Proceedings of the 5th WSEAS International Conference on Applications of Electrical Engineering, Prague, Czech Republic, March 12-14, 2006 (pp161-166)

Determination of the Sensitivities for Two-Ports Networks in Steady State Nonsinusoidal Regime

HORIA ANDREI
Faculty of Electrical Engineering
University Valahia Targoviste
18-20 Blv. Unirii, Targoviste, Dambovita
ROMANIA
http://www.valahia.ro
FANICA SPINEI
Faculty of Electrical Engineering
University Politehnica Bucharest
313 Splaiul Independentei, sector 6, Bucharest
ROMANIA
http://www.elth.pub.ro
COSTIN CEPISCA
Faculty of Electrical Engineering
University Politehnica Bucharest
313 Splaiul Independentei, sector 6, Bucharest
ROMANIA
http://www.elth.pub.ro
GEORGE ENCIU
Faculty of Industrial Engineering
University Politehnica Bucharest
313 Splaiul Independentei, sector 6, Bucharest
ROMANIA
http://www.imst.pub.ro
MIRCEA POPA
S.C. Electrica S.A. North Transilvania-SDFEE Oradea
ROMANIA
http://www.electrica.ro
MARIUS SILAGHI
Faculty of Electrical Engineering, University of Oradea
5, Armatei Romane
ROMANIA
http://www.uoradea.ro

Abstract: - The paper proposes an original method for determination of the sensitivities for two-ports networks in steady state nonsinusoidal regime. It is presented a computation algorithm for determination the sensitivities using the relative values which characterize the current and voltage harmonics, without knowing the circuit structure. The given example is necessary for the discussions of some aspects related the two-ports networks applications.

Key words: - sensitivities, two-ports, relative values, harmonics.

1. Introduction

The sensitivity has been an interesting problem for a long time in the technical literature [1], [2]. This paper presents a method for computation of the sensitivities in electrical steady state linear networks, correlating it to the new four parameters which characterize the current and respectively voltage harmonics. The given example is used for the discussion of some aspects related to the sensitivity problems.

The propose of this paper is to define some sensitivities for the two-ports networks. On consider the steady state nonsinusoidal regime of the two-ports networks and we calculate the sensitivities correlated with the relative values which characterize the current respectively the voltage harmonics.

2. The Sensitivities Definitions and Invariant Relationships

Classical sensitivity of a network function F on an independent parameter x is defined, [3] or [4], by relation

$$S(F,x)=\frac{x}{F}\frac{\partial F}{\partial x}=\frac{\partial(\ln F)}{\partial(\ln x)}. \quad (1)$$

As a rule by k-order sensitivity we understand:

$$S^{(k)}(F,x)=\frac{x^k}{F^k}\frac{\partial^{(k)}F}{\partial x^{(k)}}. \quad (2)$$

Consider a linear nonreciprocal network and the following forms of the matrix equations of the network [5]:

$$[U]=[Z]\,[J], \quad (3)$$

$$[V]=\left[Z^n\right]\left[J^n\right], \quad (4)$$

$$[I]=[Y]\,[E], \quad (5)$$

where the symbols are usual: $[Z]$ is the transfer branches impedances matrix, $[Y]$ is the transfer branches admittances matrix, $\left[Z^n\right]$ is the node-node transfer impedances matrix, $[U]$ is the branches voltages matrix, $[I]$ is the branches currents matrix, $[E]$ is the independent voltages sources matrix, $[J]$ the independent current sources matrix, $\left[J^n\right]$ is the nodes transfer matrix of sources of current and $[V]$ is the nodes voltages matrix. Thus the variation theorems lead in the case of the very small modifications to:

$$\frac{dZ_{kq}}{dY_h}=-Z_{kh}Z_{hq}, \quad (7)$$

$$\frac{dY_{kq}}{dZ_h}=-Y_{kh}Y_{hq}, \quad (8)$$

and to two analogous relations for the magnitudes appearing in equations (4).

Relations (7) and (8) lead to the following expressions of the first order sensitivities, [6],

$$S(Z_{kq},Y_h)=-\frac{Z_{kh}Y_hZ_{hq}}{Z_{kq}}, \quad (9)$$

$$S(Y_{kq},Y_h)=-\frac{Y_{kh}Z_hY_{hq}}{Y_{kq}}. \quad (10)$$

By successive derivation of the relations (9) and (10) we can obtain higher order derivatives useful for the high-order sensitivity calculation, as in [7].

