

# Blackouts in electric power transmission systems

KARAMITSOS IOANNIS and ORFANIDIS KONSTANTINOS  
Athens 16342  
GREECE

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**Abstract:** - In this paper an analysis of blackouts in electric power transmission systems is implemented and studied in simple networks with a regular structure. The proposed model describes load demand and network improvements evolving on a slow timescale as well as the fast dynamics of cascading overloads and outages. The model dynamics are demonstrated on the simple power system networks.

**Key-Words:** - Blackout, Generator, Load, Power load

## 1 Introduction

Electrical power transmission systems are complex engineering systems with many static or dynamic components. Their complete dynamical description involves detailed knowledge of each component and its coupling to the rest of the system. The power system can be modeled using two possible approaches. The most commonly used approach is a deterministic calculation that models all the components in detail. Because all of the components and the physical laws that govern their interactions are known, it is possible to develop software that simulates particular blackouts. These codes may be complicated and time-consuming, but they are feasible. This approach has proven to be effective in helping to manage the power system. However, a different perspective can be taken.

Blackouts in power systems happen quite frequently. These blackouts have a multiplicity of causes such as equipment failure, weather conditions, vandalism, and human error [1]. The dominant causes triggering blackouts cannot be written in the equations of a software code. Therefore, if we want to understand the global dynamics of power system blackouts, we need to emphasize the random character of the events that trigger them and the overall response of the system to such events. This is the approach taken in this paper. The two approaches are necessary and complement each other. They may converge in the future when the second approach is further developed. In following the second approach, it is sensible to start from a global, top-down methodology with simple models that capture the main effects only. A recent analysis of blackouts in the North American power grid [2, 3] has shown that measures of such blackouts such as megawatt hours unserved or number of customers affected show the existence of long-range dependencies. Furthermore, the probability distribution function (PDF) of the size of the blackouts has a power law scaling. This behavior

of the power transmission system is suggestive of a dynamical system close to a critical point. One possible governing principle for its dynamics is self-organized criticality [6]. We have considered a sequence of models that may reflect the dynamical properties of a self organized critical system. The simplest model was employed in reference [3]. In [3] we used a sand-pile model [7] as a black box to generate a 2 self-organized critical time series that could be compared to the time series of historical data for power grid blackouts. The sandpile was not a model for the dynamics of the power grid, but merely a means of testing the self-organized critical properties of the data. The next step was taken by constructing a power transmission model [4] based on a cellular automaton similar to the sandpile model. This model allowed studying properties of network power transmission, but it did not solve the network power flow equations. The interesting result is that these two models produce PDFs of blackout sizes that are quite similar and are also similar to the PDF determined from the historical data for North American power grid blackouts. Here we describe the implementation and results of a model [5] that takes it a step further by solving the network power flow equations. This model still remains simple, and in this paper we consider the power networks of homogeneous structure. In this way, we can vary a minimum number of parameters to explore the dynamics. However, extensions of the model are possible and easy to implement. These extensions will allow us to consider more realistic power system networks, incorporate the reliability of each component, and to vary the methods of responding to increasing power demand and improving the system. Many researches and studies should be paid to issues of voltage and reactive power control and load behavior.

This paper is organized as follows: Following the introduction, an analysis of the proposed model is

described in section 2. Then in section 3, the formulation of the power flow problem is introduced. The analytical result is discussed in section 4. Finally, a brief conclusion is deduced.

## 2 Analysis of proposed Model

In this section we calculated the mathematical model [5] for the dynamics of power transmission networks. For each network, we define two types of classes, the node class and the line class.

The node class represents the buses. They are either loads (L) or generators (G). Each node class contains the information on the type of bus, the instantaneous real power  $P_i$  (positive for generators and negative for loads), the maximum generator power  $P_{i\max}$ , and the connections with the other existing buses.

