Proceedings of the 5th WSEAS International Conference on Applications of Electrical Engineering, Prague, Czech Republic, March 12-14, 2006 (pp24-29)

Alternative Newton-Raphson Power Flow Calculation in Unbalanced Three-phase Power Distribution Systems

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Abstract: - This paper proposes an alternative approach for Newton-Raphson power flow calculation, which is based on current-balanced equations rather than a widely-used power-balanced principle, especially for power distribution systems. This concept gives the replacement of the power-balanced equations that are broadly used by conventional Newton-Raphson power flow methods. The non-linear current equations can simplify very complicated power flow problems, however new mathematical derivation of Jacobian matrices is necessary. Although the power flow equations have been modified, the alternative power flow method still has quadratic convergence. It is sufficient to enhance calculation time required by the iterative processes. To distinguish the alternative method in comparison with the conventional Newton-Raphson, 25-bus and IEEE 37-node test feeders were tested. Moreover, the results reveal that solving the power flow problems with the alternative Newton-Raphson method can considerably reduce execution time consumed by simulation programs when comparing with the conventional methods.

Key-Words: - Power flow, Newton-Raphson method, Gauss-Seidel method, Fast-decouple method, Quadratic convergence, Electric power distribution system

1 Introduction

For many decades, electrical power systems have been analyzed using the bus reference frame approach [1]. Nodal analysis [2] is typically used to obtain voltage solutions. However, electrical demands or loads are usually defined in powers, this causes non-linearity of nodal voltage equations. Since simple methods to solve linear equations fail to handle this problem, some efficient numerical methods (e.g. Gauss-Seidel, Newton-Raphson, etc), have been used [1-3]. To date, there is no objection that the Newton-Raphson power flow method is one of the most powerful algorithms, which has long history of development [4-8], and is widely used to develop commercial power-flow solution software across the world.

Although the classical Newton-Raphson method is very efficient and becomes the standard for the power flow calculation in several power system textbooks, to formulate a matrix equation requires tedious and complicated mathematical expressions. In this paper, the Newton-Raphson method is still applied as the main numerical framework. The key difference is that the mismatch to formulate the matrix equation is derived directly from current-balanced equations rather than the power-balancing [9]. This concept simplifies very long mathematical formulae to very simplistic ones. With this simplification, reduction of the overall execution time for power flow problem solving is expected.

In this paper, electric power distribution systems are of our focuses [10-13]. They have special features of radial feed configuration to distinguish them from electric power transmission systems. Besides feeding arrangement, unbalanced load services also make them special and need particular derivation of solution methods. Per-unit power flow calculation cannot be applied unlike the transmission systems.

To handle power distribution power flow problems, an intensified mathematical expression of power flow analysis is required. Therefore, in this paper the Newton-Raphson method will be modified to be able and to be suitable for solving unbalanced three-phase distribution power flow problems.

These are all in Section 2 and 3. Assessment of numerical computation resulting from the use of the developed method is carried out and then compares with the assessment of those obtained from the classical Newton-Raphson method. These assessment and comparison to confirm the effectiveness of the proposed method are in Section 4. Conclusion and discussion leading to further works are provided in Sections 5.

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2 Three-phase Power System Models

In power distribution systems, one node or so-called bus consisting of three separate busbars of phases A, B and C. However, in many feeder portions where their far-end loads require a single-phase supply, it is unnecessary to build complete three-phase power lines for them. A pair of lines will be used, therefore at the end of this feeder section only two busbars exist. With an additional assumption of earth return, any customer load may consist of a single busbar, double busbars or triple busbars. This leads to a variety of feeder models as shown in Fig. 1.

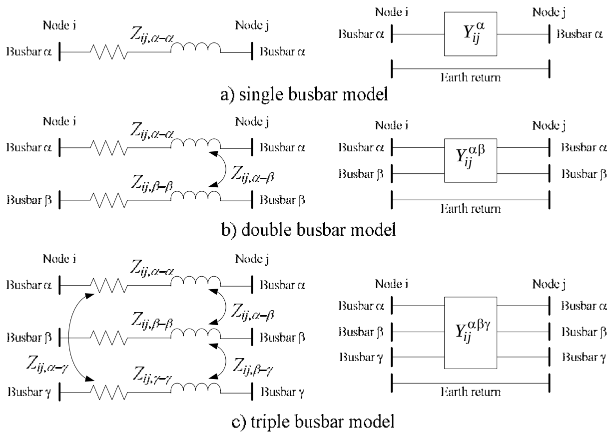

Fig. 1 Modeling of a feeder portion

Different busbar configuration of the two ends of any feeder causes different size of an admittance matrix characterizing the line portion. In the figure, α, β and γ are phase indices and they must represent one of {A, B, C}. With appropriate algebraic techniques of matrix, these matrices with different size can be rewritten into three-by-three matrices as shown in Equations 1 – 3, for single, double and triple busbar configuration, respectively. Consequently, when the system bus admittance matrix is successfully formulated, non-existing busbars can be eliminated by Kron's matrix reduction technique.

$$Y_{ij}^{\alpha} = \left[y_{ij}^{\alpha\alpha} \right] \rightarrow Y_{ij}^{\alpha\beta\gamma} = \begin{bmatrix} y_{ij}^{\alpha\alpha} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

$$Y_{ij}^{\alpha\beta} = \begin{bmatrix} y_{ij}^{\alpha\alpha} & y_{ij}^{\alpha\beta} \\ y_{ij}^{\beta\alpha} & y_{ij}^{\beta\beta} \end{bmatrix} \rightarrow Y_{ij}^{\alpha\beta\gamma} = \begin{bmatrix} y_{ij}^{\alpha\alpha} & y_{ij}^{\alpha\beta} & 0 \\ y_{ij}^{\beta\alpha} & y_{ij}^{\beta\beta} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

$$Y_{ij}^{\alpha\beta\gamma} = \begin{bmatrix} y_{ij}^{\alpha\alpha} & y_{ij}^{\alpha\beta} & y_{ij}^{\alpha\gamma} \\ y_{ij}^{\beta\alpha} & y_{ij}^{\beta\beta} & y_{ij}^{\beta\gamma} \\ y_{ij}^{\gamma\alpha} & y_{ij}^{\gamma\beta} & y_{ij}^{\gamma\gamma} \end{bmatrix} \tag{3}$$

One of special feature of the distribution power system is to have only one feeding point that is the substation. Although many types of bus arrangement are possible, the substation for power flow problem is pimply a current source in parallel with admittance as shown in Fig. 2.

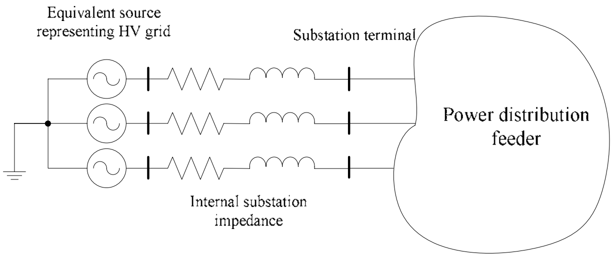

Fig. 2 Modeling of a power substation

Unlike the feeder models, the power substation consists of all three busbars, therefore a voltage vector representing HV grid is a 3×1 matrix, while its admittance is also a 3×3 matrix.

Customer demands are normally modeled as power loads. Any spot load can be single-phase, two-phase or three-phase. However the spot load draws power from busbar(s), therefore only a 3×1 column vector is sufficient to representing such a load, see Fig. 3 for detail.

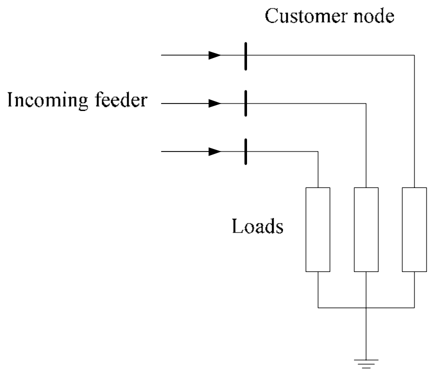

Fig. 3 Modeling of a spot load

3 Problem Formulation

A power distribution system defines as a set of several interconnected elements through between a pair of nodes as shown in Fig. 4. For simplification, most electric power apparatus in power distribution network can be classified into three major types, which are i) power source, ii) feeder line and iii) load. To analyze system characteristics, nonlinear nodal analysis is employed to formulate a set of complex power flow equations as shown in Equation 4. Also, these equations can be decomposed into real and reactive power equations as in Equations 5 and 6.

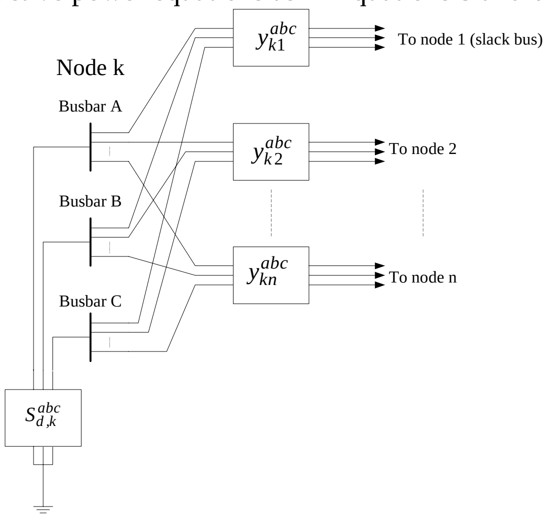

Fig. 4 Three-phase node in distribution systems

$$S_{sch,k}^{abc\,*} = P_{sch,k}^{abc} - jQ_{sch,k}^{abc} = V_k^{abc}\sum_{i=1}^{n} Y_{ki}^{abc}V_i^{abc} \quad (4)$$

$$P_{cal,k}^{p} = \sum_{i=1}^{n}\sum_{\phi=a}^{c}\left|Y_{ki}^{p\phi}V_k^{p}V_i^{\phi}\right| cos\left(\theta_{ki}^{p\phi} + \delta_i^{\phi} - \delta_k^{p}\right) \quad (5)$$

$$Q_{cal,k}^{p} = -\sum_{i=1}^{n}\sum_{\phi=a}^{c}\left|Y_{ki}^{p\phi}V_k^{p}V_i^{\phi}\right| sin\left(\theta_{ki}^{p\phi} + \delta_i^{\phi} - \delta_k^{p}\right) \quad (6)$$

where,

$S_{sch,k}^{abc} = S_{gen,k}^{abc} - S_{d,k}^{abc}$

$P_{sch,k}^{abc} = P_{gen,k}^{abc} - P_{d,k}^{abc}$

$Q_{sch,k}^{abc} = Q_{gen,k}^{abc} - Q_{d,k}^{abc}$

$S_{sch,k}^{abc}$ is scheduled complex power

$P_{sch,k}^{abc}$ is scheduled real power

$Q_{sch,k}^{abc}$ is scheduled reactive power

$S_{d,k}^{abc}$ is demand complex power

$P_{cal,k}^{abc}$ is calculated real power

$Q_{cal,k}^{abc}$ is calculated reactive power

V_k^{abc} is a three-phase voltage vector

Y_{ki}^{abc} is an $\mathrm{k^{th}}$ – column, $\mathrm{i^{th}}$ – row of $\left[Y_{bus}^{abc}\right]$

$p, \phi \in \{a, b, c\}$

The Proposed power flow method simplifies these equations by rearranging into Equations 7 – 9.

$$\left(\frac{S_{sch,k}^{abc}}{V_k^{abc}}\right)^* = I_k^{abc} = F_k^{abc} + jH_k^{abc} = \sum_{i=1}^{n} Y_{ki}^{abc}V_i^{abc} \quad (7)$$

$$F_{cal,k}^{p} = \sum_{i=1}^{n}\sum_{\phi=a}^{c}\left|Y_{ki}^{p\phi}V_i^{\phi}\right| cos\left(\theta_{ki}^{p\phi} + \delta_i^{\phi}\right) - \left|\frac{S_{sch,k}^{p}}{V_k^{p}}\right| cos\left(\delta_k^{p} - \varphi_k^{p}\right) \quad (8)$$

$$H_{cal,k}^{p} = \sum_{i=1}^{n}\sum_{\phi=a}^{c}\left|Y_{ki}^{p\phi}V_i^{\phi}\right| sin\left(\theta_{ki}^{p\phi} + \delta_i^{\phi}\right) - \left|\frac{S_{sch,k}^{p}}{V_k^{p}}\right| sin\left(\delta_k^{p} - \varphi_k^{p}\right) \quad (9)$$

where,

I_k^{abc} is mismatched complex current

$F_{cal,k}^{abc}$ is calculated real current

$H_{cal,k}^{abc}$ is calculated reactive current

Therefore, current mismatch equations are used to formulate the proposed Newton-Raphson updating equations as follows.

$$\begin{bmatrix}\Delta F \\ \Delta H\end{bmatrix} = \begin{bmatrix}\frac{\partial F}{\partial \delta} & \frac{\partial F}{\partial V} \\ \frac{\partial H}{\partial \delta} & \frac{\partial H}{\partial V}\end{bmatrix}\begin{bmatrix}\delta \\ |V|\end{bmatrix} = \begin{bmatrix}J_1 & J_2 \\ J_3 & J_4\end{bmatrix}\begin{bmatrix}\delta \\ |V|\end{bmatrix} \quad (10)$$

To update node-voltage vectors, elements of the Jacobian matrix must be calculated. Without provision of any mathematical derivation herein, Jacobian sub-matrices can be expressed as follows.

<u>Sub-matrix J_1</u>:

$$J_{1,kk}^{pp} = \frac{\partial F_k^{p}}{\partial \delta_k^{p}} = -\left|Y_{kk}^{pp}V_k^{p}\right| sin\left(\theta_{kk}^{pp} + \delta_k^{p}\right) + \left|\frac{S_{sch,k}^{p}}{V_k^{p}}\right| sin\left(\delta_k^{p} - \varphi_k^{p}\right) \quad (11a)$$

$$J_{1,ki}^{p\phi} = \frac{\partial F_k^{p}}{\partial \delta_i^{\phi}} = -\left|Y_{ki}^{p\phi}V_i^{\phi}\right| sin\left(\theta_{ki}^{p\phi} + \delta_i^{\phi}\right) \quad (11b)$$

(11b) is for (i = k and $p \neq \phi$).

Sub-matrix J_2:

$$J_{2,kk}^{pp} = \frac{\partial F_k^p}{\partial V_k^p} = \left|Y_{kk}^{pp}\right| cos\left(\theta_{kk}^{pp} + \delta_k^p\right) + \frac{\left|S_{sch,k}^p\right|}{\left|V_k^p\right|^2} cos\left(\delta_k^p - \varphi_k^p\right) \quad (12a)$$

$$J_{2,ki}^{p\phi} = \frac{\partial F_k^p}{\partial V_i^\phi} = \left|Y_{ki}^{p\phi}\right| cos\left(\theta_{ki}^{p\phi} + \delta_i^\phi\right) \quad (12b)$$

(12b) is for ($i = k$ and $p \neq \phi$).

Sub-matrix J_3:

$$J_{3,kk}^{pp} = \frac{\partial H_k^p}{\partial \delta_k^p} = \left|Y_{kk}^{pp} V_k^p\right| cos\left(\theta_{kk}^{pp} + \delta_k^p\right) - \left|\frac{S_{sch,k}^p}{V_k^p}\right| cos\left(\delta_k^p - \varphi_k^p\right) \quad (13a)$$

$$J_{3,ki}^{p\phi} = \frac{\partial H_k^p}{\partial \delta_i^\phi} = \left|Y_{ki}^{p\phi} V_i^\phi\right| cos\left(\theta_{ki}^{p\phi} + \delta_i^\phi\right) \quad (13b)$$

(13b) is for ($i = k$ and $p \neq \phi$).

Sub-matrix J_4:

$$J_{4,kk}^{pp} = \frac{\partial H_k^p}{\partial V_k^p} = \left|Y_{kk}^{pp}\right| sin\left(\theta_{kk}^{pp} + \delta_k^p\right) + \frac{\left|S_{sch,k}^p\right|}{\left|V_k^p\right|^2} sin\left(\delta_k^p - \varphi_k^p\right) \quad (14a)$$

$$J_{4,ki}^{p\phi} = \frac{\partial H_k^p}{\partial V_i^\phi} = \left|Y_{ki}^{p\phi}\right| sin\left(\theta_{ki}^{p\phi} + \delta_i^\phi\right) \quad (14b)$$

(14b) is for ($i = k$ and $p \neq \phi$).

With this computation, voltage magnitudes and phases can be updated iteratively by using the following equation where h indicates a counter for iteration.

$$\begin{bmatrix} \delta \\ |V| \end{bmatrix}^{(h+1)} = \begin{bmatrix} \delta \\ |V| \end{bmatrix}^{(h)} + \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta F \\ \Delta H \end{bmatrix}^{(h)} \quad (15)$$

In addition, a power flow solution framework cane be summarized in flow diagram of Fig. 5.

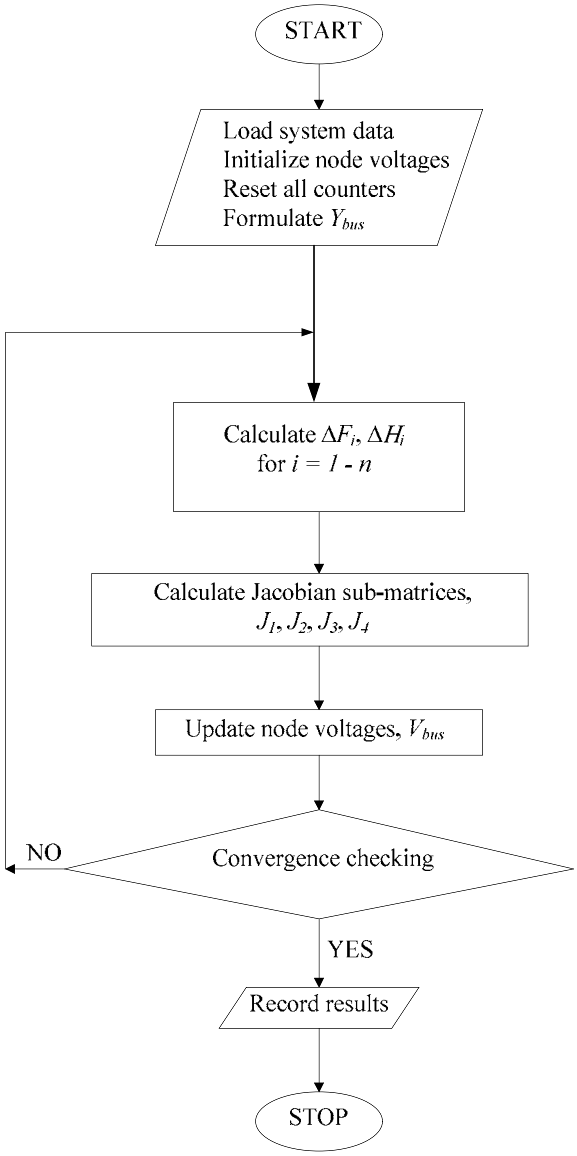

Fig. 5 Flow diagram for power flow calculation

4 Simulation Results

The effectiveness of the alternative Newton-Raphson power flow calculation was tested against 25-bus and IEEE 37-node test systems as shown in Figs 6 – 7, respectively. Table 1 shows total loads of each phase for each test system. The tests were performed by using a 2.4-GHz, 512-SDRAM Pentium 4 computer in which the power flow calculation programs were coded in MATLAB™.

Table 1 System loads for each test case

Test system	System load		
	Phase a	Phase b	Phase c
25 bus	513 kW 385 kvar	473 kW 355 kvar	493 kW 370 kvar
37 bus	727 kW 357 kvar	639 kW 314 kvar	1091 kW 530 kvar

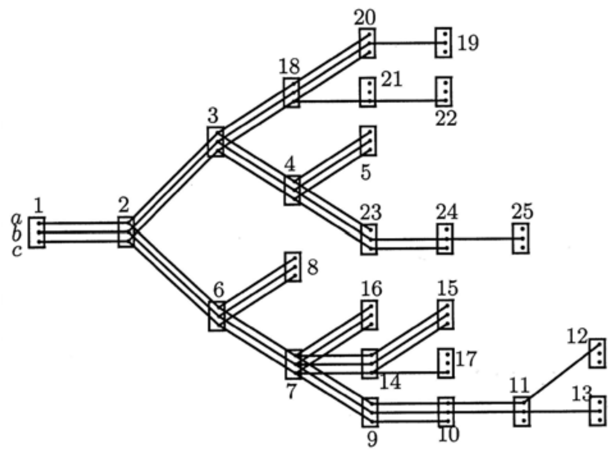

Fig. 6 25-bus test system [11]

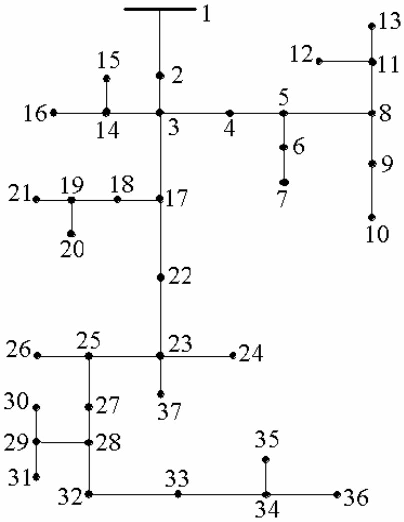

Fig. 7 IEEE 37-node test feeder [14]

To perform the tests, all initial node voltages are assumed to be 1.0 p.u. and to terminate iterative processes, maximum voltage error is set as 1×10^{-6} p.u. for both power flow methods. The results obtained are presented in Table 2.

Table 2 Simulation results

Test system	Iteration used		Execution time	
	SNR	ANR	SNR	ANR
25-bus	4	4	100%	86%
37-bus	4	4	100%	84%

* SNR denotes Standard Newton-Raphson method
ANR denotes Alternative Newton-Raphson method

As a result, the alternative Newton-Raphson power flow calculation can reduce calculation time by 15% (average value). The number of iteration used is equal for both methods. This implies that their convergence property is quite similar. The readers can observe their convergences for each test case in Figs 8 – 11.

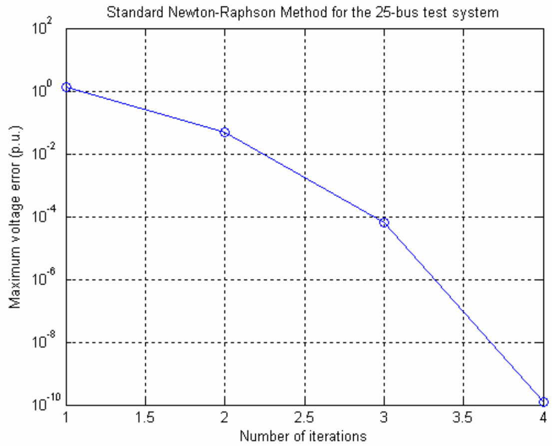

Fig. 8 Convergence of SNR for the 25-bus system

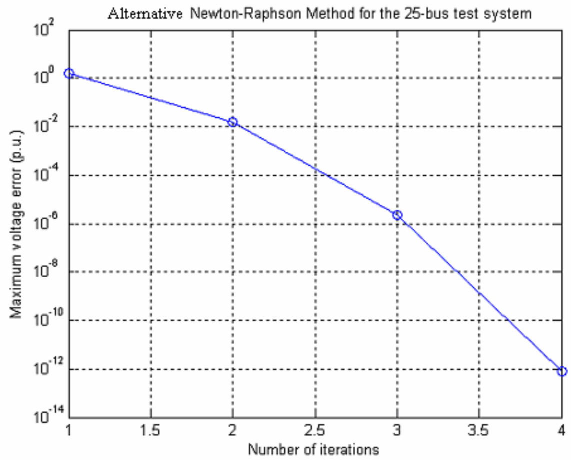

Fig. 9 Convergence of ANR for the 25-bus system

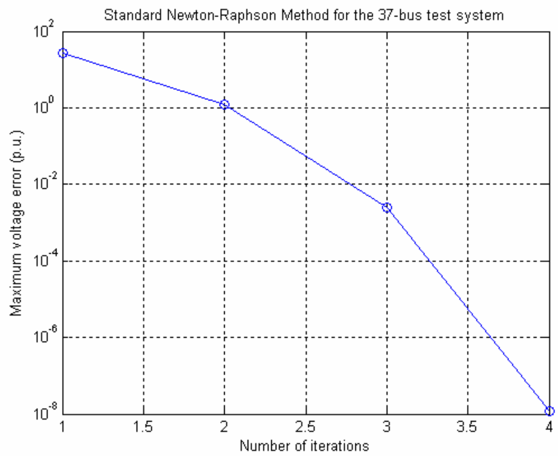

Fig. 10 Convergence of SNR for the 37-bus system

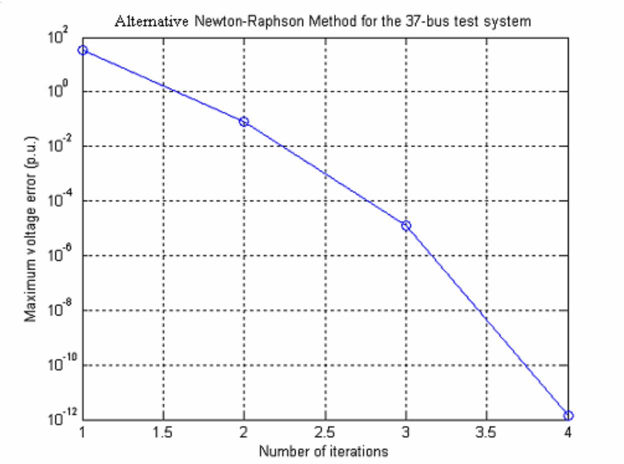

Fig. 11 Convergence of ANR for the 37-bus system

5 Conclusions

This paper proposes an alternative approach for Newton-Raphson power flow calculation, especially in electric power distribution systems. The developed method is based on the nonlinear current-balanced equations, where the derivation of Jacobian matrices and their elements are fully provided. With its simpler updating formulae, shorter execution time consumed is expected. As confirmed by simulation results, the calculation time can be reduced by 15% of the time used by the standard Newton-Raphson method. This advantage can lead to great improvement of power-flow software development in fast computational speed and effective memory usage.

6 Acknowledgement

The authors would like to acknowledge the financial support of the research grant (SUT7-711-47-12-52) from Suranaree University of Technology, Nakhon Ratchasima, THAILAND, during a period of this work.

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