Controller Design and Fault Detection Method Based on Genetic Algorithms for MIMO System

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Abstract: This paper describes a new procedure to design robust $H_{\infty}$ controller with time domain specifications and a fault detection method for MIMO system. Controller design parameters and fault detection method are calculated solving multi-objective optimization problems which are based on genetic algorithms. Simulations tests are carried out to evaluate our design procedure, and satisfactory simulation results are obtained with nonlinear mathematical model of F-16 aircraft.

Key–Words: Robust control, fault detection, genetic algorithm

1 Introduction

Control systems specifications have several levels of requirements which go from system stabilization to robust performance with respect to uncertainties and disturbances. For that, robust control theory based on techniques such as $H_2/H_{\infty}$ and $\mu/H_{\infty}$ synthesis [8],[11],[13] are specially indicated due to some type of performance and robustness requirements expressed on frequency domain may be considered a priori by designer. At the present time, there is a solid base of analytical methods and algorithms to solve these problems [8],[11],[13]; nevertheless, the design parameters selection is difficult in order to achieve design specifications based on time domain response, such as the usual parameters employed by operators in order to evaluate control system performance: overshoot, rise time and settle time. Taking into account genetic algorithms (GA) properties as optimization tool, GA are specially indicated for obtaining controller parameters directly or in order to calculate design parameters [5],[6],[9]. In this paper, the second option has been considered combined with $H_{\infty}$ control theory for a MIMO (multiple-input and multiple-output) flight control system design.

Due to a flight control system is a critical system, on-line supervision and fault detection are essential in order to increase the reliability of the system. Significant advances to residual generation in model-based fault detection and isolation (MB-FDI) approaches has been achieved [1],[2],[4],[7]. To ensure reliable operation of a control system hard faults in system components are not tolerable and must be detected before they actually occur. Hence, in a reliable system operation incipient faults must be detected and isolate as early as possible. When MB-FDI based on residual signal approach is used, mixture effects of fault effects and modelling uncertainty must be taking into account. Robust residual generation is based on the maximization of fault effects and the minimization of uncertainty effects [1],[2],[4],[7].

In this paper we propose an optimal residual approach which is based on the combination of multi-objective optimization and genetic algorithm (MOO-GA). In order to make the residual more insensitive to modelling uncertainty and more sensitive to sensor and actuator faults, multiple performance indices (MPI) are defined in the frequency domain to take into account the fact that modelling uncertainty effects and faults occupy different frequency bands. The rest of the paper is organized in sections as follows: in section two the $H_{\infty}$ controller synthesis based on GA is described, in section three the proposed procedure based on MOO-GA for FDI is presented, in section four simulations results are shown; and finally, conclusions are resumed.

2 $H_{\infty}$ Controller Design

For controller design, the closed loop system shown in Fig. 1 is considered, which consists of plant ($G$), controller $G_c$, reference signal or set-point ($r$), measurement noise ($n$), disturbances acting at the plant input ($d_i$) and at the plant output ($d_o$), control signal ($u$), output signal or process variable ($y$), and error signal ($e = r - y$); where signals are multivariable and nomi-
nal mathematical models for $G$ and $G_r$ are considered LTI (Linear Time Invariant). In this scheme, it is considered a vector $z$, which is used to include signals required to characterize the behavior of the closed-loop system, and a vector $w$ which contains external inputs (set-points, disturbances and noise). So that, the behavior of the closed-loop system is given by $z = T_z w$, where input-output transfer function $T_z$ depends on weighting transfer functions employed in the $H_\infty$ controller design problem. In Fig. 1, the respective weighting transfer functions,

