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Modelling Urban Intersections in Dioid Algebra

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Abstract: - In this paper, we introduce a new model of urban traffic networks by using Petri Nets. Based on dioid algebra, the behavior of the system is described by $(Max, +)$-linear equations. This will allow to approach the traffic control problems which rise issues of cycle times and of synchronization. As a result, we show how we can solve shared resource problems inherent in dioid modelling. Besides, the proposed model provides us with interesting performance evaluations, such as real-time counts of vehicles and bounds of sojourn time in each part of the studied crossroad.

Key-Words: - Urban traffic, analytic model, dioid algebra, Timed Event Graphs, shared resource

1 Introduction

Despite several existing models of traffic networks [1, 2, 3], there is still plenty of scope for the traffic modelling improvement. Indeed, the traffic signal control with variable cycles is often treated by means of averaged models, which are not suitable for short sections generally encountered in factory sites or in downtowns. Moreover, on the one hand, macroscopic models remain rough to consider the real-time signal problems. This is due to the fact that such models do not take into account the individual arrival of vehicles. On the other hand, microscopic models become complicated when we try to study the behavior of a set of vehicles. It is basically these facts which motivate us to introduce a new traffic model.

Since through the problem of the traffic control we have to tackle cycle time and synchronization issues, a model based on *Timed Event Graph (TEG)* allows us to carry out our modelling objective. Indeed, it is widely known that the TEG is well adapted to deal with periodic and synchronization phenomena. Besides, the TEG can be described as a system of linear equations by using the dioid algebra. This has motivated several researches dealing with modelization, performance evaluation [4, 5] and the computation of control laws for $(Max, +)$-linear systems [6].

The aim of this paper is to define a $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ model of urban crossroad compatible with an existing model of a lonely street in this dioid [7]. Thus, present paper tends to expand the scope of dioids as a powerful tool to study urban traffic networks. More precisely, it deals with live TEG endowed with invariant resource sharing systems, which means that the number of shared resources remains constant. In order to compute the instantaneous number of tokens, a new operator is introduced. As a result, a new dioid model of a elementary crossroad is proposed. This model does not require any specific controller.

The next section presents an overview of the dioid algebra and the TEG. Section 3 describes the traffic crossroad and its model. The latter is analysed in section 4.

2 Notions about dioid algebra and TEG

Since the traffic modelling presented in this paper is based on dioid algebra, we give here a brief introduction of the necessary background.

2.1 Dioid algebra

The dioid $\overline{\mathbb{Z}}_{\max}$, commonly known as $(Max, +)$ *algebra*, is the set $\overline{\mathbb{Z}} = \mathbb{Z} \cup \{-\infty, +\infty\}$ endowed with both following operations [5, 8]:

$$\begin{aligned} a \oplus b &= Max(a, b) \quad , \\ a \otimes b &= a + b \quad , \end{aligned}$$

for "scalars" $a, b \in \overline{\mathbb{Z}}$, and

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} = Max(a_{ij}, b_{ij}) \quad ,$$

$$[A \otimes C]_{ij} = \bigoplus_{k=1}^{n} a_{ik} \otimes c_{kj} = \underset{k=1,\dots,n}{Max} (a_{ik} + c_{kj}),$$

for matrices $A, B \in \overline{\mathbb{Z}}^{m \times n}$ and $C \in \overline{\mathbb{Z}}^{n \times p}$.

The dual dioid $\overline{\mathbb{Z}}_{\min}$, also called $(Min, +)$ *algebra*, is obtained by replacing Max with Min in all previous equations.

Each dioid $\mathcal{D}$ has by definition the following intrinsic properties, for all $a, b \in \mathcal{D}$:

- a neutral element, denoted ε: $a \oplus \varepsilon = a$ and $a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon$;
- an identity element, denoted $\mathbf{e}$: $a \otimes \mathbf{e} = \mathbf{e} \otimes a = a$;
- an order relation, given by: $a \preceq b \Leftrightarrow a \oplus b = b$.

