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HIGH RESOLUTION ANALOG-DIGITAL CONVERTER AND DIGITAL POLYNOMIAL FILTER APPLICATIONS

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Abstract: - The high resolution sigma-delta A/D converter with on-chip linear digital filtering intended for the measurement of wide dynamic range, low frequency signals such as those in industrial control or process control applications and digital smoothing polynomial filters are described. The A/D converter consist of a sigma-delta (or charge balancing) part, a calibration microcontroller, a clock oscillator, a digital filter and bidirectional serial communications port. The converter also contains two low level programmable-gain pseudo-differential analog input channels and one high level single-ended input channel.

The digital smoothing polynomial filtering, or Savitzky - Golay filtering defined a family of filters, which are suitable for smoothing and/or differentiating sampled data. The data are assumed to be taken at equal intervals. The smoothing strategy is derived from the least squares fitting of a lower polynomial to a number of consecutive points. The digital smoothing polynomial filter (DSPF) in cooperation with finite impulse response filter (FIR) enables better signal evaluation (signal separation and noise suppression). These signals are taken from different types of sensors. The examples of A/D converter and polynomial filter using for biomedical signal applications are also presented.

Key-Words: Sigma-delta converter, polynomial filters, biomedical signals, noise suppression.

1 Introduction

Most smoothing operations consist of the replacement of each data point by some kind of local average of neighboring data points (e.g. simple moving average smoothing function). As a side effect this type of smoothing will reduce the amplitude and broaden, narrow peaks (that is those peaks comprising only a few data points). This also means that the rising and falling edges of the peak tend to be flattened.

The digital smoothing polynomial filtering performs noise reduction while preserving higher order moments of the original spectrum [1]. The preservation of these moments corresponds to less distortion, particularly of spectral features such as absorption band heights and widths. What makes Savitzky-Golay filtering even more attractive is the straightforward and efficient manner in which the filtering is accomplished [2]. Only a linear convolution with a set of filter coefficients is required. The resulting spectrum contains less noise than the original, and exhibits less distortion than moving average filtering techniques of the same order. Other filtering techniques such as the moving average may be able to remove more noise, but as the filter order increases, more distortion is introduced. By utilizing the Savitzky-Golay technique, the philosophy assumed is to place more priority on the preservation of spectral characteristics as opposed to noise removal. Even more important to the application of derivative spectroscopy is that these filters are not restricted to smoothing.

There are filters available that also result in the computation of derivative spectra in addition to smoothing within the same convolution process. Although the filtering involves a least-squares fit of the noisy spectrum, the filters are generic for a specified filter length, interpolating polynomial order, and derivative order. That is, for given values of these parameters, the filter coefficients are the same without regard to the data in question. In fact, Savitzky and Golay published several tables of coefficients in their original publication that could be applied directly to the smoothing and differentiation of spectra. The SavitzkyGolay convolution process is exactly equivalent to a smoothing operation, least squares polynomial fit, and differentiation, but is performed in a computationally efficient one step procedure. A full mathematical development of

Savitzky-Golay filtering is given in [3], with some corrections given in [4] and [5].

Savitzky-Golay filtering (DSPF) has its roots in least squares polynomial smoothing. The objective of least squares smoothing is to find a smoothed value for each point in the spectrum based on a least squares polynomial fit of a subset of data within a window. The window contains the point to be smoothed in the center position of the window as well as several of its neighbors to either side in the spectrum. All the data within the window is used to perform the least squares fit, but only the central point is smoothed for each window position. The other points are smoothed by moving the window across the spectrum point by point, performing a least squares approximation to the windowed data at each location. A low order polynomial is typically used to perform the approximation, so the low frequency characteristics of the spectrum are approximated best by the polynomial, while the high frequency noise is lost in the approximation error. Since these low order polynomials are generally smooth, the resulting spectrum takes on the smooth characteristic. Each of these approximations defines a different polynomial that may then be used to evaluate the new, smoothed value of each point in the spectrum. In addition, to find an approximation of the derivative spectra at that same point, the power rule may be used to differentiate the polynomial prior to evaluation. Any order derivative approximation may be made, as long as the order of the polynomial used in the approximation is high enough.

If implemented as described here, a least squares approximation would have to be performed on each sample in the spectrum in succession, with each point placed at the center of the window. The values of the polynomial coefficients are linear with respect to the data within the window, so the actual fitting operation can be reduced to linear combinations of a "pre-fit" window consisting of all zeros and a single one. Savitzky and Golay's primary contribution was the development of a digital filter that automatically performs the least squares approximation when convolved with the spectrum. It has been shown that Savitzky-Golay filters of order 2M preserve all moments of the original spectrum up to the 2M+ 1 order moment while optimally attenuating noise for any integer M [6]. This is a very important property of Savitzky-Golay filters, as it guarantees that the maximal amount of noise will be removed while preserving some very important spectral characteristics such as the area underneath spectral features and the mean location of spectral features across the spectrum.

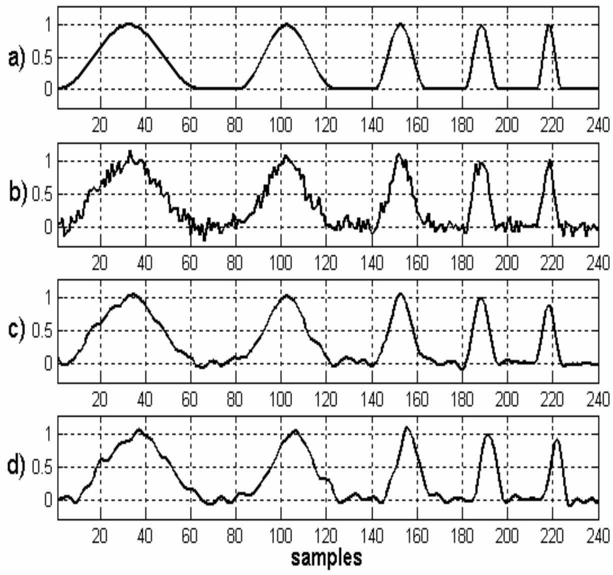

Figure 1. Filter example (From top to bottom):
a) Input signal, b) Noisy signal, c) Savitzki-Golay filter output; polynomial order = 4; number of window point = 15, d) Lowpass digital Butterworth filter output; 5-th order; relative cutoff frequency = 0.3

2 Smoothing strategy

The DSPF smoothing strategy is derived from the least squares fitting of a lower order polynomial to a number of consecutive points. For example, a cubic curve which is fit to 5 or more points in a least squares sense can be viewed as a smoothing function. The method consist of finding coefficients for the *j*th order smoothing polynomial in terms of the values of some number, $k > j+1$, of adjacent points and computing the value of the polynomial at the point to be smoothed. At first glance, it appears that the computation of the appropriate coefficients for the cubic needs to be repeated for each point. However, by solving the appropriate equations in terms of a general point set it is possible to write an expression which is a weighted sum of neighboring points with weights constant for a given polynomial order and number of points. We must solve the matrix equations:

$$\mathbf{Ax} = \mathbf{y} \tag{1}$$

where

$$A = \begin{bmatrix} i_a^0 & i_a^1 & .. & i_a^n \\ i_b^0 & .. & & .. \\ .. & & & .. \\ i_q^0 & .. & i_q^{n-1} & i_q^n \end{bmatrix} \tag{2}$$

and the i_k are the relative distances from the point we are smoothing to the y_k. An example matrix formulation with vector **x** representing the coefficient vector is:

$$\begin{bmatrix} -2^0 & -2^1 & -2^2 & -2^3 \\ -1^0 & -1^1 & -1^2 & -1^3 \\ 0^0 & 0^1 & 0^2 & 0^3 \\ 1^0 & 1^1 & 1^2 & 1^3 \\ 2^0 & 2^1 & 2^2 & 2^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} y_{-2} \\ y_{-1} \\ y_0 \\ y_1 \\ y_2 \end{bmatrix} \tag{3}$$

where the cubic, $a_0 + a_1 i^1 + a_2 i^2 + a_3 i^3$, is to be fit to 5 consecutive points, y_{-2} , y_{-1} , y_0 , y_1, y_2, so that the central point at y_0 occurs where $i = 0$.

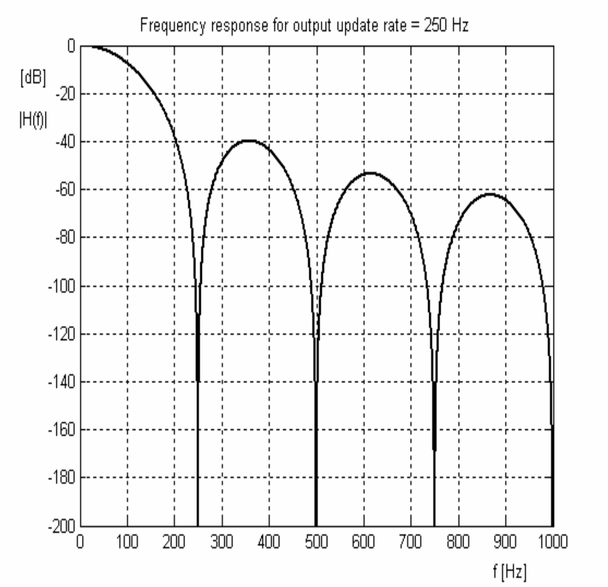

Figure 2. Frequency response of a *(sinx/x)*^{3} AD7707 digital filter for output update rate of 250 Hz.

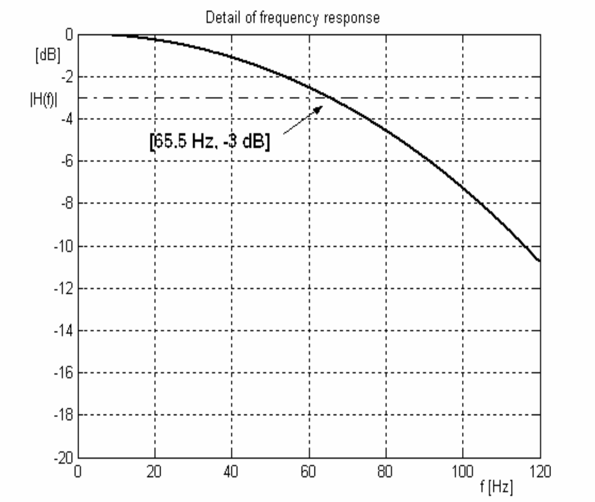

Figure 3. Detail of frequency response of a *(sinx/x)*^{3} digital filter.

As it stands, **Ax = y** is an overdeterminated system and a least squares solution to the system is a desired result. Note that the solution values for the coefficients of the cubic will be given symbolically in terms of the y_i, since no numerical values for them have been specified yet.

Least squares problems of this sort are easily solved by forming the 'normal' equations for the system. That is, we solve the overdetermined system (1) where the matrix **A** has fewer columns than rows, **y** has the same number of rows as **A** and **x** has the same number of columns as **A**.

The solution is given by (4):

$$\mathbf{x} = (\mathbf{A}^{\mathbf{T}}\,\mathbf{A})^{-1}\,\mathbf{A}^{\mathbf{T}}\,\mathbf{y} \tag{4}$$

The solution to the normal equations provides more information than was expected. Specifically, we found that the expression for the coefficient of a_0 is a weighted function of **y** which satisfies our requirement for a smoothing function. However, we may differentiate the cubic and evaluate the derivative at zero to obtain a point on the derivative of the smoothed curve as well. This derivative is simply the expression for the coefficient a_1 already obtained in solving the normal equations. Similarly, higher derivatives of the smoothed curve are available as the coefficients a_2, a_3, . , . ,.. Note, however, that these coefficient expressions must be multiplied by 0!, 1!, 2!, , as appropriate. That is, for the polynomial, p

$$\frac{d^n p}{di^n}\Big|_{i=0} = n!\,a_i \tag{5}$$

To return to the example problem (3), we solve:

$$\begin{bmatrix} 1 & -2 & 4 & -8 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} y_{-2} \\ y_{-1} \\ y_0 \\ y_1 \\ y_2 \end{bmatrix} \tag{6}$$

which gives

$$a_0 = \frac{-3y_{-2} + 12y_{-1} + 17y_0 + 12y_1 - 3y_2}{35} \tag{7}$$

$$a_1 = \frac{y_{-2} - 8y_{-1} + 8y_1 - y_2}{12} \tag{8}$$

$$a_2 = \frac{2y_{-2} - y_{-1} - 2y_0 - y_1 + 2y_2}{14} \tag{9}$$

$$a_3 = \frac{-y_{-2} + 2y_{-1} - 2y_1 + y_2}{12} \tag{10}$$

To form the derivatives, the third and fourth of these equations, (9) and (10), must be multiplied by 2! and 3!, respectively, according (5).

Sample Savitzki-Golay coefficients for order = 2 and window length = 5 are:

-0.086 0.343 0.486 0.343 -0.086

Example of Savitzki-Golay signal filtering is shown in Figure 1. It is important to note, that *filtered signal is not phase shifted* (but others digital and analog filters output is phase shifted).

In presented biomedical applications the Savitzki-Golay filter output is subtract from raw signal which form a highpass filter for signal baseline recovery. After, the highpass, signal is filtered by lowpass filter.

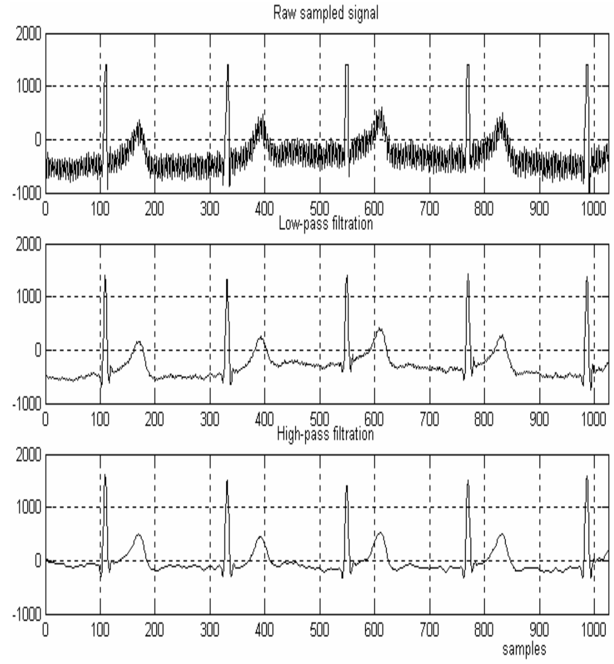

Figure 3. The example of ECG signal filtering.
Top: Raw sampled ECG stress test signal with noise and baseline distortion.
Middle: Signal after low-pass filtration by DSPF filter order of 5, window size is 17.
Bottom: Signal after low-pass and high-pass filtration for baseline restoration. Booth filters are Savitzki-Golay filters.

3 Analog/digital converter

The analog front end, based on AD7707 is tree-channel device which can accept either 2 low level input signals (+10 mV to + 1.225 V or ± 10 mV to ±1.225 V, depends on PGA setting) directly from transducer or one high level signal (+10 V or ± 10 V) and produce serial digital output [7, 8]. It employs a sigma-delta conversion technique to realize up to 16 bits of no missing codes performance [9].

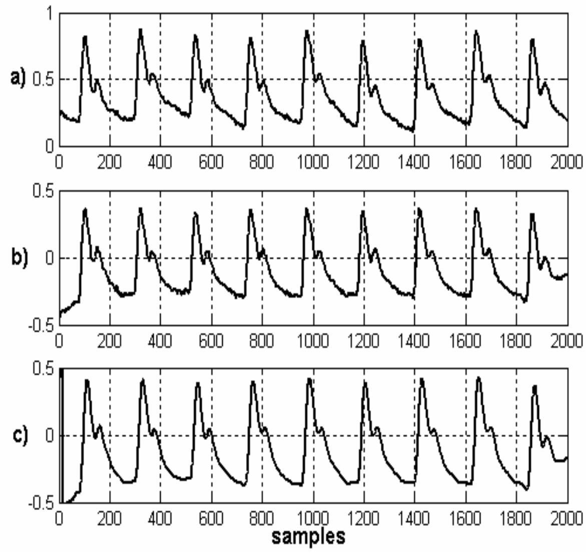

Figure 4. Example of continuous indirect arterial blood pressure monitoring based on Penaz technique:
a) Raw signal with noise and baseline wander
b) Highpass filter output
c) Highpass and Lowpass filter output

The sigma-delta modulator output is processed by an on-chip digital filter. The first notch of this digital filter can be programmed via an on-chip control register allowing adjustment of the filter cutoff (1.06 Hz to 131 Hz) and output update rate (4.054 Hz to 500 Hz). The -3 dB frequency f_{-3dB} is determined by the programmed first notch frequency according to the relationship (11):

$$f_{-3dB} = 0.262 * f_{FN} = 0.262 * f_s \quad [\mathrm{Hz}] \qquad (11)$$

where f_{FN} is filter first notch frequency and f_s is output update rate (sampling rate). The AD7707's digital filter is a low-pass filter with a *(sinx/x)*^{3} response (also called *sinc*^{3}). The transfer function for this filter is described in z-domain by:

$$H(z) = \left| \frac{1}{N} \cdot \frac{1 - z^{-N}}{1 - z^{-1}} \right|^3 \qquad (12)$$

and in the frequency domain by:

$$H(f) = \left| \frac{1}{N} \cdot \frac{\sin(\pi N f / f_s)}{\sin(\pi f / f_s)} \right|^3 \quad [\mathrm{Hz}] \qquad (13)$$

where N is the ratio of the modulator rate to the output rate (modulator rate is 19.2 kHz for Xtal=2.4576 MHz).

The frequency response of the digital filter is shown in Figure 2. Phase response is given by (14):

$$Phase(f) = -3\,\pi\,(N-2)\,f/f_s \quad [\mathrm{Rad}] \qquad (14)$$

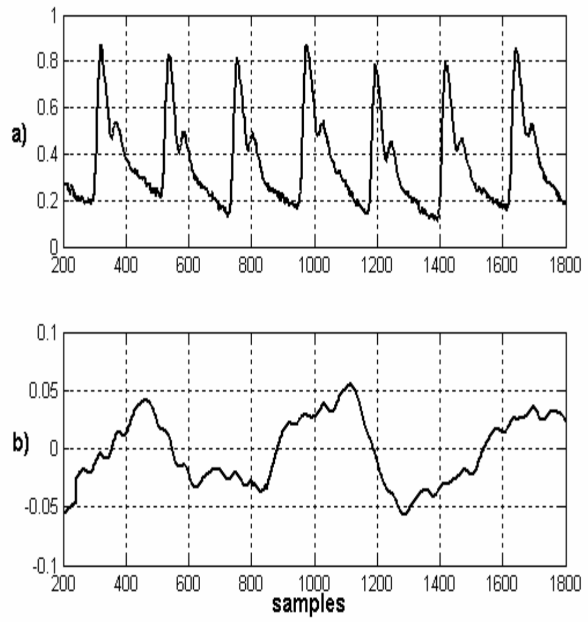

Figure 5. Breathing signal derived from continuous indirect arterial blood pressure monitoring based on Penaz technique:
a) Raw signal with noise and baseline wander
b) Breathing signal

4 Results

The example of filtering is demonstrated on ECG stress - test signal. The raw sampled ECG signal, low-pass and high-pass filtered signals are shown in Figure 3. The ECG signal is filtered by Savitzki-Golay filters. The highpass filtered signal is obtain by subtracting of two lowpass signals filtered by different order and window length of Savitzki-Golay filters. The combination of linear digital filter (e.g. finite impulse response filter and infinite impulse response filter) is also possible.

In second example (Figure 4), blood pressure, noninvasive continuous signal gained by Penaz [10, 11] method is filtered by Savitzki-Golay filters. The breathing signal is also derived from raw signal (Figure 5). It is important to note that sampling frequency is 200 Hz and signal amplitude is normalized to <0 - 1>.

5 Discussion

Moving average smoothing is equivalent to fitting the data about each point with a straight line. A more ‘gentle’ method of smoothing is to use a polynomial to fit the points. This is commonly called Savitzky–Golay, DISPO, or least-squares smoothing.

In Savitzky–Golay smoothing, the weights *w* are chosen in such a way that the smoothed data point *y* is the value of a polynomial fitted by least-squares to the raw data points. The chief advantage of the method is that peaks defined by even few data points can often smoothed with little loss in amplitude, broadening or change in slope.

Use Savitzky–Golay smoothing when you require a more gentle algorithm than moving point average smoothing provides. Note that if the peaks, and leading and falling edges are composed of many data points then there will be little advantage in using Savitzky–Golay smoothing.

6 Conclusion

The basic theory and some examples use of the Savitzky-Golay method for filtering in biomedical signal processing was demonstrated.

The amount of smoothing performed is directly dependent on the order of the interpolating polynomial and the filter length used to perform the least squares approximation. The polynomial order chosen should be the minimum necessary to reflect the derivative information accurately and to preserve spectral characteristics. Orders higher than the minimum necessary should not be used as they effectively over fit the data, representing noise and possibly introducing some extraneous oscillations.

The filter length chosen is of utmost importance in order to maintain the integrity of the derivative spectra. Several tests were made on the data using Savitzky-Golay filtering using various different filter lengths. It was found, as expected, that when more smoothing is performed using a longer filter. A wide window will result in more smoothing but at the cost of more distortion of higher frequency content. In contrast, higher-order filters can track narrower features but with loss of smoothing of low frequency content.

Savitzky-Golay filters are typically used for smoothing of signals whose frequency span is large. In general, they are not as effective at rejecting noise as standard averaging filters. However, the Savitzky-Golay filtering method has the advantage that it is easy to determine the second derivative directly. One need only find the derivative of each polynomial at the centre point. It should be noted that the Savitzky-Golay method strictly only applies to data points which are equally spaced in the independent variable. However, assuming that the data points are equally spaced simply amounts to shifting of each point to equally spaced positions. This is equivalent to adding noise to the function which may be acceptable if it is much smaller than the noise already present.

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