New Integrated Control Design Method based on Receding Horizon Control with Adaptive DA Converter

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Abstract: In this paper, a new integrated control design method based on RHC (receding horizon control) and Fluency analysis is proposed for systems with relatively fast-moving dynamics. In the Fluency analysis which has been proposed by a co-author of this paper, signal spaces are well-classified by its smoothness, and Fluency sampling functions with good properties have been proposed in each space. However, it has been difficult to apply the Fluency analysis approach for a DA (digital-to-analog) converter in sampled-data control systems, since the interpolation using sampling function needs the information of future samples. We, therefore, propose a new interpolation method using predicted information calculated on RHC controller in each step. Moreover, the way to switch the Fluency sampling functions according to the system status is proposed. This approach is realized as an adaptive DA converter. A numerical example is also included to show the proposed method enables us to improve the control performance of systems including a controller and the adaptive DA converter.

Key-Words: RHC (Receding horizon control), Sampling function, DA (digital-to-analog) converter

1 Introduction

The recent dramatic advance of computer has made it possible to control the continuous-time objects by a discrete-time controller designed on computer. At the same time, digital control has become increasingly important. In such systems, that is a kind of sampled-data control systems, analog-to-digital(AD) and digital-to-analog(DA) conversion of signals are indispensable operations. Generally, the zero-order hold is used as a DA conversion, while the way to sample the analog signal at a constant frequency is used as an AD conversion [1]. However, to improve the performance of sampled-data control systems, it’s very important to take account of the behavior of systems between sampling intervals. On this issue, some notable methods to design the discrete-time controller for continuous-time objects with AD/DA conversion have been proposed [2][3][4].

We also have studied this type of sampled-data control systems from the viewpoint of DA conversion based on Fluency analysis [5]. In the Fluency analysis which has been proposed by a co-author of this paper, signal spaces are well-classified by its smoothness, and sampling functions with sampling bases in each space are given in complying with requested specifications [6].

However, it has been difficult to apply these sampling functions to DA conversion directly in sampled-data control system, since the interpolation based on sampling function needs the information of future sampling points. Generally, in the case of DA conversion in sampled-data control, it is impossible to obtain the information of future sampling points, and DA converter has to interpolate discrete control input generated from discrete-time controller based on past information. Therefore, we have been forced to tolerate the long time-delay during the DA conversion using sampling function in our previous works. However, in the case of control of the systems with relatively fast-moving dynamics, such as robots or vehicles, the method with long time-delay is unable to be applied. Hence, we need to develop a new control method without long time-delay.

In this paper, therefore, we propose a new integrated control design method based on RHC (receding horizon control) with the Fluency analysis for such systems with relatively fast-moving dynamics. RHC is one of digital control methods which have gotten a lot of attention recently because of its high practicality [7]. In RHC algorithm, a prediction of the future system status is executed, and future control inputs based on the prediction are also calculated in each step. Hence, we propose a new idea to use these future control inputs based on the prediction for interpolation by sampling function. This idea enables us to apply sampling functions for DA conver-
ion in sampled-data control system without so long time-delay. Moreover, we propose a new method to switch the Fluency sampling function optimally according to the system status, since Fluency analysis has provided sampling functions with good properties. This approach is realized as the adaptive DA converter. A numerical example is also included to show the proposed method gives the improvement of control performance of the entire system.

2 Preliminaries
Let’s consider a discrete-time linear time-invariant system as follows,

\[ x(k+1) = Ax(k) + Bu(k) \]
\[ y(k) = Cx(k) \]

where \( u(k) \in R^n \), \( x(k) \in R^n \) and \( y(k) \in R^1 \) mean control input, state values and observed output at step \( k \) respectively, and \( A \in R^{n \times n}, B \in R^{n \times 1} \) and \( C \in R^{1 \times n} \) are coefficient matrices.

The outlines of the Fluency analysis and RHC are given briefly in next sections.

2.1 Fluency analysis
In Fluency analysis, the signal spaces are defined as the function spaces composed of piecewise polynomials of degree \((m-1)\) with \((m-2)\) times differentiability [6]. Moreover, the existence of the sampling basis in each space has been proven. For example, when \( m = 1 \), the sampling basis is equivalent to the sampling basis in the signal space composed by staircase function, and when \( m \) goes to infinity, the sampling basis is equivalent to the Shannon’s sampling basis which characterizes the band limited signal space. By selecting appropriate signal spaces characterized by parameter \( m \) according to the object, the Fluency analysis enables us to deal with signals flexibly and precisely. In the field of audio file and image processing, our new approaches based on the Fluency analysis have already been highly-acclaimed.