$$\{W_r, W_{di}, W_{do}, W_n, W_e, W_S, W_R, W_T\}$$

for signals $\{r, d_i, d_o, n, e, u, y\}$ are included in the generalized plant $P$, for which: $w = [r, d_i, d_o, n]^T$, and $z = [z_1, z_2, z_3]^T$, with $z_1 = W_S e$, $z_2 = W_R u + W_{di} d_i$, $z_3 = W_T y$. The objective is to obtain a suboptimal $H_\infty$ controller such as, $\|T_z w\|_\infty < \gamma (\gamma > 0)$, where $z = T_z w$. If $W_r, W_{di}, W_{do}$ and $W_n$ are unitary matrices, the following $T_z$ transfer function results:

$$T_z w = [W_S S \ W_R G_r S \ W_T T]^T$$

where $S = (I + G G_r)^{-1}$ the sensitivity function, $T = (I + G G_r)^{-1} G G_r$ is the complementary sensitivity function, and $R = G_r S$ is the control sensitivity function.

In conventional approach of $H_\infty$ control, controller is designed for robustness and performance specifications expressed in the frequency domain; but usual indicators based on time domain response such as overshoot, rise time and settle time, are not considered a priori. In practice, it is difficult to obtain the specified time responses using this approach, for what in this paper we present a method to satisfy time response specifications as well as robustness properties. For that, $H_\infty$ control theory and genetic algorithms ($H_\infty$-GA) are employed. We have implemented the following procedure for controller design:

1) Time domain specifications are established for rise time ($t_r$), overshoot ($M_p$), settle time ($t_s$) and stationary error ($e_{ss}$) for a step change in set-point.
2) Suboptimal $H_\infty$ problem, $\|T_z w\|_\infty < \gamma (\gamma > 0)$ is considered.
3) Weighting transfer functions are given by,

$$W_m(s) = \text{diag} \left\{ \frac{b_j}{a_j s + c_j} \right\}, \quad m = S, R, T; \quad j = 1, 2$$

With this election, and if $\gamma = 1$, performance and robustness lower bounds are guaranteed. If it is necessary, higher order transfer functions may be used.

4) Minimization through GA uses the following objective function

$$f(k) = w_1 |M_p(k) - M_{po}| + w_2 |t_r(k) - t_{ro}| + w_3 |e_{ss}|$$

where $M_{po}$, $t_{ro}$, $e_{ss}$ are respectively the specified (objective) overshoot, rising time and steady-state error; and $w_i (i = 1, 2, 3)$ are weighting factors. Here, GA are used for obtaining weighting transfer functions parameters $\{a_j, b_j, c_j\}$, such that closed loop system satisfies design specifications. Once these parameters are fixed, conventional algorithms based on algebraic Ricatti equations (ARE) or on linear matrix inequalities (LMI) can be used for solving $\|T_z w\| < \gamma$.

### 3 Residual Generator

It is supposed that the state space model of the system with faults is given by

$$x(t) = Ax(t) + Bu(t) + F_1 f(t) + M_d d(t)$$
$$y(t) = Cx(t) + Du(t) + F_2 f(t)$$

where $f(t)$ represents the fault vector, and the matrices $F_1$ and $F_2$ are fault distribution matrices which represent the influence of faults on the system, and $M_d$ the disturbance distribution matrix. In Fig. 2, the residual vector is given by

$$\epsilon(t) = Q_e C e_c(t) + Q_e F_2 f(t)$$

where $e_c(t) = x(t) - \hat{x}(t)$ is the state estimation error.

The eigenstructure assignment method is considered as a robust (in the disturbance de-coupling sense) residual generator, in which some left eigenvectors of the observer are assigned to be orthogonal to the disturbance distribution directions. In this paper, as it is shown in Fig. 2, the residual generator is based on a full-order observer approach. The basic idea is to estimate the system output from the measurements using an observer. The weighted output estimation error is then used as a residual. In order to achieve robust FDI a multi-objective optimization (MOO) problem will be solved by means of multiple (four) performance indices, in which it has been taken into account that the frequency ranges of the faults, disturbances and noise are normally different. For example, in case of an incipient fault signal, the fault information is contained within a low frequency band as the fault development is slow; however, the noise comprises mainly high frequencies signals. In this work, four frequency weighting functions, $W_j(s)$, $j = 1, 2, 3, 4$, are used:

- Performance index $J_1$: To reduce false and missed alarm rates, the effect of faults on the residual should be maximized. This is equivalent to the minimization of the following performance index:

$$J_1 = \|W_1(s)Q_1 H(s)\|_\infty$$

where $H(s) = M_2 + C(s I - A + K_\infty C)^{-1}(M_1 - K_\infty M_2)$.
Performance index $J_2$: To reduce the effects of both disturbance and initial condition by minimizing.

$$J_2 = \|W_2(s)Q_eC(sI - A + K_{oe}C)^{-1}M_d\|_\infty$$

Performance index $J_3$: To reduce the noise effect at high frequencies and to maximize the effects of faults at low frequencies, $W_3(s)$ must be selected opposite to $W_1(s)$

$$J_3 = \|W_3(s)Q_e[I - C(sI - A + K_{oe}C)^{-1}K_{oe}]\|_\infty$$

Performance index $J_4$. For stationary phase, the residual steady state value is very important in FDI. For that, the disturbance effects on residual can be considered by minimizing the following performance index:

$$J_4 = \|(A - K_{oe}C)^{-1}\|_\infty$$

In this paper, the eigenstructure assignment method is chosen to get a satisfactory gain matrix $K_{oe}$, which, at least, must guarantee the stability of the observer [1],[2],[4],[7]. A solution which minimizes multiple performance indices (MPI) can not exist in practice, and therefore some compromises and trade-offs must be considered for solving the design problem. The trade-offs are based on relative importance of objectives. The MOO has been solved using numerical search algorithms and the method of inequalities. This method replaces the minimization of the MPI by inequality constraints on the MPI. The optimization problem is transformed into a set of inequalities, with upper bounds acting as constraints $J_i \leq \rho_i$, $i = 1, 2, 3, 4$. The problem is to find a parameter set to make all performances are satisfied for acceptable constraints bounds ($\rho_i$). By adjusting the bounds, different emphasis on each of the objectives can be placed. For that, the moving-boundaries algorithm [2],[8] is employed, using a version based on GA which has been implemented in this work (MOO-GA).

**Design Procedure for FDI**

In order to design a robust FDI system we have implemented a robust residual generator design procedure, which uses genetic algorithms (GA) to solve multiple objective optimization problems. The following steps are considered in our design methodology:

Step 1: Four weighting transfer functions, \{W_i(s)\}, are selected to separate the effects of noise and disturbances of faults, where the plant dynamics will be taken into account.

Step 2: In order to search the interval of values for that satisfies all the inequalities, MOO-GA algorithm is executed and performance indices are minimized individually.

Step 3: The bounds are selected from the intervals computed in step 2; with this, relative weighting performance indices are selected. Adjusting the bounds one can place a different emphasis on each of the objectives.

Step 4: Sensor and Actuator fault residual generator is designed via MOO-GA. The observer gain matrix $K_{oe}$ is computed by means of design parameters to achieve robust FDI, a multi-objective optimization problem will be solved.

Step 5: The detection thresholds are selected taking into account incipient faults detection and false alarms rejection. Adaptive threshold and/or several levels of safety can be implemented, and it will be considered in other work.

Step 6: Simulation results of the plant are analyzed in order to evaluate the FDI system.

Step 7: If simulations results are not satisfactory, Go to step 3; else End procedure.

**4 Simulation results**

In this section, a $H_{\infty}$ controller with MB-FDI based on MOO-GA is applied and tested by simulations to F-16 aircraft. The tracking control system objective is to provide coordinated turns by causing the bank (roll) angle $\phi(t)$ to follow a desired command while maintaining the sideslip angle $\beta(t)$ at zero. The sensor fault detection filter is sensitive to roll and sideslip sensors, and the actuator fault detection filter is sensitive to ailerons and rudder actuators.

