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The Identification of Eigenfrequencies for Cylinder-Shaped Piezo Actuators

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Abstract: The analytical and numerical modelling of ultrasonic motors is very complicate because piezoelectric effect and high frequency of the actuator and small displacement of deformable bodies must be evaluated in calculations. Usually numerical modelling and simulation of multicomponent piezoelectric actuators lead to the large number of recurred calculations with different geometrical parameters of the actuator and to the problems concerning the changes of modal shape sequences during recurred calculations. The exchanges in the modal shape sequence are a general case problem concerning not only ultrasonic actuators, but also with all mechanical structures. Analysis of the modal shapes sequences exchanges of two-dimensional and cylinder piezoelectric actuators were done. Dependencies and characteristic ratios of geometrical parameters of the analysed piezoelectric actuators were defined when modal shape sequences changes.
Key-Words: ultrasonic actuator, modal shape, Multicomponent oscillations, dominating coefficients

1 Introduction

The development of new groups of the devices using the piezoactive materials for the transformation of high frequency oscillations into a continuous multidirectional motion gave an opportunity to extend the area of the creation of time constant positioning drivers, micromanipulators, micro pumps, transducers of material physical properties measurement and others. Some kinds of these devices i. e. ultrasonic motors now are used instead of electro motors and hydro drives in some industrial and medical fields. The analytical and numerical modelling of ultrasonic motors is very complicate because piezoelectric effect and high frequency of the actuator and small displacement of deformable bodies must be evaluated in calculations. The design of ultrasonic motors can be divided into following steps: dynamic studies and application of finite element method (FEM) of piezoelectric actuators, optimisation of the actuator and development of control scheme[6].

2 Construction of Ultrasonic Actuators

The shape and optimal location of the electrodes on the surface of the actuator have the great importance to vibration mode of the actuator. Using different configurations of electrodes, vibration of the main and higher resonance modes of the ultrasonic actuator could be achieved [7]. In case of optimal electrode configuration, needless harmonics of the actuator could be eliminated and the concentration of mechanical stresses could be reduced. These facts are very important in case of Multicomponent oscillations of the actuator that is analysed in this paper.

Various constructions of the actuators are used in order to achieve a particular law of movement of the actuator and the final link of kinematics pair [8, 9]. There are some basic shapes of the ultrasonic actuator such as beam, plate, cylinder, disc, ring and etc., but more complex constructions are also used in the ultrasonic motors: cross shaped various multilayer actuators with or without attached masses.

Characteristics and types of the excited Multicomponent oscillations of the ultrasonic actuator depend on the geometrical parameters, boundary conditions and direction of the polarization vector. The topology of electrodes and geometrical parameters of the actuator define the direction of the excited oscillations. In order to achieve suitable characteristics of the oscillations, particular geometrical parameters of the actuator must be calculated. It is especially important to select a suitable correlation between these parameters [5, 7].

The beam and plate shape actuators are used mostly in the ultrasonic motors.

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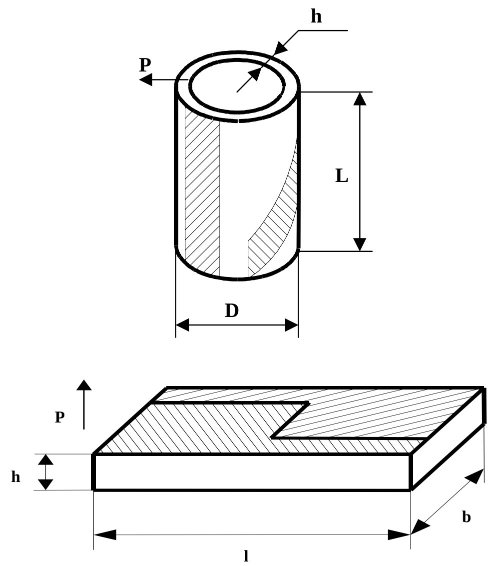

Fig.1. Constructions of ultrasonic actuators [4].

Many different types of Multicomponent oscillations can be excited using these actuators: longitudinal-flexural, longitudinal - torsion and etc. Using a variable vector of polarization, three and four components oscillations of the beam shaped actuator can be achieved [1].

The maximum of oscillation amplitudes (up to 10 μm) can be exciting in bimorfical plates. Usually these Multicomponent oscillations have longitudinal and flexural components. Some constructions of piezo actuators with different electrodes configurations are presented in Fig.1.

3 Problem Definition

Since the analysis of multidimensional piezoelectric actuator cannot be performed without considering the vibration device, most often the problems of piezoelectric actuator research are solved in an integral fashion taking into account the whole device.

Using the technical oscillation theory of the beam the longitudinal oscillations are found by solving the second order differential equation [2, 4]:

$$m(x)\frac{\partial^2 \xi}{\partial^2 t} - \frac{\partial}{\partial x}\left(E s_i \frac{\partial \xi}{\partial x} \right) = 0 \tag{1}$$

Longitudinal oscillations of the beam can be expressed as follows [2, 4]:

$$\omega_k = \frac{k}{2l}\sqrt{\frac{E}{\rho}} \tag{2}$$

E - the Jung modulus; k - the mode number of the longitudinal oscillations; l - the length of the beam; ρ mass density.

Flexural oscillations of the beam are found by solving the second order differential equation [2, 4]:

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 \xi_{len}}{\partial x^2} \right) + \rho S \frac{\partial^2 \xi_{len}}{\partial^2 t} = 0 \tag{3}$$

Flexural oscillations of the beam are described by the expression [2, 4]:

$$\omega_n = \frac{\pi h\,(n+0.5)^2}{4l^2}\sqrt{\frac{E}{3\rho}} \tag{4}$$

h - the height of the beam; n – the mode number of the flexural oscillations;

If certain values of k and n are defined, then h/l ratio of the beam could be calculated. From the equations (2) and (4) following equation could be obtained:

$$\frac{h}{l} = \frac{2\sqrt{3}\,k}{\pi\,(n+0.5)^2} \tag{5}$$

As an example:

$$k = 1;\; n = 2 \;\; \Rightarrow \;\; \frac{h}{l} = 0.1765 \tag{6}$$

But h/l ratio could be changed, for example, increasing or reducing the height of the beam. In this case k value remains the same, but n value changes. This means that the sequence of modal shapes changes when the geometrical parameters of the beam vary. For example, when the length and height ratio is $0.5<l/h<3$ we have an ordinary modal shape sequence and in other case second and third modal shape of two dimensional actuator changes

Calculations of multidimensional piezoelectric actuators, of course, have the same problem, but to solve it analytically is very difficult or even impossible [8]. Modal shape sequence exchanges often causes fatal errors in the calculation process well as unexpected errors in results [5].

Analysis of the piezoelectric actuator must be carried out taking into account the electric occurrence in the system. Based on FEM, every node of the element has one additional DOF used for electric potentials in FEM modelling. The solution applied for the equations of motion, suitable for the actuator, can be derived from the principle of minimum potential energy by means of variation functional [3]. The basic dynamic FEM equation of motion for piezoelectric transducers that are fully covered with electrodes can be expressed as [8]:

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$$\begin{aligned}&[M]\{\ddot{\delta}\}+[C]\{\dot{\delta}\}+[K]\{\delta\}-[T]\{\varphi\}=\{R(\omega_k t)\}\\&[T]^T\{\delta\}+[S]\{\varphi\}=\{Q\}\end{aligned} \tag{8}$$

where [M], [K], [T], [S], [C] - the matrices of mass, stiffness, electro elasticity, capacity and damping, respectively; $\{\delta\}, \{\varphi\}, \{R\}$ - the vectors of nodes displacements, potentials and external mechanical forces, respectively.
Here:

$$[K]=\int_V [B]^T \left[c^E\right] [B]\, dV \tag{9}$$

$$[T]=\int_V [B]^T [e]\, [B_E]\, dV \tag{10}$$

$$[S]=\int_V [B_E]^T \left[\ni^s\right] [B_E]\, dV \tag{11}$$

$$[M]=\rho\int_V [N]^T [N]\, dV \tag{12}$$

$$[C]=\alpha[M]+\beta[K] \tag{13}$$

where [B], $[B_E]$ – the matrices of geometry used for evaluation of displacements and potential, respectively; [N] – the function of the shape used for evaluation of the mass matrix. The damping matrix [C] is derived using mass and stiffness matrices by assigning constants α and β.

Usually only the first equation from system (2) is used in the modelling process; because it is considered that the source of current is powerful enough and ensures defined values of the electric potentials.

4 An Algorithm of Modal Shape Identification

Dominating coefficients is the way to define the type of oscillations of the actuator and to sort modal shapes by the dominating type of the oscillations for example longitudinal, flexural or torsion.

When the modal frequencies analysis of Multicomponent actuators is done using FEM, dominating components of the oscillations can be found referring to the energetic method of the oscillation analysis, because amplitudes raised to the second power are proportional to the energy of the oscillations [8]. In that way the ratios (dominating coefficients) of the components of amplitudes in all directions can be found and the direction with the maximum of amplitudes can be defined.

Let's calculate the following sum [7]:

$$S_p^b=\sum_{i=1}^{r}\left(A_{ip}^b\right)^2, \quad r=\frac{q}{p} \tag{14}$$

The dominating coefficients of the model can be expressed as follows [7]:

$$m_{jk}^b=\frac{S_j^b}{S_k^b}, \quad j\neq k \tag{15}$$

The physical meaning of dominating coefficients is the ratios between different oscillation energy components in the directions of coordinate axes. The sum S_k^b defines the energy of the oscillation of the *b* natural frequency in *k* direction and the dominating coefficient m_{jk}^b defines the relation of the oscillation energies in *i* and *j* directions of *b* natural frequency. Dominating coefficients have the following characteristic [7]:

$$m_{kj}^b=\frac{1}{m_{jk}^b}, \quad j\neq k \tag{16}$$

Based on dominating coefficients we could determinate the type of dominating oscillations and also define the level of correlation of Multicomponent oscillations as follows:

$$m_{jk}^b=\tau \quad\Rightarrow\quad \lg\tau=1, 2, 3\ ... \tag{17}$$

Dominating coefficients is the method to define the type of oscillations of the actuator and to sort modal shapes by dominating type of the oscillations for example longitudinal, flexural or torsional. In order to finally identify the modal shape additional characteristic criteria must be applied, because the values of dominating coefficients vary when the geometrical parameters change. These criteria are the nodes points or lines number of the modal shape.

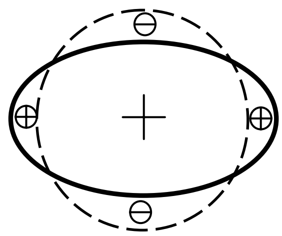

Fig. 2 Modal shape identification of cylinder shape piezo actuator.

During calculations the number of node points or lines could be found referring to the sign of the oscillations amplitude alternating around the equilibrium attitude (Fig. 2).

The exchanges in the modal shape sequence are a general case problem concerning not only piezoelectric actuators, but also with all mechanical structures [8].

5. Processing and Results

The aim of the analysis was to determine the dependencies of modal shape sequence from geometrical parameters of the actuator

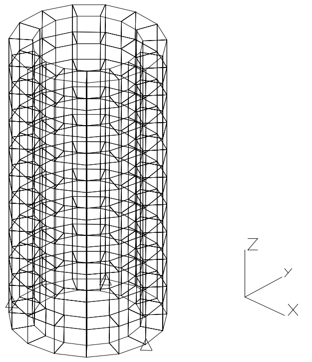

Fig.3 Finite elements model of cylinder piezo actuator.