2.2 State equations of a TEG

A TEG is a *Petri Net (PN)* in which each place has exactly one upstream and one downstream transition. We are able to write the $(Max, +)$- or $(Min, +)$-linear *state equations* that describes the firings of the transitions of a TEG since it respects the *Just In Time (JIT)* operational rule. For each transition x, we can either evaluate the *dater function* $x(k)$, which gives the date of its k^{th} firing, or the *counter function* $x(t)$, which gives the number of times it has been fired until the date t.

We illustrate this by means of the following example.

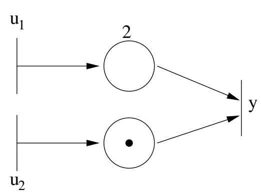

Figure 1: A simple TEG.

Consider the TEG model of figure 1. We want to describe the behavior of transition y, then:

- the associated dater is given by

$$\begin{aligned} y(k) &= Max(u_1(k) + 2, u_2(k-1)) \quad , \\ &= 2 \otimes u_1(k) \oplus u_2(k-1) \text{ in } \overline{\mathbb{Z}}_{\max}; \end{aligned}$$

- the associated counter is given by

$$\begin{aligned} y(t) &= Min(u_1(t-2), u_2(t) + 1) \quad , \\ &= u_1(t-2) \oplus 1 \otimes u_2(t) \text{ in } \overline{\mathbb{Z}}_{\min}. \end{aligned}$$

Like the z-transform for series in classical algebra, the γ- and δ-*transforms* allow to translate daters and counters functions, respectively, to formal series:

- γ is the *backward shift operator in the event domain, e.g.* $x(k-1) \xrightarrow{\gamma} \gamma x(\gamma)$,
- δ is the *backward shift operator in the time domain, e.g.* $x(t-1) \xrightarrow{\delta} \delta x(\delta)$.

The behavior of transition y of figure 1 is described by the following equation in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$:

$$y(\gamma, \delta) = \delta^2 u_1(\gamma, \delta) \oplus \gamma^1 u_2(\gamma, \delta) \quad .$$

The behavior of a whole TEG can be described by state equations that are linear in a dioid of formal series in two commutative variables γ, δ with exponents in $\mathbb{Z}$ and with boolean coefficients [4]. This dioid, denoted by $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ allows to study both time and event domains at a time [5]. Usually, we group the series, which describe the firing of the

- n source transitions into the *input vector* U,
- p well transitions into the *output vector* Y,
- m other transitions into the *state vector* X.

State equations look like the following:

$$\begin{cases} X(\gamma, \delta) = A \otimes X(\gamma, \delta) \oplus B \otimes U(\gamma, \delta) \ , \\ Y(\gamma, \delta) = C \otimes X(\gamma, \delta) \ . \end{cases}$$

2.3 Notations and additional results

The *causal projection* Pr_+ allows to compute a causal series by "removing" the monomials with non-positive exponants. A less naive definition is given in [6].

The operator '$*$' defined by

$$a^* = \bigoplus_{k \in \mathbb{Z}} a^k$$

is called *Kleene star*.

It is well known that the $\otimes$-multiplication of a dioid $\mathcal{D}$ is rarely invertible. However, let us draw the reader's attention to the fact that the *residuation theory* provides a "pseudo-division" [5]. We denote

$$R_a^\sharp(y) = y \varnothing a = \frac{y}{a} = \bigoplus \{x \in \mathcal{D} \mid x \otimes a \preceq y\}$$

the *residuated* of the mapping $R_a : x \mapsto x \otimes a$.

Since substraction of classical algebra is equivalent to the right division in $(Min, +)$ algebra [9], in any case where at least one counter $c(t)$ has an infinite value (*e.g.* $c(t) = \varepsilon$), right division is ambiguous. To overcome this inherent difficulty, which does not occur in the classical algebra, a new operator is introduced, which is defined as follows.

Definition 1 *Let $\triangleright$ be an operator defined as follows:*

$$c \xrightarrow{\triangleright} c^\triangleright = \begin{cases} \mathbf{e} & \text{if } c = \varepsilon, \\ c \otimes 1 & \text{elsewhere}, \end{cases}$$

for a scalar $c \in \overline{\mathbb{Z}}_{\min}$.

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We draw the reader attention to the fact that this operator preserves markings if it is applied to all transition counters of a live TEG [10].

3 Elementary crossroad modelling

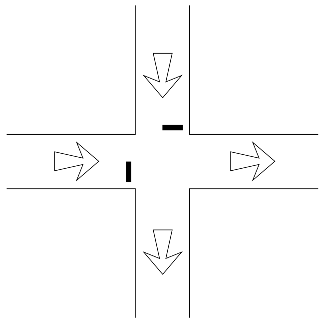

Figure 2: An elementary crossroad.

In order to model a whole crossroad by a TEG, each street is treated separately and discretized into segments where the characteristic variables of flow depend only on the time but not on the position into the segment [7]. Thus, both streets have a shared segment, which models the crossroad itself, and which is our interest. We take into consideration its dimensions (shared length can be different for each street) and the maximal flow rate. The density is naturally limited in the crossroad since we allow only one car to cross the shared surface at a time.

3.1 TEG modelling

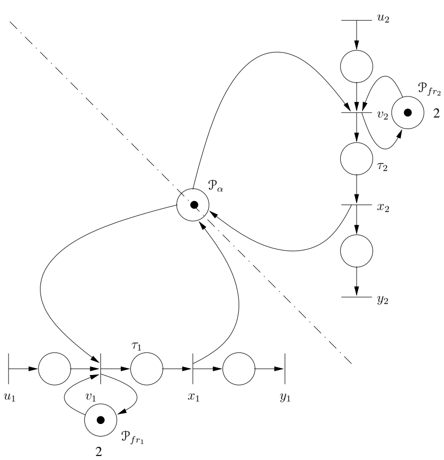

Figure 3: PN model of an elementary crossroad.

The model represents the averaged behavior of cars evolving through an elementary crossroad. Along paths delimited by transitions u_i and y_i, representing street i, tokens representing cars evolves accross the model part. Places $\mathcal{P}_{fr_i}$ model the flow rate (one car every two time units seems to be the usual value in the litterature [11]). Place $\mathcal{P}_\alpha$ represents the shared surface on which at most one car can travel. A token is present in this place at a date t *iff* crossroad is empty at such a date. In this representation, we can observe the counters for each transition. Table 1 details the semantics of the model.

Label	Meaning
u_i	input transition: $u_i(t)$ is the number of cars that have entered street i until t
y_i	output transition: $y_i(t)$ is the number of cars that have left street i until t
v_i	$v_i(t)$ is the number of times cars coming from street i have engaged in the crossroad until t
x_i	$x_i(t)$ is the number of times cars coming from street i have left the crossroad until t
τ_i	crossing time: it is computed from the length of the crossroad along street i and from the average speed of vehicles (fixed)

Table 1: Semantics of figure 3.

3.2 modelling in the dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$

Since the PN model without crossroad is a TEG, the PN model of figure 3 is split into two PN parts as it is shown in figure 4, *i.e.* streets PN part, which holds all sub-TEG representing streets appart from each other, and the crossroad one. The latter, denoted $\mathcal{G}_\alpha$, is the one of interest.

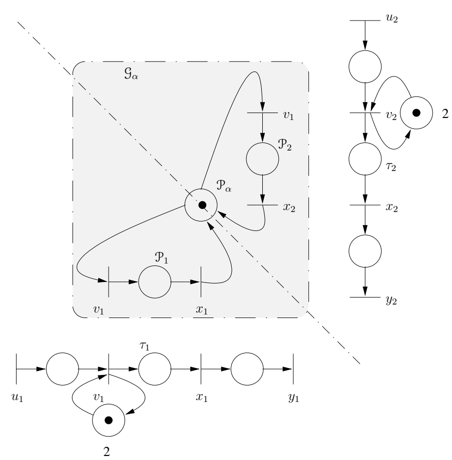

Figure 4: Split model of an elementary crossroad.

In order to study the behavior of cars driving through the crossroad, transitions v_i and x_i are duplicated. This aims at focusing the analysis on the crossroad, regardless of streets external influence. Thus, we consider $\mathcal{G}_\alpha$ as the model of an elementary crossroad. The structural analysis of $\mathcal{G}_\alpha$ provide us with the following lemma.

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Lemma 1 (Constraint) *Let two streets cross at an intersection. We note the counters of their transitions representing cars engaging and leaving the crossroad are denoted $v_i(t)$ and $x_i(t)$, respectively. These counters verify the following inequality:*

$$\sum_{i=1}^{2}\big(v_i(t) - x_i(t)\big) \le 1, \ \forall t \quad . \tag{1}$$

Proof: From the $\mathcal{P}$-invariant property of $\mathcal{G}_\alpha$, we have:

$$\sum_{i=1}^{2}\mathcal{M}_t(\mathcal{P}_i) + \mathcal{M}_t(\mathcal{P}_\alpha) = 1 \quad , \tag{2}$$

where $\mathcal{M}_t(\mathcal{P})$ is the instantaneous marking of place $\mathcal{P}$ at date t. It is well known [9] that $\sum_{i=1}^{2}\big(v_i(t) - x_i(t) + \mathcal{M}_0(\mathcal{P}_i)\big) + \mathcal{M}_t(\mathcal{P}_\alpha) = 1$. Hence, since $\mathcal{M}_t(\mathcal{P}_\alpha)$ is positive for all t and $1 - \sum_{i=1}^{2}\mathcal{M}_0(\mathcal{P}_i) = 1$, then the conclusion follows. ∎

It is important to note that this lemma is very powerful to constraint the dealt system. Indeed, it provides us with a relation which implies only counters of transitions involved in $\mathcal{G}_\alpha$. However, from theoretical and practical points of view, it is interesting to check that the constraint (1) is in agreement with the kind of the behavior of an elementary crossroad. To do this, it suffices to show that:

- (i) the constraint denies the entrance for a car if there is already a car driving through the crossroad,
- (ii) if the crossroad is free, the constraint must allow the entrance of a car,
- (iii) the constraint must not forbid a car to leave the crossroad.

Note that, assertion (iii) is obviously verified since the counter of output transitions $x_i(t)$ are multiplied by -1 in (1). That means each firing of such a transition relaxes the constraint. Hence, we only have to prove (i) and (ii), which is the main concern of the following lemma.

Lemma 2 *Let an elementary crossroad PN model. Then under the constraint (1):*

- (a) *no transition v_i can fire despite a car is driving through the crossroad,*
- (b) *a transition v_i is allowed to fire if the crossroad is free.*

Proof: The proof of (a) is by contradiction. Indeed, suppose that all transitions v_i can be fired despite a car is driving through the crossroad. In such a case, we have [9]:

$$\sum_{i=1}^{2}\big(v_i(t) - x_i(t) + \mathcal{M}_0(\mathcal{P}_i)\big) \ > \ r \quad .$$

But $1 = \sum_{i=1}^{2}\mathcal{M}_0(\mathcal{P}_i) + 1$, it follows that $\sum_{i=1}^{2}(v_i(t) - x_i(t)) + \sum_{i=1}^{2}\mathcal{M}_0(\mathcal{P}_i) > 1 + \sum_{i=1}^{2}\mathcal{M}_0(\mathcal{P}_i)$. Hence, we have $\sum_{i=1}^{2}(v_i(t) - x_i(t)) > 1$, which contradict (1). Thus, it is not possible to fire v_i if a car is driving through the crossroad. In order to prove (b), it suffices to prove that $\mathcal{M}_t(\mathcal{P}_\alpha) > 0$ implies the strict inequality of (1). Indeed, if the crossroad is free, it follows:

$$\mathcal{M}_t(\mathcal{P}_\alpha) \ > \ 0 \quad . \tag{3}$$

With this in mind, the number $\mathcal{M}_t(\mathcal{P}_\alpha)$ that let us know wether the crossroad is free or not at date t, equals to $1 - \sum_{i=1}^{2}\big(v_i(t) - x_i(t) + \mathcal{M}_0(\mathcal{P}_i)\big)$. This means that the crossroad is free *iff* no car is crossing from any street. Thus, from (2) we can check that $\mathcal{M}_t(\mathcal{P}_\alpha) = 1 + \sum_{i=1}^{2}\mathcal{M}_0(\mathcal{P}_i) - \sum_{i=1}^{2}\mathcal{M}_0(\mathcal{P}_i) - \sum_{i=1}^{2}\big(v_i(t) - x_i(t)\big)$. Which implies that $\mathcal{M}_t(\mathcal{P}_\alpha) = 1 - \sum_{i=1}^{2}\big(v_i(t) - x_i(t)\big)$. From (3), we obtain $\sum_{i=1}^{2}\big(v_i(t) - x_i(t)\big) < 1$. Hence the conclusion follows. ∎

In summary, lemma 1 leads to the following theorem.

Theorem 1 *The behavior of an elementary crossroad with two streets is described by the following state equations:*

$$\begin{cases} X = \begin{pmatrix} \delta^{\tau_1} & \varepsilon \\ \varepsilon & \delta^{\tau_2} \end{pmatrix} V \quad , \\ Y = \begin{pmatrix} \mathbf{e} & \varepsilon \\ \varepsilon & \mathbf{e} \end{pmatrix} X \quad , \end{cases}$$

under the constraints

$$\begin{cases} \bigotimes_{i=1}^{2} \dfrac{\mathcal{C}^{\triangleright}_{(\Delta_i V)}(t)}{\mathcal{C}^{\triangleright}_{(\Delta_i X)}(t)} \succeq 1, \ \forall t \ge 0 \quad , \\ \begin{pmatrix} \gamma\delta^2 & \varepsilon \\ \varepsilon & \gamma\delta^2 \end{pmatrix} V \oplus \begin{pmatrix} \mathbf{e} & \varepsilon \\ \varepsilon & \mathbf{e} \end{pmatrix} U \preceq V \quad , \end{cases}$$

where $\Delta_i^\top \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n$ is defined by

$$[\Delta_i]_j = \begin{cases} \mathbf{e} & \text{if } j = i, \\ \varepsilon & \text{elsewhere}. \end{cases}$$

$\mathcal{C}_{\Delta_i V}$ is the input (control) transition counter inventorying cars from street i that have been engaging in the crossroad and $\mathcal{C}_{\Delta_i X}$ is the one inventorying cars of street i that have been leaving the crossroad.

4 Performance measures

We can retrieve from the proposed TEG model of the elementary traffic system some of the main performance measures, which are:

1. Instant count of vehicles driving in each segment of the road,
2. Fundamental diagram.

4.1 State of traffic

Since in a TEG, each place has exactly one upstream and one downstream transition, we can easily compute the instantaneous markings from the counters describing their behavior.

Proposition 1 *The marking $\mathcal{M}_t(\mathcal{P})$ of a place $\mathcal{P}$ between two transitions x_a and x_b in a live TEG at instant $t \geq 0$ is given in $\overline{\mathbb{Z}}_{\min}$ by*

$$\mathcal{M}\{\mathcal{P}\}(t) = \mathcal{M}\{\mathcal{P}\}(0) \otimes \frac{x_a^{\triangleright}(t)}{x_b^{\triangleright}(t)} , \quad (4)$$

with $x_a(t) \prec \top$ and $x_b(t) \prec \top$.

Proof: There are two cases to consider:

1. Both $x_a(t)$ and $x_b(t)$ are finite. In this case, we know from [9] that the marking $\mathcal{M}\{\mathcal{P}\}(t)$ of a place $\mathcal{P}$ between two transitions x_a and x_b is given in $\overline{\mathbb{Z}}_{\min}$ by

$$\mathcal{M}\{\mathcal{P}\}(t) = \mathcal{M}\{\mathcal{P}\}(0) \otimes \frac{x_a(t)}{x_b(t)} .$$

Since the operator $\triangleright$ preserves marking [10], we have for finite values of $x_a(t)$ and $x_b(t)$:

$$\mathcal{M}\{\mathcal{P}\}(0) \otimes \frac{x_a^{\triangleright}(t)}{x_b^{\triangleright}(t)} = \mathcal{M}\{\mathcal{P}\}(0) \otimes \frac{x_a(t)}{x_b(t)} ;$$

2. $x_a(t)$ and/or $x_b(t)$ is infinite. In this case, we have to check that $\mathcal{M}\{\mathcal{P}\}(t)$ equals either
   - $\mathcal{M}\{\mathcal{P}\}(0)$ if $x_a(t) = x_b(t) = \varepsilon$,
   - $\mathcal{M}\{\mathcal{P}\}(0) + (x_a(t) + 1)$ if $x_a(t) \preceq \mathbf{e}$ and $x_b(t) = \varepsilon$,
   - $\mathcal{M}\{\mathcal{P}\}(0) - (x_b(t) + 1)$ if $x_a(t) = \varepsilon$ and $x_b(t) \preceq \mathbf{e}$.

   Indeed, if we compute $x_a^{\triangleright}(t)$ and $x_b^{\triangleright}(t)$ for the above values of $x_a(t)$ and $x_b(t)$, the conclusion follows from (4).

■

The instant marking of the places $u_i \rightarrow v_i$ gives exactly the current number of vehicles that are waiting to engage in crossroad on lane i, respectively.

Parameter	Value
τ_c	60
τp_i	30 sec
λ_i	0.20 vehicle/sec

Table 2: Parameter values for the simulation.

4.2 Fundamental diagram

In order to build the fundamental diagram, we simulate the model of the street several times with different constant input flow rates:

- The mean density $\overline{\rho}$ is given by the following equation:

$$\overline{\rho} = \frac{\sum_i \rho_i \times (t_{i+1} - t_i)}{\mathcal{T}}$$

  where ρ_i is the instantaneous density in the time interval $t_{i+1} - t_i$ and $\mathcal{T}$ is the simulation duration. ρ_i is obtained as follows:

$$\rho_i = \frac{m_i}{l} \text{ at } t = t_i$$

  where m_i is the current marking and l is the addition of the two shared surface lengths of studied lane.

- The mean flow rate $\overline{q}$ is given by the count of the output firings $\mathcal{C}_{y_i}(\mathcal{T})$ divided by the simulation duration:

$$\overline{q} = \frac{\mathcal{C}_{x_i}(\mathcal{T})}{\mathcal{T}} .$$

4.3 Simulation

The main objective of the following simulation is to confirm that the model takes into account the conflict situation and corresponds to the behaviour of an intersection. Hence, we assume that the vehicle arrivals are stochastic and the cycle time, τ_c, is constant, as well as, both phase, (p_1 and p_2) durations (τ_{p_1}, τ_{p_2}, respectively). p_1 and p_2 corresponds to both stages where vehicles of lane 1 and 2 are authorized to enter the intersection, respectively. The vehicle arrivals are exponentially distributed and λ_i is the arrival rates at the lane i.

Figure 5 shows the variation of cumulative number of vehicles to enter intersection by time. The number of vehicles of each lane is presented separately, solid line for the lane 1 and dashed line for the other one.

One can immediately observe vehicles of both lanes cannot enter intersection at the same time. Indeed, when the cumulative number of vehicles of lane 1 increases the one of the lane 2 is still constant and vice-versa. Such a respect of the resource sharing is provided by the introduced constraint.

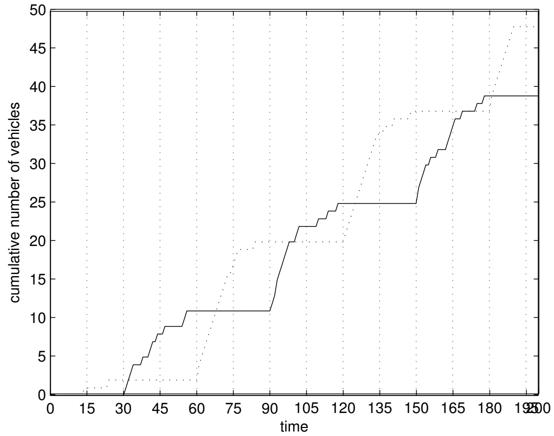

Figure 5: Variation of cumulative number of vehicles in intersection.

5 Conclusion

This paper proposes an approach to model elementary crossroads using PN under constraints and dioid theories. Thus, it enlarges the scope of this powerful algebra for the analysis of urban traffic. Furthermore, the proposed model is extensible and allows to consider several streets meeting at a single crossroad. Combined with a compatible model of an elementary street [7], we may be able to model several successive crossroads in a further paper. We think that there are several other issues that deserve further investigation. One is the extension of the model to include the traffic light control.

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