Let's consider the steady state linear network. To illustrate the relative contributions of all harmonics of current and voltage, we can use two formulae equivalents, [8], of the Fourier series. If the receptor is linear and supplied with a nonsinusoidal voltage u(t) whose the development in a Fourier's series is truncated only at n terms, the characterization of the receptor introducing a φ_k phase angle on each harmonic at the input terminals can be done as follows:

$$u(t)=U\sum_{k=1}^{n}b^{(k)}\sqrt{2}\sin(kwt+\alpha_k) \quad (11)$$

$$i(t)=I\sum_{k=1}^{n}a^{(k)}\sqrt{2}\sin(kwt+\alpha_k-\varphi_k) \quad (12)$$

where the relative values of each k - harmonics magnitude of the voltage $b^{(k)}$ and current $a^{(k)}$ as compared to the root – mean – square value of the voltage, U, respectively current, I, have been taken down as:

$$b^{(k)}=\frac{U^{(k)}}{U};\quad a^{(k)}=\frac{I^{(k)}}{I}. \quad (13)$$

Otherwise, [9], [10], if mark with $\mu^{(k)}$ the fractions of the magnitude of each k - harmonics of voltage compared to the fundamental value $U^{(1)}$,

$$\mu^{(k)}=\frac{U^{(k)}}{U^{(1)}}, \quad (14)$$

respectively, with $\varepsilon^{(k)}$ the fractions of the magnitude of each k - harmonics of current compared to the fundamental value $I^{(1)}$

$$\varepsilon^{(k)}=\frac{I^{(k)}}{I^{(1)}}, \quad (15)$$

we get the second set of formulae:

$$u(t)=U^{(1)}\sum_{k=1}^{n}\mu^{(k)}\sqrt{2}\sin(kwt+\alpha_k) \quad (16)$$

$$i(t)=I^{(1)}\sum_{k=1}^{n}\varepsilon^{(k)}\sqrt{2}\sin(kwt+\alpha_k-\varphi_k) \quad (17)$$

These four new relative parameters verify the conditions [6]:

$$\sum_{k=1}^{n} a^{(k)^2} = \sum_{k=1}^{n} b^{(k)^2} = 1 , \qquad (18)$$

$$b^{(i)^2} = \frac{\mu^{(i)^2}}{\sum_{k=1}^{n}\mu^{(k)}} \; ; \; a^{(i)^2} = \frac{\varepsilon^{(i)^2}}{\sum_{k=1}^{n}\varepsilon^{(k)}} , \; \forall i = 1,...,n. \quad (19)$$

In this case, for two ports networks, we can define two magnitude voltage and current transfer function [11]:

$$F_U = \frac{U_{out}^2}{U_{in}^2} = \frac{\sum_{k=1}^{n} U_{out}^{(k)^2}}{\sum_{k=1}^{n} U_{in}^{(k)^2}} = \frac{U_{out}^2 \sum_{k=1}^{n} b_{out}^{(k)^2}}{U_{in}^2 \sum_{k=1}^{n} b_{in}^{(k)^2}} = \frac{U_{out}^{(1)^2} \sum_{k=1}^{n} \mu_{out}^{(k)^2}}{U_{in}^{(1)^2} \sum_{k=1}^{n} \mu_{in}^{(k)^2}} , \qquad (20)$$

$$F_I = \frac{I_{out}^2}{I_{in}^2} = \frac{\sum_{k=1}^{n} I_{out}^{(k)^2}}{\sum_{k=1}^{n} I_{in}^{(k)^2}} = \frac{I_{out}^2 \sum_{k=1}^{n} a_{out}^{(k)^2}}{I_{in}^2 \sum_{k=1}^{n} a_{in}^{(k)^2}} = \frac{I_{out}^{(1)^2} \sum_{k=1}^{n} \varepsilon_{out}^{(k)^2}}{I_{in}^{(1)^2} \sum_{k=1}^{n} \varepsilon_{in}^{(k)^2}} , \qquad (21)$$

where: $U_{out}, U_{in}, I_{out}, I_{in}$ represent the root-mean-square values of output and input voltage, respectively current; $U_{out}^{(1)}, U_{in}^{(1)}, I_{out}^{(1)}, I_{in}^{(1)}$ represent the values of fundamental of output and input voltage respectively current; $b_{out}, b_{in}, \mu_{out}, \mu_{in}$ represent the relative values of harmonics of output and input voltage; $a_{out}, a_{in}, \varepsilon_{out}, \varepsilon_{in}$ represent the relative values of harmonics of output and input current.