The non-linear F-16 model is linearized at nominal flight condition (true air speed $V_T=502$ ft/s, 0 ft altitude, 300 psf dynamic pressure). The non-linear mathematical model of the F-16 aircraft used in simulations are given in [10],[3],[12]. The basic lateral states are sideslip $\beta$, bank angle $\phi$, roll rate $p = \dot{\phi}$, and yaw rate $r = \psi$, where $\psi$ is the yawing angle. Additional states $\delta_a$ and $\delta_r$ are introduced by the aileron and rudder actuators, both modelled as first order system. Additional modifications carried out in the plant model used for controller design are the following: 1) input/output variables scaling, 2) pre-compensator for balancing the singular values at low frequencies, 3) integrator in each control channel, so that the closed-loop steady-state error will be zero for step changes in set-point and constant disturbances. The resulting augmented plant, including aircraft states and integrators, has the following state vector: $x = [\beta \ \phi \ p \ r \ \delta_a \ \delta_r \ e_{\phi} \ e_{\beta}]^T$, where $e_{\phi} = r_{\phi} - \phi$, and $e_{\beta} = r_{\beta} - \beta$. The following time response specifications have been considered: $M_p = 3\%$, $t_{rs} = 0.2$ seconds and $\epsilon_{tas} = 0$. With $H_{\infty}$-GA procedure the following experimental values are obtained: $M_p = 3.7\%$, $t_r = 0.21$ seconds, which
are near enough to target specifications. Robustness analysis is carried out by means of: 1) calculating robustness indicators based on singular values and structured singular values which result when multiplicative uncertainty (multiplicative stability margin, $\text{MSM}$) at the plant input ($\text{MSM}_i = 79.8\%$), at the plant output ($\text{MSM}_o = 87.1\%$) or both ($\text{MMS}_s = 35\%$), are considered. These indicators are useful during controller design, so that a low bound is considered for satisfactory robustness. 2) Making tests simulations for different flight conditions. In Fig. 3, bank and roll angles are shown for unit step change in set-point. The results for a fixed controller and three different flight conditions (nominal and $\pm 20\%$ true air speed variations) are satisfactory too.

**FDI system.** Once the controller design is carried out, the fault detection filter is considered. The fault detection filter is sensitive to ailerons and rudder actuators, and to roll and sideslip sensors. 

**Sensor residual generator.** An observer is designed to generate sensor residual signal for FDI. For that, the previous proposed method for FDI with seven steps is applied: (step 1):

$$W_1 = \frac{200}{(s + 5)(s + 40)}, \quad W_2 = W_4 = 1, \quad W_3 = W_1^{-1}$$

which place emphasis on the residuals at low frequencies and on noise at high frequencies. To apply the method of inequalities, we begin searching for the values intervals that satisfy the inequalities $J_i < \rho_i$ ($i = 1, 2, 3, 4$). MOO-GA algorithm is executed and performance indices are minimized individually. Table 1 lists the MPI for different observer gains. In this table, $K^*_o$ represents the observer gain matrix $K^*_o$ which minimizes $J_i$. It can be seen that a design which minimizes a particular performance index makes all other performance indexes unacceptably large (step 2).

In order to use the method of inequalities to solve this problem, a set of MPI bounds, $\{\rho_i\}$, is chosen as shown in the table 1 (bounds row) (step 3). Sensors fault residual generator is designed via MOO-GA (step 4). The detection threshold (step 5) are fixed experimentally, and the selected values are shown in Fig. 4 (for sensors fault detection) and in Fig. 5 for actuators fault detection. The inertial navigation system (INS) detects aircraft motion and provides acceleration, velocity, present position, pitch, roll and true heading to related systems. Typical inertial navigation unit contains a gyro stabilized platform which contains three accelerometers and two gyros which are isolated from external angular motion by a set of four gimbals. Each accelerometer is mounted so that the unit is sensitive to motion on a specific axis. The accelerometers provide acceler-
Figure 3: Closed-loop system response for set-point change and three flight conditions