The line class contains the information on the nodes (i) and (j) that the line connects, the instantaneous power flow  $F_{ij}$ , the maximum power flow  $F_{ij\max}$ , and the line impedance  $z_{ij}$ . In our model we assume that only one line allows connecting two given nodes. This assumption of the model allows the consideration of any interconnected network with  $N=G+L$ , where N is the number of network nodes, G is the number of generators, and L is the number of loads. The present implementation does not allow the network to be disconnected and islanding cannot be studied. As discussed in [7], the direct current (dc) power flow equations can be written in the following form:

$$F=Ax P \quad (1)$$

Where F is a vector whose L components are the line power flows  $F_{ij}$  between the nodes, P is a vector whose N-1 components are the power injected at each node  $P_i$ , and A is a matrix that depends on the network structure and impedances. The reference generator power  $P_0$  is not included in the vector P in order to avoid the singularity of A as a consequence of the overall power balance.

The dynamic evolution of the network involves two timescales. There is a slow timescale of days to years over which power demand changes and improvements to the system are made. There is also a fast timescale of minutes to hours over which a cascade of overloads and outages may take place.

This cascade may lead to a blackout or back to normal operation. For simplicity, the daily peak load is chosen as representative of the loading during each day, and the events are computed based on that peak load. The timing of events in the cascade is neglected so that the cascade modeling moves through a possible sequence of states of the network rather than simulating the evolution of the cascade in time.

## 3 Formulation of the power flow problem

The dynamic of the long-term evolution of the network is carried out by a simple set of rules. At the beginning of day t, we apply the following rules:

Rule 1: Increase the power electricity demand.

All loads are multiplied by a fixed parameter  $\bar{\kappa}$ , which is the average daily rate of increase in electricity demand. On the basis of the past rate of growth of electricity consumption, we estimated the parameter to be  $\bar{\kappa} = 1.00005$ . This value corresponds to a yearly growth rate of approximately 2%.

$$P_i(t) = \bar{\kappa} P_i(t-1) \text{ for } i \in L \quad (2)$$

The maximum generator power is increased at the same rate:

$$P_i^{\max}(t) = \bar{\kappa} P_i^{\max}(t-1) \text{ for } i \in G \quad (3)$$

Rule 2: Power transmission improvement

We assume a steady improvement in the transmission capacity of the grid network in response to the outages and blackouts. This improvement is implemented through an increase of the maximum line flow  $F_{ij\max}$  for the lines that have overloaded during a blackout on the previous day. That is,

$$F_{ij}^{\max}(t) = \lambda F_{ij}^{\max}(t-1) \quad (4)$$

if line ij overloads during a blackout. We consider ( $\lambda$ ) to be a constant and it is the main control parameter of the model.

**Rule 3:** Daily power fluctuations.

To provide the daily fluctuations in power demand, all load powers are multiplied by a random number  $\rho$ , such that  $\frac{1}{\gamma} \leq \rho \leq \gamma$ .

The range of the parameter  $\gamma$  is from 1 to 1.5. We also assign a probability  $p_0$  for a random outage of a line. We represent the line outage by multiplying the line impedance by a large number  $\theta_1$  and dividing the line maximum flow  $F_{ij} \max$  by another large number  $\theta_2$ . The values of  $\theta_1$  and  $\theta_2$  are of the order 10000 in our calculations. After applying these three rules to the network parameters, we solve the power flow problem using linear programming.

Using the input power demand and power transmission rules updated as indicated above, we solve the power flow equations (1). In addition we assume that all generators run at the same cost and that all loads have the same priority to be served. However, we set up a high cost for load shed by setting  $W = 1000$ .

The formulation of the linear problem (LP) is the following with the minimization of the cost function:

$$\min Cost = \sum_{i \in G} P_i(t) - W \sum_{j \in L} P_j(t) \quad (5)$$

Subject to constraints:

$$0 \leq P_i \leq P_i^{\max} \quad \text{for } i \in G \quad (6)$$

$$P_j \leq 0 \quad \text{for } j \in L \quad (7)$$

$$|F_{ij}| \leq F_{ij}^{\max} \quad (8)$$

$$\sum_{i \in G \cup L} P_i = 0 \quad (9)$$

This linear programming problem is numerically solved using the simplex method as implemented in [8].