Analysis of modal shapes sequence of the cylinder piezoelectric actuator must be done depending on cylinder wall thickness and internal radius ratio (Fig. 4 because two different sets of cylinder modal shapes are defined.

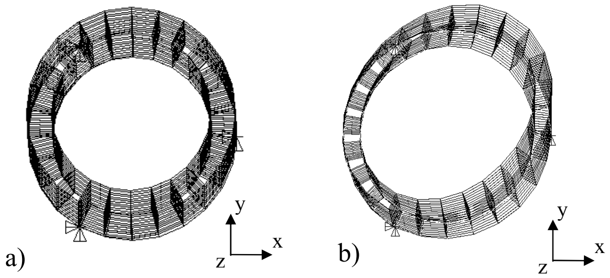

Fig. 4 First modal shape of cylinder piezoelectric actuator: a) $0.5 < h/R_{vid}$ (2,66 kHz), b) $h/R_{vid} < 0.25$ (2,87 kHz).

When the ratio of the wall thickness and radius is $0.25 < h/R_{vid} < 0.5$, transition from thin layered cylinder modal shapes to thick layered cylinder modal shapes happen. This process independs from cylinder boundary conditions. When cylinder geometrical parameters meet aforementioned ratio the sequence of modal shapes is unstable and it changes when any of parameters changes (Fig. 5). Dominating coefficient indexes 1, 3 correspond radial and axial directions respectively.

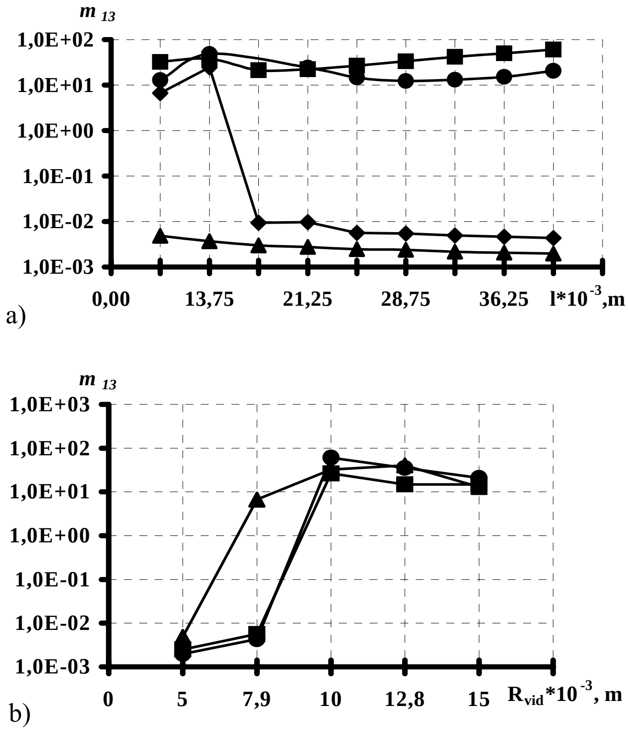

Fig. 5 Dependence of the dominating coefficient m_{13} of cylinder piezoelectric actuator for the third natural frequency on: a) length; b) internal radius. Wall thickness of the cylinder actuator is 0.0025m.

During numerical analysis it was determined that increasing of the length of the cylinder, increase the amplitudes of dominating oscillations, but hasn't influence to modal shape sequence changes. Internal radius of the cylinder actuator defines the ratio of the wall thickness and radius, so its influence to the modal shape sequence reveal when ratio reach already mentioned values.

6 Conclusions

Analysis of modal shapes sequence of cylinder – shaped piezoelectric actuators were done.

During numerical analysis it was determined that increasing of the length of the cylinder, increase the amplitudes of dominating oscillations, but hasn't influence to modal shape sequence changes. It was determined that exchanges of modal shapes sequences depend on length and width ratio for two-dimensional actuator and on the wall thickness and internal radius for cylinder actuator.

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