnosis through adaptive testing. Lecture Notes in Computer Science, Springer-Verlag, Berlin Heidelberg New Jork **3220**, 2004, pp. 12–21

[13] Harper, R.: Correcting computer-based assessments for guessing. Journal of Computer Assisted Learning, **19**(1), 2003, pp. 2–8

[14] http://www.aleks.com

[15] http://wundt.uni-graz.at/projects/rath