ations for system computations. The gimbals position provide 360 freedom of rotation. The gyros provide the stabilization of the platform to maintain accurate outputs. The INS must be alignment before take-off, a bad alignment provides excessive tolerance for errors and must be considered a fault in roll and sideslip angle sensors since sideslip is computed by

\[ \beta = \frac{\dot{\nu} V_T - \nu \dot{V}_T}{V_T^2 \cos \beta}, \]

where \( \nu \) is the lateral velocity and \( V_T \) is the true airspeed.

In simulations (step 6), the set-point for roll angle is a unit step and for sideslip angle is zero. To take into account noise in sensors and actuators, control channels are perturbed by means of band-limited white noise. In order to detect incipient faults in sensors and actuators, incipient faults are modelled by means of 0.01 deg/s ramp signal. The simulated fault is added to the roll angle sensor. To illustrate the small nature of the incipient fault, Fig. 4 shows the plot of both, sensor (faulty) and observer measurements of the roll angle. The fault takes place at the 2 sec., the residual signal activates the alarm when the signal reaches the threshold (t = 5 sec). If a fault is added to the sideslip angle sensor, the simulation result of the FDI filter can be seen in Fig. 4, where the residual shows a noisier signal due to the fact that \( \beta \) is computed by means of a discrete time version. Incipient fault alarm take places at 5.3 sec, when the error is smaller than 0.01 degrees.

Actuator residual generator. Similar procedure is applied to the observer design to generate actuator residual signal for the FDI system. Typical lateral and rudder control system consists of the control stick, pedals, high speed stop unit, spring feel unit, trim actuator, cables, control rods, hydraulic actuators and control surfaces. One or more of these elements can be degraded due to fatigue and must be considered a fault in actuators command position.

A fault takes place when an actuator (ailerons or rudder) has a loss in effectiveness. Fig. 5 shows the plots of both ailerons and rudder actuator faults. Respective faults take place at 2 sec., the residual signals activate alarms when the signals reach the threshold at 3.0 and 3.4 sec, respectively.

In order to test the FDI system robustness and behavior for flight conditions different to nominal case,
closed loop simulations are made with a fixed observer. This can be seen in Fig. 5, where the fault takes place at 2 sec and satisfactory behavior is obtained. In order to achieve incipient fault detection and false alarm rejection, the threshold must be increasing for non-nominal operation of FDI system. For greater variations in flight conditions a new observer must be designed for good performance.

Table 1: MPI for sensor fault design

<table>
<thead>
<tr>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( J_3 )</th>
<th>( J_4 )</th>
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<td>29570.1</td>
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<tr>
<td>( K_2^* )</td>
<td>13.5</td>
<td>1276.6</td>
<td>2034.6</td>
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<td>( K_3^* )</td>
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<td>10310.1</td>
<td>2003.6</td>
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<tr>
<td>( K_4^* )</td>
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<td>950.0</td>
<td>2007.1</td>
</tr>
<tr>
<td>( \rho_i )</td>
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<td>3000.0</td>
<td>3000.0</td>
</tr>
</tbody>
</table>

5 Concluding remarks

In this paper a systematic procedure for \( H_\infty \) controller and MB-FDI system design has been presented, where controller and FDI system are designed using genetic algorithms (GA). GA are employed for obtaining the weighting transfer functions in the \( H_\infty \) controller design, so that time response specifications are satisfied; and for multi-objective optimization which is used in the incipient fault detection procedure. There is a combination of a robust control technique (\( H_\infty \)) with a robust FDI technique, using a robust numerical optimization method (GA) for parameter tuning. Incipient fault detection, suitable time responses as well as satisfactory robustness properties are obtained in simulations with F-16 aircraft.

References: