Proceedings of the 5th WSEAS International Conference on Applications of Electrical Engineering, Prague, Czech Republic, March 12-14, 2006 (pp84-89)

Optimizing Voltage-Frequency Control Strategy for Single-Phase Induction Motor Drives

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Abstract: - Single-phase induction motors are widely used in household electric motor applications. Although most electric appliances require a few amount of kilo-watt input, minimizing power losses during their operation gives a great benefit resulting in nationwide electric energy used by householders. In this paper, a model-based simulation for loss minimization using the space-phasor theory is proposed. Instead of a fixed 220 V, 50 Hz AC power source, a variable-voltage,variable-frequency (VVVF) control scheme is assumed to function the AC supply. By formulating and solving a constrained optimization of single-phase induction motor drives with VVVF, operation of the single-phase induction motor with minimum power losses can be attained. To evaluate this control strategy, test cases of a single-phase induction motor drive with variable mechanical loads are situated to evaluate its performance.

Key-Words: - single-phase induction motor, optimization, variable-voltage-variable-frequency control, sequential quadratic programming, loss minimization, space-phasor theory

1 Introduction

To date, three-phase induction motors have been increasingly important for industrial electric motor applications. It should note that there still exist DC motors in some limited applications, e.g. motors for vehicles. Apart from a large-size electric motor drive, single-phase induction motors are widely used in household electric motor applications. This application typically consumes power of a fractional horse power up to around ten horse powers. Although most electric appliances require a few amount of kilo-watt input, minimizing power losses during their operation gives a great benefit resulting in nationwide electric energy used by householders.

In general, single-phase motors are controlled by a thyristor-phase controller or a variable resistor. This is quite simple, but it is not efficient in terms of energy consumption. To achieve this goal, complex control strategy cannot be avoid as long as ac machines are involved. One of widely-used control schemes is variable-voltage, variable-frequency (VVVF) [1,2]. It can be applied for motor control in many forms. For example, the constant volt-per-hertz scheme is one of the most popular methods used for practical implementation. However, following this control strategy does not guarantee minimum loss operation. Therefore, adjustable frequency and voltage of the power supply is more flexible and can lead to more economical operation of household electric appliances.

In this paper, the control scheme of VVVF in order to minimize total power losses in a single-phase induction motor. There are five main sections to illustrate the proposed strategy. Section 2 is Modeling of Single-phase Induction Motors, Section 3 is Problem Formulation, Section 4 is Simulation Results and Section 5, the last section, is Conclusion.

2 Modeling of Single-phase Induction Motors

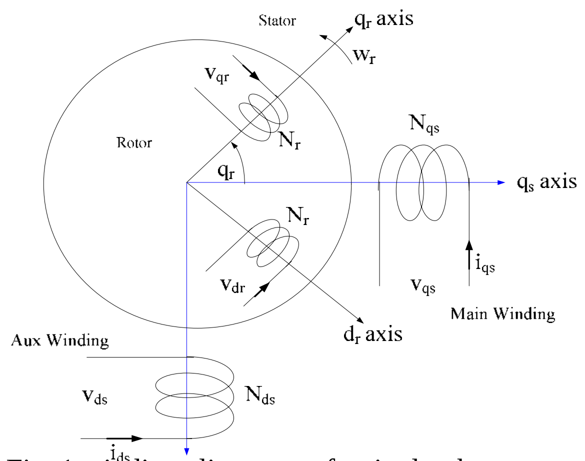

Fig. 1 winding alignment of a single-phase motor

Single-phase induction motors can be characterized by several different models. The space-phasor approach [2,3] is the method used in this paper. With this model, motor currents, torque and speed can be observable. The space-phasor model is very complicated and needs more space for explanation. However, in this paper only a brief description is presented as follows.