In order to solve the time evolution problem, the initial conditions are chosen to be a feasible solution of the linear program (i.e., a solution satisfying the constraints). As the time evolution proceeds, we can reach a solution of the linear program that

requires load shed or leads to overload of one or more lines. At this point, a cascade may be triggered, and the evolution moves to the fast timescale. Cascading overloads may start if one or more lines are overloaded in the solution of the linear program.

We consider a line to be overloaded if the power flow through the line is within 1% of  $F_{ij} \max$ . Each overloaded line is outaged with probability  $p_1$ . Once one or more lines are outaged, the solution is recalculated. This process can lead to multiple iterations and the process goes on until a solution is found with no more outages. A blackout is defined as a cascading event in which the load shed is larger than a small value, typically 10-5 times the total power demand.

## 4 Analytical Results

We have considered several network structures, such as ring, and tree networks. For the ring and tree networks, we have considered different numbers of couplings between the nodes. We have examined the sensitivity of the results to these different network structures. Moreover, we assume the network with tree topology have three connections per node.

A reason to consider these networks is that their simple structure makes it easy to generate networks of different sizes. These networks have allowed us to carry out detailed scaling studies by varying the size and number of connections. Varying the size of the network allows the separation of scales needed to study finite size systems. The scaling studies are important in determining algebraic falloff of the PDFs of cascading events. For the numerical results presented in this paper, the network parameters are given in the following table:

Table I: Different network topologies

Network Topology	No of Nodes	No of Links
Ring 45	45	68
Tree 99	99	149
Tree200	200	300

For these networks, we have arbitrarily assigned a generator at every tenth bus and loads at every other bus. For a fixed rate of average increase of the power demand ( $\bar{\kappa} = 1.00005$ ), the effective power served depends on the rate of improvements  $\lambda$  in the grid network. If the improvement rate is lower, there

are more blackouts, and on average the power served is lower. This result is presented in the following figure.

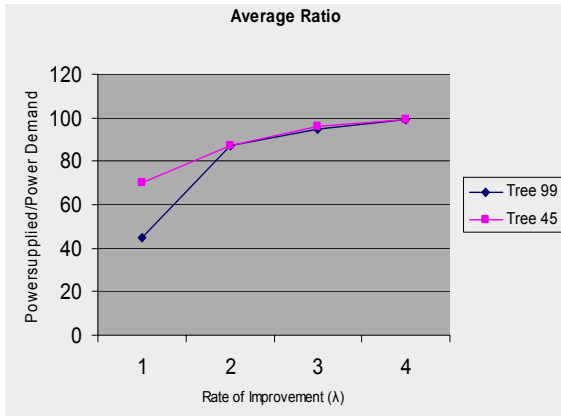


Figure 1: Average ratio of power supplied to the power demand as a function of the rate of improvement for two different network topologies

Once the rate of improvement  $\lambda$  is given, there is a self-regulation process by which the system produces the number of blackouts that it needs to stimulate the response needed to meet demand. This is a necessary condition for the dynamical equilibrium of the system. The rate of increase in power demand for the overall system is essentially given by

$$R_D \approx (\bar{\kappa} - 1)L \quad (10)$$

The system response is

$$R_R \approx (\lambda - 1)f_{blackout} \langle l_o \rangle L \quad (11)$$

where  $f_{blackout}$  is the frequency of blackouts and  $\langle l_o \rangle$  is a weighted average of lines overloaded in a blackout.

## 5 Conclusion

In this paper we have presented a model and some initial results of a dynamical model for blackouts in power transmission systems. The proposed model has the potential of incorporating inhomogeneities of the system and making the model more realistic. Due to the simplicity of the present model realization, the model shows very rich dynamics over both long and short timescales. We have focused on the main properties of the cascading

events. The cascading events involve lines limiting, line outages, and possible load shed. When load shedding happens, we define the cascade as a blackout. Blackout frequency and size depend on the rate of improvement of the network. The frequency and size of the blackouts depend weakly on the topology of the network, at least for the three topologies considered here. Finally the distribution of the blackout sizes is a weak function of the topology.

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