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A modified HDWTSVD image coding system

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Abstract: In this paper, we propose a modification of the HDWTSVD algorithm to encode monochromatic images that combines DWT and SVD techniques. At the encoder side, the input image is divided into blocks (tiles) of 64x64 pixels. A criterion based on the average standard deviation of 8x8 subblocks is used to choose DWT or SVD. If the tile exhibits a high average standard deviation, it is compressed by using SVD otherwise by DWT. The Daubechies 9/7 filter bank factored into lifting steps is used for the DWT. The maximum number of levels of subband decomposition for each tile is 3. The proposed algorithm is applied to image coding and its performance is discussed and compared with the HSVDDCT algorithm.

Keywords: image coding using wavelets, singular value decomposition, HC-RIOT, discrete wavelets, vector quantization .

1 INTRODUCTION

The phenomenal increase in the generation, transmission, and use of digital images in many applications is placing enormous demands on the storage space and communication bandwidth. Data compression algorithms are a viable approach to alleviate the storage and bandwidth demands. They are key enabling components in a wide variety of information technology applications that require handling a large amount of information. From text and image representation in digital libraries to video streaming over the Internet, current information transmission and storage capabilities are made possible by recent advances in data compression.

Linear transforms are the basis of many techniques used in image processing, images analysis, and image coding. Subband transforms are a subclass of linear transforms, which offer useful properties for these applications.

The basic idea of compression using the DWT is, as discussed before, to exploit the local correlation that exists in most of the images for building an aproximation. In the first generation wavelets, the Fourier transform is used to build the space-frequency localization. However, in the second generation wavelets, this can be done in the spatial domain and can reduce the computational complexity of the wavelet transform by a factor of two [1].

The Singular Value Decomposition (SVD) technique provides optimal energy-packing efficiency for any given image, but its application is very limited due to the computational complexity associated with the computation of eigenvalues and eigenvectors [2]. The best results for SVD image compression have been obtained by combining SVD and vector quantization (VQ) of the singular vectors [3], [4].

SVD is used in order to have a better performance of the transformation in the areas of the image where the correlation is low and DWT where the correlation is high. The original system was called “A Hybrid DWT-SVD Image-Coding Algorithm” (HDWTSVD) and this is a modification to the original HDWTSVD.

2 SVD Coding

The SVD is a transform suitable for image compression because it provides optimal energy compaction for any given image [2]. A good representation of the image can be achieved by taking only a few largest eigenvalues and corresponding eigenvectors. A (NxN) matrix $\boldsymbol{A}$ is decomposed to form two orthogonal matrices, $\boldsymbol{U}$ and

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$\boldsymbol{V^T}$, representing the eigenvectors, and a diagonal matrix $\boldsymbol{\Sigma}$ representing the eigenvalues.

$$\boldsymbol{A} = \boldsymbol{U \Sigma V}^{\boldsymbol{T}} \quad (1)$$

where *r* is the rank of $\boldsymbol{A}$

$$\Sigma^T \Sigma = diag(\lambda_1, \lambda_2......\lambda_r, 0,0,0,0) = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_r^2, 0,0,0,0) \quad (2)$$
$$\lambda_1 \geq \lambda_2 ... \geq \lambda_r > \lambda_{r+1} = ... = \lambda_N = 0$$

We can calculate the columns $\boldsymbol{v}(n)$ of $\boldsymbol{V}$ by solving

$$(\boldsymbol{A'} - \lambda(n)\boldsymbol{I})\, \boldsymbol{v}(n) = \boldsymbol{0} \qquad n = 1,.....,r \quad (3)$$

where $\boldsymbol{A'} = \boldsymbol{A^T A}$. The columns of $\boldsymbol{U}$ are

$$\boldsymbol{u}(n) = \frac{1}{\sqrt{\lambda(n)}} \boldsymbol{A v}(n) \qquad n = 1,.....,r \quad (4)$$

The original block can be estimated by retaining the *q* largest eigenvalues and corresponding eigenvectors

$$\hat{A} = \sum_{n=1}^{q} \sqrt{\lambda(n)}\, \boldsymbol{u}(n)\, \boldsymbol{v}^{\boldsymbol{T}}(n) \qquad q \leq r \quad (5)$$

The square error is equal to the sum of the discarded eigenvalues

$$\sum_{m=1}^{r} \sum_{n=1}^{r} \left| X(m,n) - \hat{X}(m,n) \right|^2 = \sum_{n=q+1}^{r} \lambda(n) \quad (6)$$

and the energy contained in *q* retained eigenvalues is

$$Energy = \sum_{n=1}^{q} \lambda(n) \quad (7)$$

SVD yields two matrices of eigenvectors and one of eigenvalues. VQ techniques have been used successfully to encode the eigenvectors and scalar quantization techniques (SQ) to encode the eigenvalues [5].

3 THE PROPOSED ALGORITHM

Figure 1 shows the proposed algorithm. The decision threshold on what transform to use is based on the average standard deviation criterion (ASTD) [6].

3.1 Encoder

The block diagram of the modified codec HDWTSVD is shown in Fig. 1. Before encoding, the source image is divided into tiles of 64x64 pixels. Each tile is encoded independently. The ASTD is first calculated on the source tile by calculating the standard deviation of 8x8 subblocks of a tile. If the ASTD is high then, the mean of the 8x8 subblock is subtracted and SVD is applied to decompose the subblock into two orthogonal matrices $\boldsymbol{U}$ and $\boldsymbol{V^T}$, containing the eigenvectors, and a matrix $\boldsymbol{W}$ containing the square root of $\boldsymbol{\Sigma}$ or eigenvalues of the original subblock. These matrices are stored to calculate the optimum number of eigenvalues/eigenvectors to quantize from a mean square error point of view. This operation is performed in the adaptive reconstruction stage. The mean of the subblock is quantized uniformly with 8 bits.

In the adaptive reconstruction and comparison stage, the eigenvalues are rearranged in decreasing order (from the highest to the lowest) and discarded progressively, by setting them to zero, from the lowest to the highest until a MSE for the subblock is met (after exhaustive tests the MSE set for this unit was 5). The total MSE of the subblock is calculated each time an eigenvalue is discarded by using equation (6).

After meeting the MSE requirement for a subblock, the resulting eigenvalues are coded using uniform scalar quantizers of 8,8,7,7,6,6, and 4 bits respectively and their corresponding eigenvectors are sent to the decision stage. The eighth eigenvalues/eigenvectors are discarded. The adaptive reconstruction and comparison stage helps us to discard adaptively the eigenvalues and eigenvectors that do not introduce a significant visual error.

In the decision stage, the first three eigenvectors of matrices $\boldsymbol{U}$ and $\boldsymbol{V^T}$ are vector quantized using codebooks of 256, 128 and 32 codewords. If the MSE of the indexed codeword is below 0.01, 0.1 and 0.4 respectively, with respect to the original eigenvectors, then the indices are transmitted. If the MSE is above these thresholds, the components of the first eigenvector are uniformly quantized to 7 bits and the components of the second and third eigenvectors are uniformly quantized to 5 bits each and sent to the encoded file. One extra bit is included to indicate to the decoder that the data corresponds to an index of the codebook or to the quantized eigenvector. The codebooks for the fourth and fifth eigenvectors are of length 32 each. The sixth, seventh and eighth codebooks are of length 2.

The decision on the size of the codebooks for eigenvectors 5,6 and 7 was based on Table 1. In this, test images Lena, Goldhill and Peppers (512 x 512, 8 bit PCM) were used. It shows the values for two different allowed MSE of the last three eigenvectors. This table was obtained by using, in a first test, an allowed MSE per eigenvector of: 0.01, 0.1, 0.4, 100.0, 100.0, 0.001, 0.001 and 0.001, and in a second test, an allowed MSE per eigenvector of 0.01, 0.1, 0.4, 100.0, 100.0, 100.0, 100.0, 100.0. The codebooks sizes for the test were 256, 128, 32, 32, 32, 2, 2, 2.

A MSE of 100.0, means that the index representing the minimum MSE will be sent without regards to the amount of MSE. The codebooks used were of size 256, 128, 32, 32, 32, 2, 2, 2.

SVD is calculated only once and the matrices are saved. The reconstructed subblock is calculated by using a copy of these matrices and by copying data from the original matrices each time it is necessary to reconstruct a new subblock with a different number of eigenvalues/eigenvectors. The first bit is part of the header of the encoded tile to indicate SVD or DWT.

1). Allowed MSE per eigenvector: 0.01, 0.1, 0.4, 100.0, 100.0, 0.001, 0.001, 0.001			
	Lena	Goldhill	Peppers
Bitrate	0.7	0.55	0.362
PSNR	38.55	38.44	38.21
2). Allowed MSE per eigenvector: 0.01, 0.1, 0.4, 100.0, 100.0, 100.0, 100.0, 100.0			
Bitrate	0.65	0.48	0.360
PSNR	38.30	37.93	38.21

Table 1. Comparison for two different MSE of eigenvectors 6, 7 and 8.

If the standard deviation of the source tile is low, the mean of the tile is subtracted and quantized to 8 bits. The quantized mean is part of the header of the encoded tile. The resulting tile is encoded using a 9/7 Daubechies filter bank factored using lifting steps to help reduce computational complexity [1]. Before each filtering stage, the tile, or the corresponding subband, is extended using symmetric periodic extension to reduce the block-artifact in the reconstructed tile [7] and to avoid coefficients expansion. Each 64x64 tile is decomposed into a maximum of three-levels; the level zero or last decomposition level is an 8x8 block. The subband coefficients are non-integer. Therefore they are rounded to the nearest integer, which causes a minimal loss of PSNR [8].

The 9/7 Daubechies filter bank was chosen because it is biorthogonal and uses symmetric odd length filters (linear phase), which is required to preserve the symmetry of the data along subbands. The transform is separable. Therefore, filtering along rows followed by filtering along columns is performed. The resulting subbands, after one, two or a maximum of three levels of decomposition, are encoded using HC-RIOT. Each encoded tile will start with a '1' if it was encoded using SVD or '0' if DWT was used. Choice of the DWT or SVD codebooks sizes for eigenvectors (also VQ or SQ) etc, have been decided after extensive simulations using the test data.

3.2 Decoder

At the decoder side the decision to decode using HC-RIOT [8] or inverse VQ/SQ is performed by reading the first bit of each encoded tile. If the bit is a '1' then the 8x8 eigenvectors and eigenvalue matrices $\boldsymbol{U}$, $\boldsymbol{V}^T$, and $\boldsymbol{W}$ are constructed. The reconstruction process starts by reading a second bit. If it is a '0' then the entry indices for the first codebook is recovered, if it is a '1' the quantized eigenvector is recovered and dequantized. This process is repeated for the second and third eigenvectors. The remaining eigenvectors are reconstructed by recovering the entry indexes as well as the mean value. This porcess is repeated until the 64x64 tile is recoverd.

If the first bit of the encoded tile is a '0' the decoding process uses HC-RIOT. The decoding process starts with the maximum threshold and after each pass the threshold is divided by 2 until the minimum threshold is reached. The entire process can be stopped after decoding the total number of bits indicated by the encoder or after reaching a desired distortion in the recovered 64x64 tile. A synthesis filter bank is applied to the recovered subbands in order to recover the tile. An approximation of the tile is recovered after adding back the mean, which is part of the header of the encoded tile.

4 RESULTS

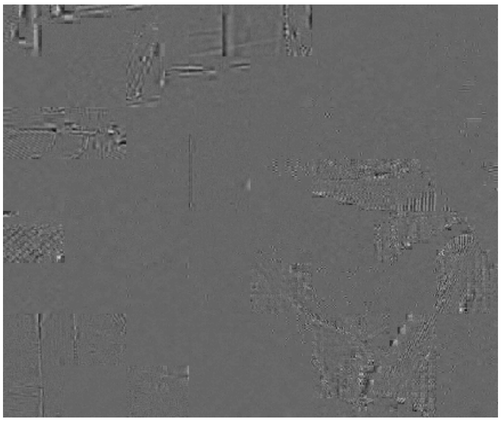
In this section we present the performance of the HDWTSVD and compare with the SVD and JPEG baseline. We used the PSNR as a measure of the distortion in terms of the mean square error (MSE).

$$\text{PSNR} = 10\log_{10}\frac{(255)^2}{\text{MSE}} \quad \text{dB} \qquad (8)$$

We used 512 x 512 8-bit monochromatic "Barbara" image". The recovered and the error images are shown in figure 2 for bit rates above and below 1 bpp.

(a)

(b)



(c)

(d)

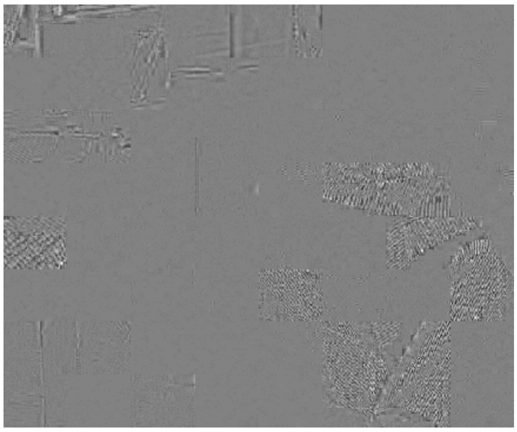


Figure 2. Recovered images of: (a) Barbara at 1.05 bpp, 35.56 dB and (b) error image, (b) Barbara at 0.51 bpp, 30.1 dB and (b) error image.

At low bit rate the contribution in PSNR is given by the area compressed using SVD and the low bit rate obtained is due to the wavelet transform in areas of high correlation.

Figure 3 shows the comparison between the modified HDWTSVD and the HDCTSVD using thresholding coding [5] for (a) Barbara and (b) Peppers.

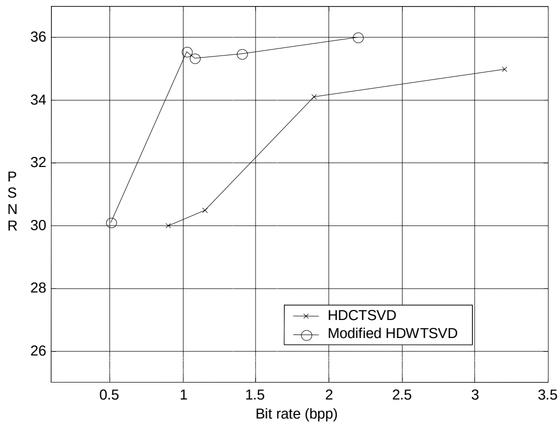


Figure 3. Comparison between the Modified HDWTSVD and HDCTSVD for Barbara.

6. CONCLUSIONS

We have presented a modified version of the original HDCTSVD system. This system gives better performance for low and high bit rates than its predecessor. Tiling an image is another way to take advantage on the correlation and combined techniques can result in high compression ratios while maintaining the image quality. We can achieve even more compression at low bit rates by implementing an adaptive multistage VQ to encode the eigenvectors. An adaptive multistage VQ will help us to reduce the bit rate at low resolutions in the areas compressed by SVD while keeping the same bit rate and quality of tiles compressed by DWT. This algorithm outperforms the previous HDCTSVD algorithm. In this research the PSNR is taken as a measurement of the distortion but is not an indication of the true image quality.

References:

[1] I. Daubechies and W. Sweldens, "Factoring Wavelet Transforms into Lifting Steps," Princeton University, Sept. 1996.

[2] A.K. Jain, Fundamentals of Digital Image Processing. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[3] C.M. Goldrick, W. Dowling, and A. Bury, "Image Coding Using the Singular Value Decomposition and Vector Quantization," in Image Processing and its Applications, pp.296-300, IEE, 1995.

[4] T. Saito and T. Komatsu, "Improvement on Singular Value Decomposition Vector Quantization," Electronics and Communications in Japan, Part 1, vol. 73, pp. 11-20, 1990.

[5] A. Dapena and S. Ahalt, "A Hybrid DCT-SVD Image-Coding Algorithm," IEEE Trans. CSVT, vol. 12, pp.114-121, Feb. 2002.

[6] H. Ochoa, K.R. Rao, "A Hybrid DWT-SVD Image-Coding System (HDWTSVD) for Monochromatic Images," WSEAS Transaction n Circuit and Systems, vol. 4, pp. 419 – 424, Dec. 2005.

[7] M.J. Smith and S. L. Eddins, "Analysis/Synthesis Techniques for Subband Image Coding," IEEE Trans. Acoust., Speech, and Signal Process., vol. 38, pp.1446-1456, July 1990.

[8] Y. F. Syed, A Low Bit Rate Wavelet-Based Image Coder for Transmission Over Hybrid Networks. Doctoral Dissertation, UTA, 1999.

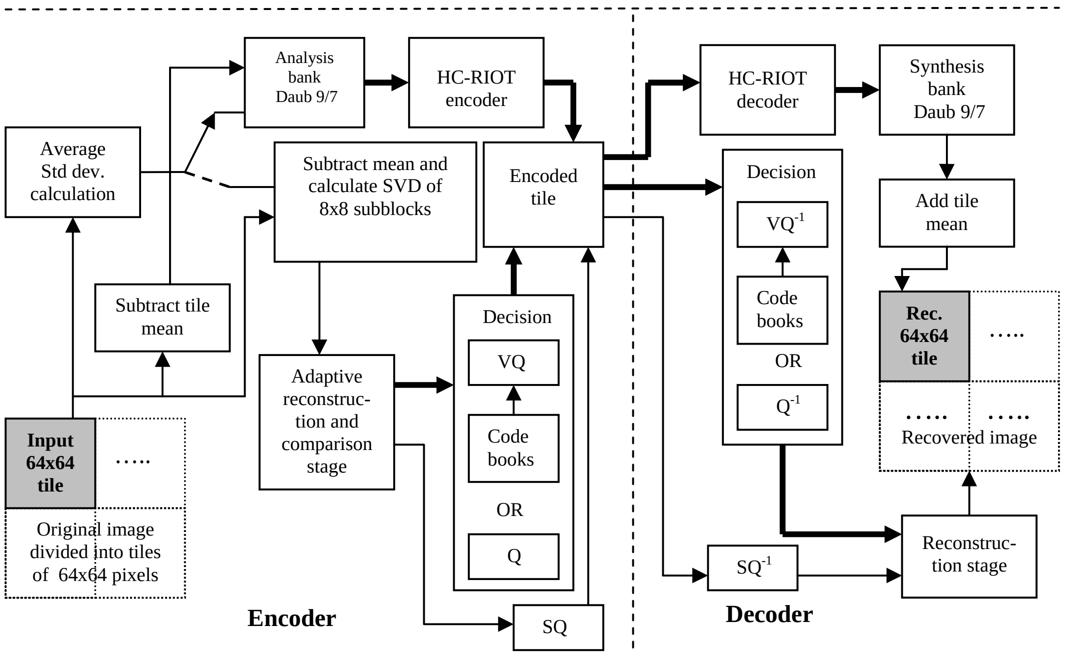


Figure 1. Modified hybrid DWT-SVD algorithm (HDWTSVD).