Figure 1 describes winding alignment of a single-phase induction motor consisting of main and auxiliary windings with their induced voltages and currents. As shown in the figure, a stationary reference frame which is along the axis of the main stator winding is defined and used for mathematical analysis throughout this paper. It is essential to inform that all quantities especially on the rotor need to be transferred to the stator axis. This can be performed by using the following transform matrix.

$$\begin{bmatrix} V_{qr}^{s} \\ V_{dr}^{s} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} v_{qr}^{r} \\ v_{dr}^{r} \end{bmatrix} \tag{1}$$

The superscripts *s* and *r* indicate the reference axis in which the variable belongs to. Briefly, the induced voltages on the stator and rotor windings are summarized in Equations 2 and 3, respectively

$$\begin{bmatrix} V_{qs}^{s} \\ V_{ds}^{s} \end{bmatrix} = \begin{bmatrix} r_{qs} + pL_{qsqs} & pL_{qsds} \\ pL_{dsqs} & r_{ds} + pL_{dsds} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} + \begin{bmatrix} L_{qsqr} & L_{qsdr} \\ L_{dsqr} & L_{dsdr} \end{bmatrix} p \begin{bmatrix} i_{qr}^{r} \\ i_{dr}^{r} \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} V_{qr}^{r} \\ V_{dr}^{r} \end{bmatrix} = \begin{bmatrix} r_{qr} + pL_{qrqr} & pL_{qrdr} \\ pL_{drqr} & r_{dr} + pL_{drdr} \end{bmatrix} \begin{bmatrix} i_{qr}^{r} \\ i_{dr}^{r} \end{bmatrix} + \begin{bmatrix} L_{qrqs} & L_{qrds} \\ L_{drqr} & L_{drds} \end{bmatrix} p \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \end{bmatrix} \tag{3}$$

By using the transform matrix mentioned above, all state variables can be transformed into the stator direct axis as follows.

$$\frac{d}{dt}[i] = [A][i] + [B][v] \tag{4}$$

where,

$$[A] = [D]^{-1}[E]$$

$$[B] = [D]^{-1}$$

$$[i] = [i_{qs}, i_{ds}, i'_{qr}, i'_{dr}]^T$$

$$[v] = [v_{qs}, v_{ds}, v'_{qr}, v'_{dr}]^T$$

$$[E] = \begin{bmatrix} -r_{qs} & 0 & \omega_r L_{mqs}\sin\theta_r & \omega_r L_{mqs}\cos\theta_r \\ 0 & -r'_{ds} & -\omega_r L_{mqs}\cos\theta_r & \omega_r L_{mqs}\sin\theta_r \\ \omega_r L_{mqs}\sin\theta_r & -\omega_r L_{mqs}\cos\theta_r & -r'_r & 0 \\ \omega_r L_{mqs}\cos\theta_r & \omega_r L_{mqs}\sin\theta_r & 0 & -r'_r \end{bmatrix}$$

$$[D] = \begin{bmatrix} L'_{lqs} + L_{mqs} & 0 & L_{mqs}\cos\theta_r & -L_{mqs}\sin\theta_r \\ 0 & (L'_{lds} + L_{mqs}) & L_{mqs}\sin\theta_r & L_{mqs}\cos\theta_r \\ L_{mqs}\cos\theta_r & L_{mqs}\sin\theta_r & (L'_{lr} + L_{mqs}) & 0 \\ -L_{mqs}\sin\theta_r & L_{mqs}\cos\theta_r & 0 & (L'_{lr} + L_{mqs}) \end{bmatrix}$$

where,

r_{qs} is stator resistance of the main winding
r_{ds} is stator resistance of auxiliary winding
L_{ls} is leakage inductance of the main winding
L_{lS} is leakage inductance of the auxiliary winding
L_{mqs} is mutual inductance on the stator q-axis
r_r is rotor resistance
L_{lr} is leakage inductance of the rotor q-axis

As can be seen, the two mechanical quantities, ω_r and θ_r, cause the need for additional two equations which can be obtained from Newton's second law of motion as shown in Equation 5.

$$\begin{bmatrix} \frac{d\omega_r}{dt} \\ \frac{d\theta_r}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{B_m}{J_m} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_r \\ \theta_r \end{bmatrix} + \begin{bmatrix} \frac{P}{2J_m} \\ 0 \end{bmatrix} [T_e - T_L] \tag{5}$$

Combine Equations 4 and 5, Equation 6 is formed.

$$\begin{bmatrix} \frac{d[i]_{4x1}}{dt} \\ \frac{d\omega_r}{dt} \\ \frac{d\theta_r}{dt} \end{bmatrix} = \begin{bmatrix} [A]_{4x4} & \cdots\ 0\ \cdots & \\ \vdots & -\frac{B_m}{J_m} & 0 \\ 0 & & \\ \vdots & 1 & 0 \end{bmatrix} \begin{bmatrix} [i]_{4x1} \\ \omega_r \\ \theta_r \end{bmatrix} + \begin{bmatrix} [B]_{4x4} & \cdots\ 0\ \cdots & \\ \vdots & \frac{P}{2J_m} & 0 \\ 0 & & \\ \vdots & 0 & 0 \end{bmatrix} \begin{bmatrix} [v]_{4x1} \\ T_e - T_L \\ 0 \end{bmatrix} \tag{6}$$

where,

J_m is motor's moment of inertia
B_m is damping coefficient

Applying a numerical time-stepping method to solve a set of differential equations, motor currents, angular speed and position can be calculated numerically.

3 Problem Formulation

To minimize power losses during the operation of a single-phase induction motor drive, a constrained nonlinear optimization problem must be formulated and then solved for a local minimum. Before

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discussion of the optimization framework, the power losses resulting from any operating point can be expressed by Equations 7 and 8.

$$P_{loss} = P_{cu} + P_{rot} \quad (7)$$
$$P_{cu} = r_{qs} i_{qs}^2 + r_{ds} i_{ds}^2 + r_r i_{qr}^2 + i_{dr}^2 \quad (8)$$

where,

P_{cu} denotes motor copper losses
P_{rot} denotes rotational losses of the motor

It notes that in this calculation the rotational loss is assumed to be constant and simply equal to no-load losses.

In order to minimize the power losses, VVVF control strategy is used. This gives two control variables, frequency (f) and voltage (V). By adjusting these parameters independently, a set of optimal values for frequency and voltage supply to the motor at any particular load is achieved via an appropriate optimization technique. Without provision of optimization techniques, a general form of loss minimization for a problem of single-phase induction motor drives is summarized as follows.

$$\begin{aligned} &\textit{Minimize} && P_{loss} = F(f, V) \\ &\textit{Subject to} && f_{min} \le f \le f_{max} \\ & && V_{min} \le V \le V_{max} \end{aligned}$$

To solve the optimization problem, a sequential quadratic programming (SQP) [4] is recommended. If tools for computer simulation are prompt to be used, the optimization framework with the SQP can be described step-by-step as follows.

START
Step 0:
All input data are loaded.
All variables are initial. $x = [f_0 \;\; V_0]^T$
Set counters

Step 1:
If any stopping criterion has not been met
Then go to step 2, *otherwise* step 3

Step 2:
Approximate the nonlinear optimization problem by using quadratic sub-problem representing by gradient and Hessian of the objective function as follows

$$\textit{Minimize} \qquad \tfrac{1}{2} d_k^T H_k d_k + \nabla f_k^T d_k$$

where,
d_k is a descent search direction
∇f_k is the gradient of the objective function
H_k is the Hessian matrix

Subsequently, the approximated quadratic problem is solved for an optimal search direction d_k at each iteration, consecutively.
Update for a solution of the next iteration by
$x_{k+1} = x_k + d_k$
Return to step 1

Step 3:
Display results

STOP

4 Simulation Results

In this section, demonstration by simulation is illustrated. A single-phase induction motor is assumed to be operated with an adjustable AC power supply source. The voltage magnitude and frequency of the supply are both adjustable (50 – 250 V_{rms}, and 10 – 100 Hz). Also, it is an ideal sinusoidal voltage source. For the test motor, parameters [5] are given below.

P = 4 poles
r_{qs} = 1.3 Ω
r_{ds} = 2.6 Ω
r_r = 2.01 Ω
L_{mqs} = 0.2785 H
L_{lr} = 0.0074 H
L_{lqs} = 0.0053 H
L_{lds} = 0.0074 H
J_m = 0.1
B_m = 0.005

In this paper, MATLAB optimization toolbox is used as an optimizer to seek an optimal operating point in association with mechanical load torques. Four load conditions are situated for demonstration as given below.

Load 1:	4 N·m
Load 2:	8 N·m
Load 3:	12 N·m
Load 4:	16 N·m

These loads are constant torque and assume to be applied on the motor shaft at t = 1.0 second after start. Let f = 50 Hz and V = 220 V_{rms} be the initial solution for all load scenarios. The optimal solutions obtained by the SQP optimizer of MATLAB toolbox (*fmincon* function) for each load scenario are shown in Table 1.

Table 1 Optimal solution for each load condition

Load (N.m)	Power losses (W)	Optimal solution		Iteration used
		Voltage (V)	Frequency (Hz)	
4.0	626.93	219.42	75.00	9
8.0	782.58	218.41	75.00	10
12.0	859.90	229.90	62.50	7
16.0	1029.90	220.38	56.25	17

For clarification, comparison between the base case (220 V, 50 Hz power supply) against the optimal operating point presented in Table 1 is in Table 2.

Table 2 Comparison between the base case and the optimal operating point

Load (N.m)	Base case Loss	Minimum power loss	Power loss reduction
4	758.72 W	626.93 W	17.37%
8	819.65 W	782.58 W	4.52%
12	921.77 W	859.90 W	6.71%
16	1070.20 W	1029.90 W	3.77%

It is the fact that most motors are typically designed to have the maximum efficiency at its full-load condition. The reader can observe that the test motor also behaves in this manner. The full-load torque of this motor is 16.0 N·m, as a result this operating point is close to the optimum (only 3.77% of the power losses can be reduced). In addition, Figs 1 – 4 show convergences for each test case. These graphs illustrate a sequence of solution movement, iteration-by-iteration, until the local minimum has been reached.