The first order sensitivity of such steady state linear network, related to the input parameters $b_{in}^{(j)}, \mu_{in}^{(j)}, a_{in}^{(j)}, \varepsilon_{in}^{(j)}$, can be expressed in the form of [12]:

$$S(F_U, b_{in}^{(j)}) = \left| \frac{b_{in}^{(j)}}{F_U} \frac{\partial F_U}{\partial b_{in}^{(j)}} \right| = \frac{2 b_{in}^{(j)^2}}{\sum_{k=1}^{n} b_{in}^{(k)^2}} , \qquad (22)$$

$$S(F_U, \mu_{in}^{(j)}) = \left| \frac{\mu_{in}^{(j)}}{F_U} \frac{\partial F_U}{\partial \mu_{in}^{(j)}} \right| = \frac{2 \mu_{in}^{(j)^2}}{\sum_{k=1}^{n} \mu_{in}^{(k)^2}} , \qquad (23)$$

$$S(F_I, a_{in}^{(j)}) = \left| \frac{a_{in}^{(j)}}{F_I} \frac{\partial F_I}{\partial a_{in}^{(j)}} \right| = \frac{2 a_{in}^{(j)^2}}{\sum_{k=1}^{n} a_{in}^{(k)^2}} , \qquad (24)$$

$$S(F_I, \varepsilon_{in}^{(j)}) = \left| \frac{\varepsilon_{in}^{(j)}}{F_I} \frac{\partial F_I}{\partial \varepsilon_{in}^{(j)}} \right| = \frac{2 \varepsilon_{in}^{(j)^2}}{\sum_{k=1}^{n} \varepsilon_{in}^{(k)^2}} , \qquad (25)$$

where $j = 1,...,n.$

Numerous authors show that in the electrical linear and nonlinear networks the sensitivities satisfy the invariant relationships or inequalities, some of which allow deducing the lower and upper bounds of the sensitivity index.

Generally speaking, a significant property of linear networks is to existence of sensitivity invariants, which have the form:

$$\sum_{k=1}^{n} S(F, x_i) = \lambda , \qquad (26)$$

where λ is a constant. A simple method for deriving sensitivity invariants employs the concept of homogeneity [2].

If a function $F = F(x_1, x_2, ..., x_n)$ is homogenous and of order λ, the following relationship holds:

$$\frac{x_1}{F}\frac{\partial F}{\partial x_1} + ... + \frac{x_n}{F}\frac{\partial F}{\partial x_n} = \lambda . \qquad (27)$$

The above-presented relations allow the direct establishment of some invariant sensitivity relationships [13].

If the network functions is a dimensionless ratio (in which the denominator is usually given) we get the following relationships

$$F = \frac{I_k}{J_q} = Y_k Z_{kq} ,$$

$$\sum_{h=1}^{l} S(F, Y_h) = \sum_{h=1}^{l} \frac{Y_h}{Y_k Z_{kq}} (-Y_k Z_{kh} Z_{hq}) + 1 = 0 \quad (28)$$

and

$$F = \frac{U_k}{E_q} = Z_k Y_{kq} ,$$

$$\sum_{h=1}^{l} S(F, Z_h) = \sum_{h=1}^{l} \frac{Z_h}{Z_k Y_{kq}} (-Z_k Y_{kh} Y_{hq}) + 1 = 0 . \quad (29)$$

Similarly, for the functions defined by the relations (22),…,(25) then homogeneity given by the relative values $(b_{in}^{(1)},..,b_{in}^{(n)}), (\mu_{in}^{(1)},..,\mu_{in}^{(n)}), (a_{in}^{(1)},..,a_{in}^{(n)}) or (\varepsilon_{in}^{(1)},..,\varepsilon_{in}^{(n)})$ is of order 2 (λ =2).

Using, for example, the parameters defined by the relative values $(b_{in}^{(1)},..,b_{in}^{(n)})$, the condition (27)

Proceedings of the 5th WSEAS International Conference on Applications of Electrical Engineering, Prague, Czech Republic, March 12-14, 2006 (pp161-166)

of the sensitivity invariants will be computed as follows:

$$\sum_{j=1} S(F_U, b_{in}^{(j)}) = \sum_{j=1}^{n} \frac{2 b_{in}^{(j)^2}}{\sum_{k=1}^{n} b_{in}^{(k)^2}} = 2. \qquad (35)$$

A similarly result is obtained if using the other set of relative values $(\mu_{in}^{(1)},..,\mu_{in}^{(n)}),(a_{in}^{(1)},..,a_{in}^{(n)}) or (\varepsilon_{in}^{(1)},..,\varepsilon_{in}^{(n)})$.

3.Two-Ports Networks in Steady State Nonsinusoidal Regime

The two-ports networks may be defined as a part of an electrical system, having two pairs of terminals, 1-1' and 2-2', for connection of sources and loads, shown in fig.1. There may be active and passive two-ports. For example, the analogues filters and the power transmission lines may be classed as a passive

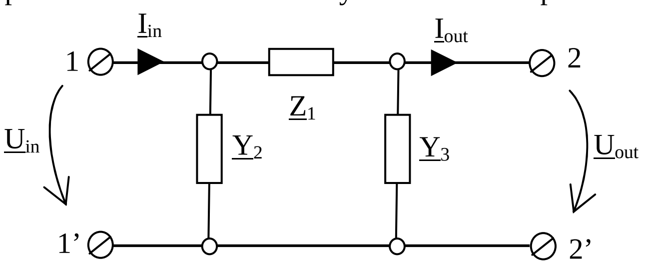

Fig.1

two-ports.

The passive two-ports may be represented by an equivalent three-element Π-network or an equivalent three-element T-network.

The two-port Π-network in steady-state nonsinusoidal regime can be described by the following equations

$$\underline{U}_{in}^{(k)} = (1+\underline{Z}_1^{(k)}\underline{Y}_3^{(k)})\underline{U}_{out}^{(k)} + \underline{Z}_1^{(k)}\underline{I}_{out}^{(k)}$$

$$\underline{I}_{in}^{(k)} = (\underline{Y}_2^{(k)} + \underline{Y}_3^{(k)} + \underline{Z}_1^{(k)}\underline{Y}_2^{(k)}\underline{Y}_3^{(k)})\underline{U}_{out}^{(k)} + (1+\underline{Z}_1^{(k)}\underline{Y}_2^{(k)})\underline{I}_{out}^{(k)} \qquad (36)$$

or equivalent

$$\underline{U}_{in}^{(k)} = \underline{A}^{(k)}\underline{U}_{out}^{(k)} + \underline{B}^{(k)}\underline{I}_{out}^{(k)}$$
$$\underline{I}_{in}^{(k)} = \underline{C}^{(k)}\underline{U}_{out}^{(k)} + \underline{D}^{(k)}\underline{I}_{out}^{(k)} \qquad (37)$$

where the impedance and admittance parameters $\underline{Z}_1, \underline{Y}_2, \underline{Y}_3$, respectively the transfer (circuit) parameters $\underline{A}, \underline{B}, \underline{C}, \underline{D}$, depend of all the frequencies, $k = 1,..,n$.

For example, the voltage amplification for the output port open-circuited is defined for each frequency:

$$\underline{A}^{(k)} = \left.\frac{\underline{U}_{in}^{(k)}}{\underline{U}_{out}^{(k)}}\right|_{\underline{I}_{out}^{(k)}=0}. \qquad (38)$$

Using relation (38) we can calculate the voltage transfer function define by (20)

$$F_U = \frac{U_{out}^2}{U_{in}^2} = \frac{U_{in}^2 \sum_{k=1}^{n} \frac{b_{in}^{(k)^2}}{\left|\underline{A}^{(k)}\right|^2}}{U_{in}^2} = \sum_{k=1}^{n} \frac{b_{in}^{(k)^2}}{\left|\underline{A}^{(k)}\right|^2} \qquad (39)$$

and the sensitivity related to the input parameters $b_{in}^{(j)}, j = 1,..,n,$

$$S(F_U, b_{in}^{(j)}) = \left|\frac{b_{in}^{(j)}}{F_U}\frac{\partial F_U}{\partial b_{in}^{(j)}}\right| = \frac{2 b_{in}^{(j)^2}}{\left|\underline{A}^{(j)}\right|^2 \sum_{k=1}^{n} \frac{b_{in}^{(k)^2}}{\left|\underline{A}^{(k)}\right|^2}}. \qquad (40)$$

4.Example

For the linear two-ports whose output and input voltages and currents contain the same number of harmonics, we can calculate the sensitivity related to the variation of magnitude of input harmonics, without knowing the circuit structure.

For example, we consider a linear two-ports network, shown in fig.2, in steady state nonsinusoidal regime, whose input voltage contains the first $n = 3$ harmonics:

$$u_{in}(t) = 110\sqrt{2}\sin(\omega t + 40.1^0) + 10\sqrt{2}\sin(2\omega t + 120.5^0) +$$
$$+ 40\sqrt{2}\sin(3\omega t).$$

For calculate its sensitivity related to the parameter $b_{in}^{(3)}$ of the 3-rd harmonic input voltage, we use two procedures.