In this paper, we assume the signal spaces with \( m = 1, 2, 3 \), since the effectiveness of these three spaces is known in a series of our studies. Furthermore, in the case that \( m \) is 4 or more, it’s difficult to apply to fast-moving dynamic systems due to the bigger calculation amount of interpolation. Fig.1 shows the Fluency sampling functions \((m = 1, 2, 3)\) we use in this paper and their interpolations. Especially, the sampling function with \( m = 3 \) in fig.1, which is called as the compactly supported Fluency sampling function with 2nd degree, has already been put to practical use in a DA converter of DVD-Audio [8]. The interpolated signal in the closed-open interval \([(i−1)\tau, i\tau)\) using the compactly supported Fluency sampling function with 2nd degree is calculated as follows,

\[ u(t) = \sum_{l=i-2}^{i+1} \left\{ u(l) \cdot \frac{1}{c} \psi(t - l\tau) \right\} \]

where \( u(t) \) and \( u(l) \) are analog signal and digital signal respectively, and \( \tau \) is sampling interval.

2.2 RHC (receding horizon control)
RHC is an online powerful control method which solves a finite horizon open-loop optimal problem with respect to each sampling frequency [7]. Although RHC has been used in systems with relatively slow-moving dynamics such as petrochemical plants due to its big calculation amount, the recent advance of computer performance has made it possible to apply for the systems with relatively fast-moving dynamics.

Now, let’s consider the finite-time constrained optimal control problem with the state space model as follows,

\[ \min_{\{u(k)| \cdots , u(k+N-1)|\}} J(k) = \left\| x(k+N|k) \right\|^2_P + \sum_{i=0}^{N-1} \left\{ \left\| x(k+i|k) \right\|^2_Q + \left\| u(k+i|k) \right\|^2_R \right\} \]
subject to:

\[ u(k) \in U, \quad x(k) \in X \quad (5) \]

where \( P, Q \) and \( R \) are positive definite matrices, and \( N \) is the length of prediction horizon. Eq.(5) means constraint conditions for the control input and the state values. In practice, since this problem is equivalent to the quadratic programming problem, the optimal solution \( \{ \hat{u}(k|k), \cdots, \hat{u}(k+N-1|k) \} \) is easily solved. Finally, only the first solution \( \hat{u}(k|k) \) is used as a control input for control object at step \( k \), and then, the current step goes on to next step.

Several kinds of RHC method have been also proposed until now [9][10].

3 Integrated control design method

In this section, the detail of the proposed integrated control design method based on RHC and Fluency analysis is given.

3.1 Interpolation based on sampling function using predicted control inputs

As we mentioned above, the interpolation based on sampling function needs the information about future sampling points. However, it’s generally impossible to know the future without any foresight information. We therefore propose a new idea to use the predicted control inputs obtained by RHC for interpolation.

In RHC, the optimal control inputs \( \{ \hat{u}(k|k), \hat{u}(k+1|k), \cdots, \hat{u}(k+N-1|k) \} \) are calculated in each step, and only the first control input \( \hat{u}(k|k) \) is used as a real control input. Therefore, we consider to use the other optimal control inputs \( \{ \hat{u}(k+1|k), \hat{u}(k+2|k), \cdots \} \) as virtual future sampling points. Actually, it is only necessary to use the optimal control inputs which are needed for interpolation according to the sampling function. Fig.2 shows this way using the compactly supported Fluency sampling function with 2nd degree. Only \( \hat{u}(k+1|k) \) is used as a virtual future sampling point in this case. By using the predicted future control inputs for interpolation, it becomes possible to reduce the time-delay in DA conversion, and the total time-delay to be needed is just only computation time of optimization in current step.

Of course, it needs to take account that there is a difference between virtual future sampling points and real sampling points like \( \hat{u}(k+1|k) \neq u(k+1) \) in future step. However, we consider that this point is not a critical problem because the influence on interpolated waveform due to prediction error is not so big compared to the scale of prediction error. Although the differentiability which each sampling function has is lost at sampling points, this also does not become a critical problem compared to the zero-order hold, and it is possible to keep a certain level of smoothness.