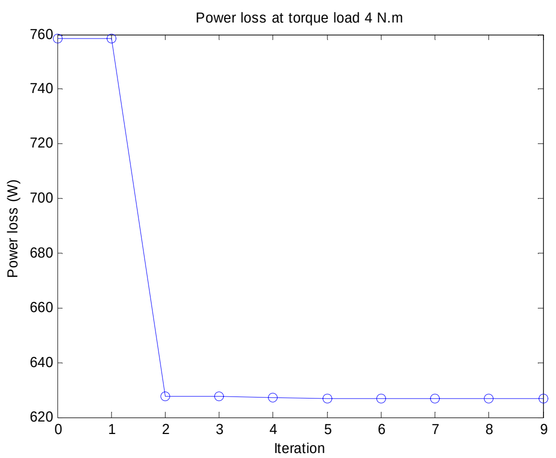

Fig. 1 Convergence for 4 N·m load condition

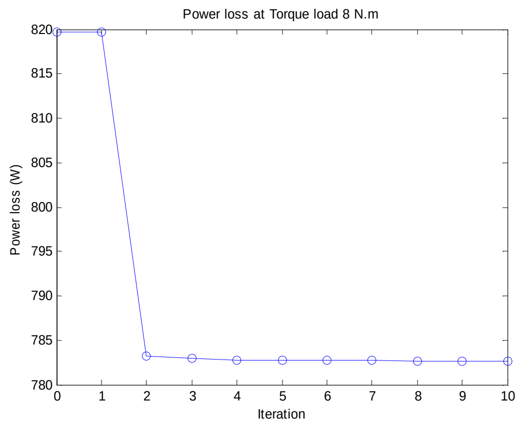

Fig. 2 Convergence for 8 N·m load condition

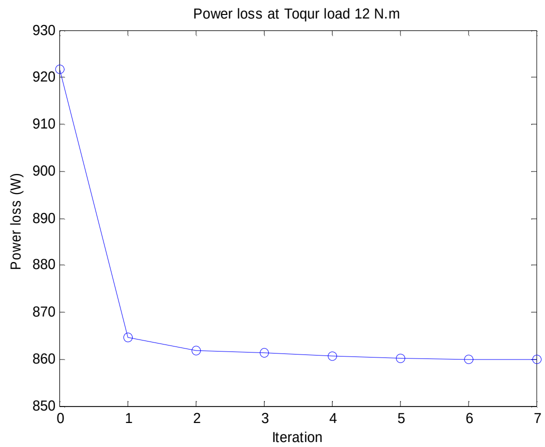

Fig. 3 Convergence for 12 N·m load condition

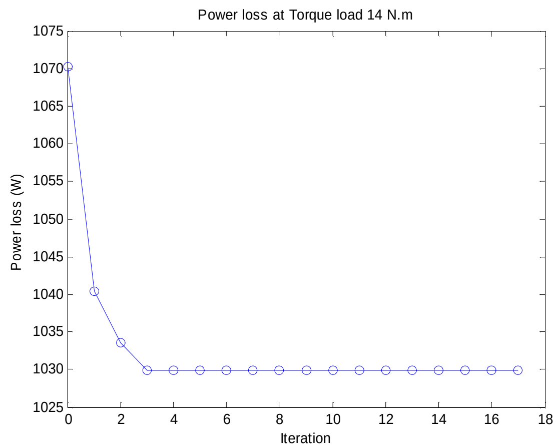

Fig. 4 Convergence for 16 N·m load condition

To observe motor characteristics, comparison between base-case operation and optimal operation is performed. Stator winding currents, mechanical torque and speed responses of the test motor under both supply conditions are depicted in Figs 5 – 11. Figs 5 – 7 present the responses of the base case while Figs 8 – 10 illustrate those of the optimal operation. Furthermore, Fig. 11 is the comparison between the supply current of the base case and the optimal operation.

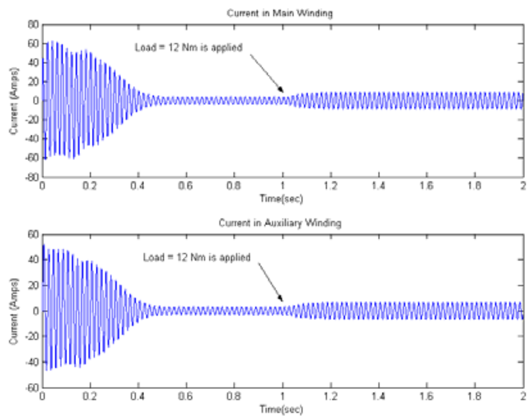

Fig. 5 Winding currents of the base case

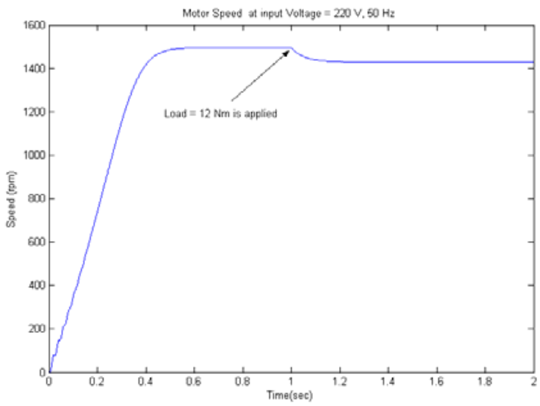

Fig. 6 Speed of the base case

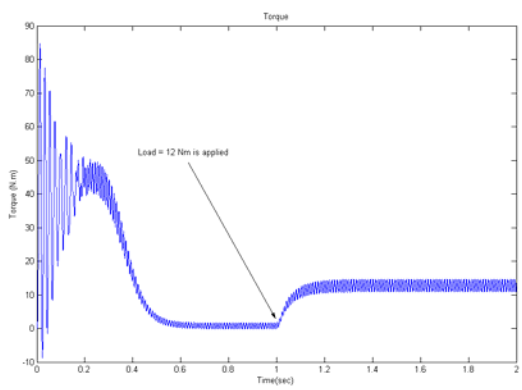

Fig. 7 Torque of the base case

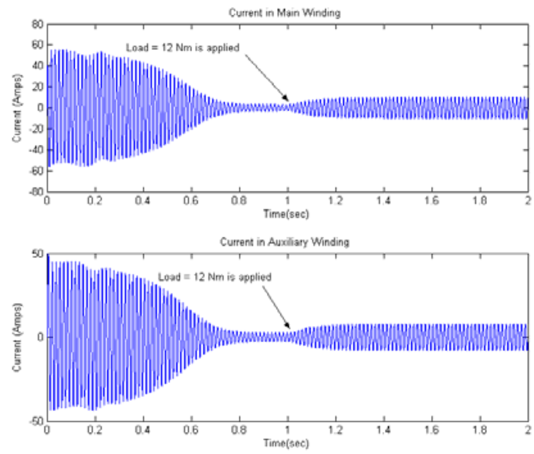

Fig. 8 Winding currents of the optimal

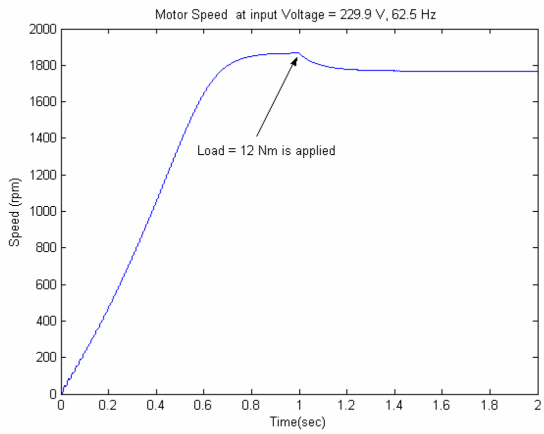

Fig. 9 Speed of the optimal

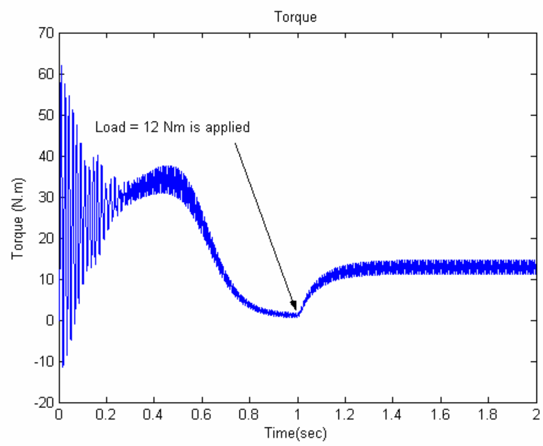

Fig. 10 Torque of the optimal

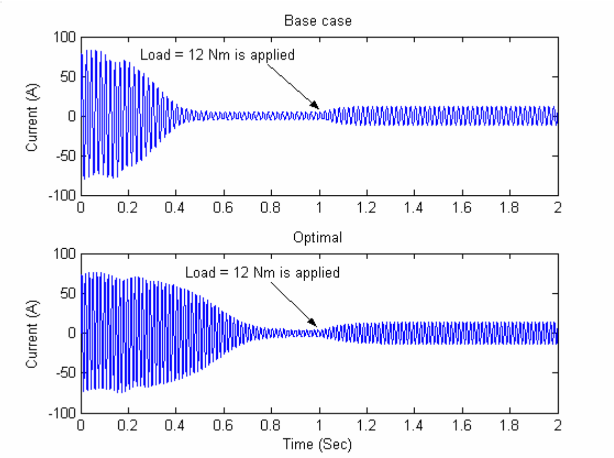

Fig. 11 Supply currents of both supply conditions

The results can confirm that with careful design of the supply source single-phase induction motor drives are loss-minimized in order to cut-off electric energy used by consumer loads in residential section, office or small-size factory. Furthermore, the proposed control scheme gives a great advantage when the motor drive serves a partial load. This can guarantee the operation with minimum power losses.

5 Conclusion and Future Work

This paper presents a voltage-frequency control strategy through a constrained optimization in order to minimize power losses of a single-phase induction motor drive. By adjusting the supply voltage and frequency, this scheme is able to provide the supply condition suitable for any particular load torque. Not only the full-load operation but also any partial load of the system, the energy losses are minimized effectively. With MATLAB optimization toolbox, relevant constrained optimization problems can be solved easily.

This work reveals that there exists at least one operating point which is said to be a local minimum for loss minimization problems. This point gives minimum power loss for a single-phase induction motor drive. However, to generate the sinusoidal power supply with optimal voltage and frequency according to any specified load is too expensive and impractical. To enhance this research and to bring this control strategy for practical implementation, a square-wave inverter is alterative. However, due to harmonic enrichment, an optimal solution of the square wave power supply may be shifted from the optimal solution of the sinusoidal one. This needs careful study and observation of characteristics for a single-phase induction motor fed by square wave inverter in order to minimize power losses at each particular load condition.

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