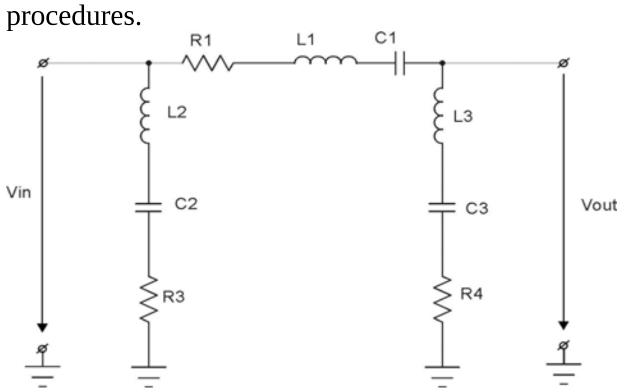

Fig.2

1) Considering that we know, by measurements, for input voltage the $n-1$ relative values of b_{in} parameters

$b_{in}^{(1)} = 93.64\%$, $b_{in}^{(2)} = 8.51\%$.

According with relation (18), results

$$b_{in}^{(3)} = \sqrt{1 - b_{in}^{(1)^2} - b_{in}^{(2)^2}} = 34.05\% .$$

We can calculate, using relation (22), the first order sensitivity of this two-ports network related-for example-to the relative value $b_{in}^{(3)}$:

$$S(F_U, b_{in}^{(3)}) = \left| \frac{b_{in}^{(3)}}{F_U} \frac{\partial F_U}{\partial b_{in}^{(3)}} \right| = \frac{2 b_{in}^{(3)^2}}{\sum_{k=1}^{n} b_{in}^{(k)^2}} = 23.2\% .$$

2) Considering that we know, by measurements, for input voltage the $n-1$ relative values of b_{in} parameters

$b_{in}^{(1)} = 9364 \cdot 10^{-4}$, $b_{in}^{(2)} = 851 \cdot 10^{-4}$, and the values of the network elements

$R_1 = 20\Omega, L_1 = 0.4H, C_1 = 20\mu F, R_2 = 500\Omega, L_2 = 0.1H,$
$C_2 = 100\mu F, R_3 = 10\Omega, L_3 = 5mH, C_3 = 5\mu F.$

In this case we use a PSPICE, [14], algorithm for calculate the following values.

According with relation (18), results

$$b_{in}^{(3)} = \sqrt{1 - b_{in}^{(1)^2} - b_{in}^{(2)^2}} = 3405 \cdot 10^{-4} .$$

Using (36), (37), and (38) we can express the voltage amplification

$$\underline{A}^{(k)} = \frac{\underline{U}_{in}^{(k)}}{\underline{U}_{out}^{(k)}} \Bigg|_{\underline{I}_{out}^{(k)} = 0} = 1 + \underline{Z}_1^{(k)} \underline{Y}_3^{(k)} =$$

$$= 1 + [R_1 + j(k\omega L_1 - \frac{1}{k\omega C_1})][\frac{1}{R_3} - j(\frac{1}{k\omega L_3} - k\omega C_3)],$$

$k = 1,2,3.$

From relation (40) results:

$$S(F_U, b_{in}^{(3)}) = \frac{2 b_{in}^{(3)^2}}{\left| \underline{A}^{(3)} \right|^2 \sum_{k=1}^{3} \frac{b_{in}^{(k)^2}}{\left| \underline{A}^{(k)} \right|^2}} = 23.2\% .$$

We obtain the same value of sensitivity using the both procedures, but the second procedure it is more complicated.

The numerical value of sensitivity proves a smaller increase, 23.2%, of sensitivity compared to a parameter $b_{in}^{(3)} = 34.05\%$. The two-ports network functioning like an analogue filter for the third harmonic of voltage.

5. Conclusions

For the linear steady state networks whose output and input voltages and currents contain the same number of harmonics, we can calculate the sensitivity related to the variation of magnitude of input harmonics, without knowing the circuit structure.

In comparison of this method, in the other works of the literature the sensitivities are calculated using the circuit parameters or the transfer (circuit) functions.

It is of practical and efficient to express the sensitivities of the linear steady state networks, especially in the electronic amplifiers and filters, related to the relative values of the input harmonics magnitude, $b, \mu, a,$ and ε, so that the former should not depend on the circuit parameters.

In this paper we define new sensitivities and we establish relations between these sensitivities and the new relative values of harmonics. These relations verify the invariants general conditions.

The relative values $b, \mu, a,$ and ε can be easily measured and thus a number of restrictions can be established as regards their variation limits.

One of the future directions of this work is the calculation of the sensitivity of the power generator at different harmonics, because it is of great practical interest in order to obtain a more efficient evaluation of the receptors affected by the harmonics within the network.

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