3.2 Switching Fluency sampling functions

Fluency analysis has provided various sampling functions with good properties. Therefore, we consider switching these Fluency sampling functions optimally according to the system status in the adaptive DA converter. Fig.3 shows the proposed integrated control design. As this figure shows, the adaptive DA converter has internal model with sampling interval \( \tau/d \). Please take notice that this internal model 2 is different from internal model 1 with sampling interval \( \tau \) of controller. The interval to be interpolated is also divided to \( d \) sections, and the dividing points \( u_m(j; k), (j = 1, 2, \cdots, d-1) \) on interpolated waveforms are used for the selection of parameter \( m \), that indicates the degree of Fluency sampling functions. Fig.4 shows the difference of the interpolation and dividing points according to the sampling function when \( m \) is limited from 1 to 3 and \( d = 5 \). The dividing points \( u_m(j; k) \) are calculated as follows,

\[
\begin{align*}
  u_m(j; k) &= \sum_{l=k-\alpha}^{k+\alpha-1} \left\{ u(l) \cdot m \psi \left( \frac{(k-1)\tau + \frac{\tau}{d} \cdot j - l\tau}{\tau} \right) \right\} \\
  (j = 1, 2, \cdots, d-1) \quad (6)
\end{align*}
\]

where \( \alpha \) is the number of samples which the sampling function needs for interpolation, and it is adjusted according to the sampling function.
Then, we summarize the algorithm to switch the Fluency sampling functions for the adaptive DA converter as follows,

(step1) Step = \( k \).
(step2) The dividing points \( u_m(j; k) \) are calculated as eq.(6).
(step3) The predicted state values \( x_m(j+1; k) \) in this interval are calculated using internal model of DA converter and the dividing points \( u_m(j; k) \).
(step4) If the interpolation wave exceeds the constrained conditions of control input due to the overshoot or undershoot, this \( m \) is excluded.
(step5) The evaluation values using evaluation function \( J_m(k) \) are calculated in each \( m \).
(step6) The parameter \( m \) whose evaluation value is the smallest is selected as an interpolation way in this interval, and then \( k = k + 1 \) and go back to (step1).

In this paper, the evaluation function in (step 5) is used as follows,

\[
J_m(k) = \sum_{j=1}^{d-1} \left\{ ||x_m(j+1; k)||_Q^2 + ||u_m(j; k)||_R^2 \right\}
\]

(7)

where \( Q_1 \) and \( R_1 \) are positive definite matrices.

From several test simulation results, it seems to be appropriate that the divided number of interval, \( d \), set 5 due to the trade-off of computation time and precision. If \( d = 5 \), the calculation amount in the adaptive DA converter is also vanishingly small compared to the calculation in RHC controller keeping a certain level of accuracy.

It is considered that the proposed method enables us to give the improvement of control performance of the entire system. Longer the sampling interval, this improvement of control performance is more conspicuous.

4 Numerical example

In this section, a numerical example is given to demonstrate the effectiveness of the proposed method. As the numerical example, let’s consider a control problem of the inverted pendulum. Moreover, the following three methods are compared through a simulation.

1. LQ optimal control with zero-order hold.
2. RHC with zero-order hold.
3. Proposed method.

LQ optimal control is one of efficient control methods for systems with relatively fast-moving dynamics [1].

4.1 Inverted pendulum

As fig.5 shows, inverted pendulum control problem is to control the wheel truck position without taking
controller and the adaptive DA converter are derived by a general discretization of the continuous-time model with the sampling interval $\tau = 0.05s$ and $\tau/d = 0.01s$ respectively. In the simulation, we use a discrete-time model with the sampling interval 0.0005s as a control object and the DA conversion means 100 times up-sampling. Control inputs are also constrained as $-0.3 \leq u(t) \leq 0.3$, and 0.02s time-delay is added to the proposed adaptive DA converter as waiting time for optimized calculation. Moreover, since it’s usually difficult to obtain the state value $x_1$ due to air resistance, measurement error and so on, $x_1$ is assumed to be perturbed by a disturbance. This disturbance acts as a force to take down the bar.

Figs.6 through 8 show the appearances of the simulation. Fig.6 shows added disturbance and the control input signals and observed output signals are shown in fig.7 and fig.8 respectively. From this simulation results, we can easily see that both input and output by the proposed method converged to the equilibrium position 0 faster than the other ones. The LQ controller cannot give its ability into full play due to constraint conditions. From comparison between the RHC with zero-order hold and the proposed method, there is no doubt about the effectiveness of the proposed adaptive DA converter. In the case of the systems with relatively fast-moving dynamics like the inverted pendulum, the tiny difference of DA conversion causes a big influence for result. If the sampling interval of controller is set longer, this tendency becomes clearer. Therefore, we think the proposed method is efficient for any other general systems with relatively fast-moving dynamics.

5 Conclusion

In this paper, a new integrated control design method based on RHC and Fluency analysis for the systems with relatively fast-moving dynamics has been proposed. The proposed method gives us improvements of control performance of systems including a controller and the adaptive DA converter.
As future works, we need to develop the selection method of the best sampling function according to control object. Moreover, we need to make an experiment using a real control object and make sure the effectiveness of the proposed method.

References: