

Adaptive Control and Simulation of A Seven-Link Biped Robot For The Combined Trajectory Motion and Stability Investigations

A. BAGHERI, M. E. FELEZI, P. N. MOUSAVI

Department of Mechanical Engineering
Guilan University, Rasht, IRAN

Abstract: In this article, simulation (trajectory planning) and control of seven link biped robot on combined trajectory (three phases), have been investigated. Using mathematical interpolation for trajectory planning and designing software for the robot simulation and implementation from adaptive control process, the joint's required torques also the ZMP diagrams to stability judgment will be obtained.

Key-words: Seven Link Biped Robot, Adaptive Control, Model Reference Method, Mathematical Interpolation, Combined Trajectory

1 Introduction

To date, the most concern has been on biped robot single phase [1] trajectory planning and control, whereas in this article combined trajectory planning and control of seven link biped robot have been focused. Designing software with aid of the robot breakpoints which will be passed (Trajectory Planning), the robot kinematics, dynamic and control parameter with implementation from adaptive control process in order to extracting the needed joint's torques also yielding ZMP diagrams which play as an indicator to the robot stability will be represented.

2 Dynamics of the Robot

Using the Lagrangian relation [2] as following:

$$\tau_i = \frac{d}{dt} \left(\frac{\partial \ell}{\partial \dot{q}_i} \right) - \frac{\partial \ell}{\partial q_i}$$

$$\ell = T - U$$

which ℓ indicates the differences between the system's kinetic and potential energies. Constructing the kinetic and potential energies:

$$T = \sum_{i=0}^6 T_i = \sum_{i=0}^6 \frac{1}{2} (m_i \dot{v}_{ci}^2 + I_{ci} \omega_i^2)$$

$$U = \sum_{i=0}^6 m_i g^0 h_{ci}$$

Which ${}^i v_{ci}, I_{ci}$ are link's center of mass velocity and inertia, respectively that are obtained from chain rule as following procedure:

$${}^{i+1} \omega_{i+1} = {}^{i+1} R^i \omega_i + \omega_{i+1} \hat{Y}_{i+1}$$

$${}^{(i+1)} v_{c(i+1)} = {}^{(i+1)} R_i [{}^i v_i + {}^i \omega_i \times {}^i P_{i+1}] + {}^{i+1} \omega_{i+1} \times {}^{i+1} P$$

In abbreviation, the link's center of mass velocities relative to mass center's coordinates are extracted as following:

$${}^0 v_{c0} = \begin{bmatrix} 0 \\ l_{f.mc} \omega_0 \\ 0 \end{bmatrix} \quad (1)$$

$${}^1 v_{c1} = \begin{bmatrix} \omega_0 l_e \sin(q_1 - \varphi) \\ \omega_0 l_e \cos(q_1 - \varphi) + l_{c1} (\omega_1 + \omega_0) \\ 0 \end{bmatrix} \quad (2)$$

$${}^2 v_{c2} = \begin{bmatrix} \omega_0 l_e \sin(q_2 - \varphi) + l_1 (\omega_0 + \omega_1) \sin(q_2 - q_1) \\ \omega_0 l_e \cos(q_2 - \varphi) + l_1 (\omega_0 + \omega_1) \cos(q_2 - q_1) + \dots \\ l_{c2} (\omega_0 + \omega_1 + \omega_2) \\ 0 \end{bmatrix} \quad (3)$$

$${}^3V_{c3} = \begin{bmatrix} \alpha l_e \sin(\varphi - q_3) + l_1(\alpha_0 + \alpha) \sin(q_1 - q_3) + \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \sin(q_2 - q_3) \\ -\alpha l_e \cos(\varphi - q_3) - l_1(\alpha_0 + \alpha) \cos(q_1 - q_3) - \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \cos(q_2 - q_3) + l_{c3}(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3) \\ 0 \end{bmatrix} \quad (4)$$

$${}^4V_{c4} = \begin{bmatrix} \alpha l_e \sin(\varphi - q_4) + l_1(\alpha_0 + \alpha) \sin(q_1 - q_4) + \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \sin(q_2 - q_4) - \dots \\ l_3(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3) \sin(q_3 - q_4) \\ -\alpha l_e \cos(\varphi - q_4) - l_1(\alpha_0 + \alpha) \cos(q_1 - q_4) - \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \cos(q_2 - q_4) + \dots \\ l_3(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3) \cos(q_3 - q_4) + \dots \\ l_{c4}(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \\ 0 \end{bmatrix} \quad (5)$$

$${}^5V_{c5} = \begin{bmatrix} \alpha l_e \sin(\varphi - \pi/2 + q_b - \beta_{fswing}) + \dots \\ l_1(\alpha_0 + \alpha) \sin(q_1 - \pi/2 + q_b - \beta_{fswing}) + \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \sin(q_2 - \pi/2 + q_b - \beta_{fswing}) - \dots \\ l_3(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3) \sin(q_3 - \pi/2 + q_b - \beta_{fswing}) - \dots \\ l_4(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \sin(q_4 - \pi/2 + q_b - \beta_{fswing}) \\ -\alpha l_e \cos(\varphi - \pi/2 + q_b - \beta_{fswing}) - \dots \\ l_1(\alpha_0 + \alpha) \cos(q_1 - \pi/2 + q_b - \beta_{fswing}) - \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \cos(q_2 - \pi/2 + q_b - \beta_{fswing}) + \dots \\ l_3(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3) \cos(q_3 - \pi/2 + q_b - \beta_{fswing}) + \dots \\ l_4(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \cos(q_4 - \pi/2 + q_b - \beta_{fswing}) + \dots \\ l_{cfswing}(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \\ 0 \end{bmatrix} \quad (6)$$

$${}^{torso}V_{ctorso} = \begin{bmatrix} \alpha l_e \sin(\varphi - q_{torso} - \pi/2) + \dots \\ l_1(\alpha_0 + \alpha) \sin(q_1 - q_{torso} - \pi/2) - \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \sin(q_{torso} + q_2 + \pi/2) \\ \alpha l_e \cos(\varphi - q_{torso} - \pi/2) + \dots \\ l_1(\alpha_0 + \alpha) \cos(q_1 - q_{torso} - \pi/2) + \dots \\ l_2(\alpha_0 + \alpha_1 + \alpha_2) \cos(q_{torso} + q_2 + \pi/2) + \dots \\ l_{torso}(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_{torso}) \\ 0 \end{bmatrix} \quad (7)$$

$$G_3 = g(0.5m_3 + m_4 + m_5)l_3 \cos(q_3 - \lambda) \quad (10)$$

$$G_4 = g(0.5m_4 + m_5)l_4 \cos(q_4 - \lambda) \quad (11)$$

$$G_5 = -gm_5 l_{cfswing} \cos(\pi/2 + q_{fswing} - \lambda - \beta_{fswing}) \quad (12)$$

$$G_{tor} = 0.5gm_{torso} l_{torso} \cos(\pi - q_{torso} - \lambda) \quad (13)$$

After of the simplification of the relations, the seven link biped robot's equation becomes as below:

$$\tau_i = H(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q_i) \quad (14)$$

where $i = 0, 2, \dots, 6$ and H, C, G are mass inertia, coriolis and gravitational matrix of system, which can be written as following:

$$H(q) = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} & h_{17} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} & h_{27} \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & h_{36} & h_{37} \\ h_{41} & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} & h_{47} \\ h_{51} & h_{52} & h_{53} & h_{54} & h_{55} & h_{56} & h_{57} \\ h_{61} & h_{62} & h_{63} & h_{64} & h_{65} & h_{66} & h_{67} \end{bmatrix} C(q, \dot{q}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & c_{27} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & c_{37} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & c_{47} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & c_{57} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & c_{67} \end{bmatrix} G(q) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_{tor} \end{bmatrix}$$

Avoiding from bulk calculations, just the diagonal components are displayed in following:

$$h_{11} = [m_1(l_{c1}^2 + l_{c1}l_e \cos(q_1 - \varphi))] + [m_2(l_1^2 + l_{c2}^2 + l_1l_e \cos(q_1 - \varphi) + \dots \\ l_{c2}l_e \cos(q_2 - \varphi) + 2l_1l_{c2} \cos(q_2 - q_1))] + [m_3(l_1^2 + l_2^2 + l_{c3}^2 + \dots \\ l_1l_e \cos(q_1 - \varphi) + l_2l_e \cos(q_2 - \varphi) - l_{c3}l_e \cos(-q_3 + \varphi) + \dots \\ 2l_1l_2 \cos(q_2 - q_1) - 2l_1l_{c3} \cos(q_1 - q_3) - 2l_2l_{c3} \cos(q_2 - q_3))] + \dots \\ [m_4(l_1^2 + l_2^2 + l_3^2 + l_{c4}^2 + l_1l_e \cos(q_1 - \varphi) + l_2l_e \cos(q_2 - \varphi) - \dots \\ l_3l_e \cos(q_3 - \varphi) - l_{c4}l_e \cos(-q_4 + \varphi) + 2l_1l_2 \cos(q_2 - q_1) - \dots \\ 2l_1l_3 \cos(q_3 - q_1) - 2l_1l_{c4} \cos(q_1 - q_4) - 2l_2l_3 \cos(q_3 - q_2) - \dots \\ 2l_2l_{c4} \cos(q_2 - q_4) + 2l_{c4}l_3 \cos(q_3 - q_4))] + [m_5(l_1^2 + l_2^2 + \dots$$

Now using the above stated terms to obtaining the system's kinetic energies and implementation from simplified potential expressions as following:

$$G_1 = -g(0.5m_1 + m_2 + m_3 + m_4 + m_5 + m_{torso})l_1 \cos(q_1 - \lambda) \quad (8)$$

$$G_2 = g(0.5m_2 + m_3 + m_4 + m_5 + m_{torso})l_2 \cos(\pi - q_2 + \lambda) \quad (9)$$

$$\begin{aligned}
 & l_3^2 + l_4^2 + l_{cfswing}^2 + l_1 l_e \cos(q_1 - \varphi) + l_2 l_e \cos(q_2 - \varphi) - \dots \\
 & l_3 l_e \cos(q_3 - \varphi) - l_4 l_e \cos(q_4 - \varphi) - \dots \\
 & l_{cfswing} l_e \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) + 2l_1 l_2 \cos(q_2 - q_1) - \dots \\
 & 2l_1 l_3 \cos(q_3 - q_1) - 2l_1 l_4 \cos(q_4 - q_1) - \dots \\
 & 2l_1 l_{cfswing} \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dots \\
 & 2l_2 l_3 \cos(q_3 - q_2) - 2l_2 l_4 \cos(q_4 - q_2) - \dots \\
 & 2l_2 l_{cfswing} \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 & 2l_3 l_4 \cos(q_4 - q_3) + 2l_4 l_{cfswing} \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 & 2l_3 l_{cfswing} \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing})) + [m_{tor} (l_1^2 + \dots \\
 & l_2^2 + l_{ctorso}^2 + l_1 l_e \cos(q_1 - \varphi) + l_2 l_e \cos(q_2 - \varphi) + \dots \\
 & l_e l_{ctorso} \cos(-q_{torso} - \varphi - (\pi/2)) + 2l_1 l_2 \cos(q_2 - q_1) + \dots \\
 & 2l_1 l_{ctorso} \cos(-q_{torso} - q_1 - (\pi/2)) + 2l_2 l_{ctorso} \cos(q_{torso} + q_2 + (\pi/2))] + \dots \\
 & I_1 + I_2 + I_3 + I_4 + I_5 + I_{torso}
 \end{aligned}$$

$$\begin{aligned}
 h_{22} = & [m_2 (l_{c2}^2 + l_1 l_{c2} \cos(q_2 - q_1))] + [m_3 (l_2^2 + l_{c3}^2 + \dots \\
 & l_1 l_2 \cos(q_2 - q_1) - l_1 l_{c3} \cos(q_1 - q_3) - 2l_2 l_{c3} \cos(q_2 - q_3))] + \dots \\
 & [m_4 (l_2^2 + l_3^2 + l_{c4}^2 + l_1 l_2 \cos(q_2 - q_1) - l_1 l_3 \cos(q_3 - q_1) - \dots \\
 & l_1 l_{c4} \cos(q_1 - q_4) - 2l_2 l_3 \cos(q_3 - q_2) - 2l_2 l_{c4} \cos(q_2 - q_4) + \dots \\
 & 2l_{c4} l_3 \cos(q_3 - q_4))] + [m_5 (l_2^2 + l_3^2 + l_4^2 + l_{cfswing}^2 + \dots \\
 & l_1 l_2 \cos(q_2 - q_1) - l_1 l_3 \cos(q_3 - q_1) - l_1 l_4 \cos(q_4 - q_1) - \dots \\
 & l_1 l_{cfswing} \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) - 2l_2 l_3 \cos(q_3 - q_2) - \dots \\
 & 2l_2 l_4 \cos(q_4 - q_2) - 2l_2 l_{cfswing} \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 & 2l_3 l_4 \cos(q_4 - q_3) + 2l_4 l_{cfswing} \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 & 2l_3 l_{cfswing} \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing})) + [m_{tor} (l_2^2 + l_{ctorso}^2 + \dots \\
 & l_1 l_2 \cos(q_2 - q_1) + l_1 l_{ctorso} \cos(-q_{torso} - q_1 - (\pi/2)) + \dots \\
 & 2l_2 l_{ctorso} \cos(q_{torso} + q_2 + (\pi/2))] + I_2 + I_3 + I_4 + I_5 + I_{torso}
 \end{aligned}$$

$$\begin{aligned}
 h_{33} = & [m_3 (l_{c3}^2 - l_2 l_{c3} \cos(q_2 - q_3))] + [m_4 (l_3^2 + l_{c4}^2 - \dots \\
 & l_2 l_3 \cos(q_3 - q_2) - l_2 l_{c4} \cos(q_2 - q_4) + 2l_{c4} l_3 \cos(q_3 - q_4))] + \dots \\
 & [m_5 (l_3^2 + l_4^2 + l_{cfswing}^2 - l_2 l_3 \cos(q_3 - q_2) - l_2 l_4 \cos(q_4 - q_2) - \dots \\
 & l_2 l_{cfswing} \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) + 2l_3 l_4 \cos(q_4 - q_3) + \dots \\
 & 2l_4 l_{cfswing} \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 & 2l_3 l_{cfswing} \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing})) + I_3 + I_4 + I_5
 \end{aligned}$$

$$\begin{aligned}
 h_{44} = & [m_4 (l_{c4}^2 + l_{c4} l_3 \cos(q_3 - q_4))] + [m_5 (l_4^2 + l_{cfswing}^2 - \dots \\
 & l_2 l_4 \cos(q_4 - q_2) + l_3 l_4 \cos(q_4 - q_3) + \dots \\
 & 2l_4 l_{cfswing} \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 & l_3 l_{cfswing} \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing})) + I_4 + I_5
 \end{aligned}$$

$$h_{55} = [m_5 (l_{cfswing}^2 + l_4 l_{cfswing} \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing})) + I_5$$

$$h_{66} = 0$$

In the above stated relations, the geometrical and inertial properties of system are as following (see figures (1), (2) and (3)):

$$\phi = \beta_{m.c} + q_f, \quad \varphi = \lambda + q_f + \beta_{m.c}$$

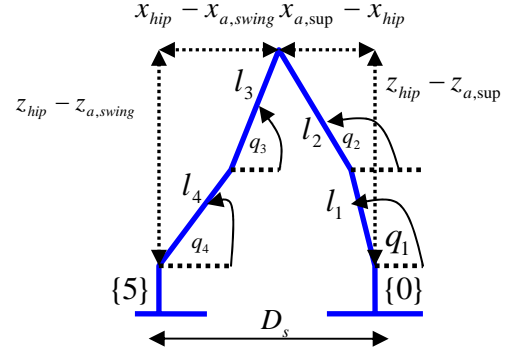


Fig.1- The link's angles and configuration

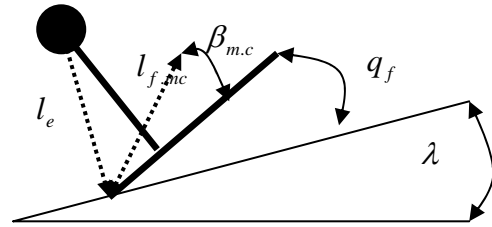


Fig. 2- The support foot parameters

Regarding to figure (2), the similar parameters for swing foot are assumed such as β_{fswing} and $l_{cfswing}$. The used parameters for simulation are listed in below:

- T_c : Total traveling time, including single and double support phases.
- T_d : Double support phase time, which is approximately equaled to 20% of T_c .
- T_m : The time which ankle joint has maximum height during walking cycle.
- k : Step number
- H_{ao} : Ankle joint maximum height
- L_{ao} : The horizontal traveled distance between ankle joint maximum height position and start point.
- D_s : Step length

- q_b, q_f : Foot lift and contact angles with level ground
- q_{gs}, q_{gf} : The ground initial terrain angles
- λ : Surface slope

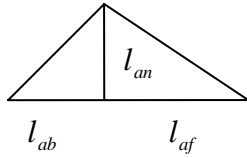


Fig. 3- The foot configuration

3 Adaptive Control

After obtaining the robot's domain nonlinear dynamic relations, control process is used to force the robot to trace the pre-defined trajectory. Adaptive control [3], [4] is one of powerful method that is applicable on trajectory control systems. The main concept of adaptive method refers to online control. In most systems, the domain controlling parameters are pre-known, regarding to physical and geometrical sight. A system is called "known" when the number of poles, zeros and corresponding locations are pre-defined, although is "unknown" while the domain pole and zero's locations are unknown. For this reasons, in conventional controlling systems, a common control process is implemented, whereas for unknown types a nonlinear control procedure is needed that mostly is called *adaptive control*. It's obvious that in the method, the process tries to accommodate with environmental conditions to trace the desired tasks. So, the first priority refers to the unknown parameters estimation to complete a process which is so called "Perfect tracking". In figure(4), a block diagram of adaptation control (Model Reference Method) process is represented:

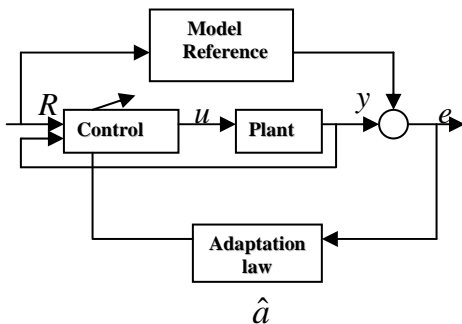


Fig. 4: Block diagram of adaptation control (MRAC)

Mainly, adaptive control is used to handle unknown systems, whereas can be implemented to control known type which in this manner is assumed an alternative of computed torque method. In general, biped robot is known system, so following procedure is implemented to control process. Similar to other nonlinear systems, linearization plays main role in control process. Following parameters are used to perform the mentioned operation. Defining combined tracking error as below:

$$s = \ddot{q} + \Lambda \tilde{q} = \dot{q} - \dot{q}_r \quad (15)$$

which \dot{q}_r, \tilde{q} are reference velocity of q and tracking error, respectively. Then, linearization based on new known matrix $Y = Y(q, \dot{q}, \ddot{q}, \dot{q}_r, \ddot{q}_r)$ in which:

$$\hat{\tau} = H(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + g(q) = Y(q, \dot{q}, \ddot{q}, \dot{q}_r, \ddot{q}_r)\hat{a}$$

Following control law is used to perform adaptive control for unknown systems:

$$\tau = \hat{\tau} - ks = Y\hat{a} - K_D s \quad (16)$$

where K_D is strictly positive definite matrix. For certain seven link biped robot, estimated parameters are replaced with the known types as below:

$$\tau = Ya - K_D s \quad (17)$$

For seven link biped robot, assuming given mass property constants in below:

$$\begin{aligned} a_1 &= (.5m_1 + m_2 + m_3 + m_4 + m_5 + m_{torso})l_1^2 \\ a_2 &= (.5m_2 + m_3 + m_4 + m_5 + m_{torso})l_2^2 \\ a_3 &= (.5m_3 + m_4 + m_5)l_3^2 \\ a_4 &= (.5m_4 + m_5)l_4^2 \\ a_5 &= (.5m_5)l_{cfswing}^2 \\ a_6 &= (.5m_{torso})l_{ctorso}^2 \\ a_7 &= I_1 + I_3 + I_2 + I_4 + I_5 + I_{torso} \\ a_8 &= l_1 l_e (.5m_1 + m_2 + m_3 + m_4 + m_5 + m_{torso}) \\ a_9 &= l_2 l_e (.5m_2 + m_3 + m_4 + m_5 + m_{torso}) \end{aligned}$$

$$\begin{aligned}
 a_{10} &= -l_3 l_e (.5m_3 + m_4 + m_5), a_{11} = -l_4 l_e (.5m_4 + m_5) \\
 a_{12} &= l_1 l_2 (.5m_2 + m_3 + m_4 + m_5 + m_{torso}) \\
 a_{13} &= -l_1 l_3 (.5m_3 + m_4 + m_5), a_{14} = -l_1 l_4 (.5m_4 + m_5) \\
 a_{15} &= -l_2 l_3 (.5m_3 + m_4 + m_5), a_{16} = -l_2 l_4 (.5m_4 + m_5) \\
 a_{17} &= l_3 l_4 (.5m_4 + m_5), a_{18} = m_{torso} l_e l_{cltorso} \\
 a_{19} &= m_{torso} l_1 l_{cltorso}, a_{20} = m_{torso} l_2 l_{cltorso}, a_{21} = -m_5 l_{cfswing} l_e \\
 a_{22} &= -m_5 l_1 l_{cfswing}, a_{23} = -m_5 l_2 l_{cfswing}, a_{24} = m_5 l_3 l_{cfswing} \\
 a_{25} &= m_5 l_4 l_{cfswing}, a_{26} = I_3 + I_2 + I_4 + I_5 + I_{torso} \\
 a_{27} &= I_3 + I_4 + I_5, a_{28} = I_4 + I_5, a_{29} = I_5, a_{30} = I_{torso} \\
 a_{31} &= a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 \\
 a_{32} &= a_2 + a_3 + a_4 + a_5 + a_6 + a_{26} \\
 a_{33} &= a_3 + a_4 + a_5 + a_{27}, a_{34} = a_4 + a_5 + a_{28} \\
 a_{36} &= a_5 + a_{29}, a_{37} = a_6 + a_{30}, c_1^* = g / l_e
 \end{aligned}$$

Then rewriting relation (15) in following form:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_{tor} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} & h_{17} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} & h_{27} \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & h_{36} & h_{37} \\ h_{41} & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} & h_{47} \\ h_{51} & h_{52} & h_{53} & h_{54} & h_{55} & h_{56} & h_{57} \\ h_{61} & h_{62} & h_{63} & h_{64} & h_{65} & h_{66} & h_{67} \end{bmatrix} \begin{bmatrix} \ddot{q}_0 \\ \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_{torso} \end{bmatrix} + \begin{bmatrix} c_{11} & \dots & c_{17} \\ \vdots & \ddots & \vdots \\ c_{61} & \dots & c_{77} \end{bmatrix} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_{torso} \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_{tor} \end{bmatrix}$$

The known matrix components are extracted as below:

$$\begin{aligned}
 y_{1,1} &= \cos(q_1 - \varphi) \ddot{q}_{r0} + (\dot{q}_{1r} - \dot{q}_{0r}) \dot{q}_0 \sin(q_1 - \varphi) - c_1^* \cos(q_1 - \lambda), \\
 y_{1,2} &= \cos(q_2 - \varphi) \ddot{q}_{r0} + (\dot{q}_{0r} - \dot{q}_{2r}) \dot{q}_0 \sin(q_2 - \varphi) \\
 y_{1,3} &= \cos(q_3 - \varphi) \ddot{q}_{r0} + (\dot{q}_{0r} - \dot{q}_{3r}) \dot{q}_0 \sin(q_3 - \varphi) \\
 y_{1,4} &= \cos(q_4 - \varphi) \ddot{q}_{r0} + (\dot{q}_{0r} - \dot{q}_{4r}) \dot{q}_0 \sin(q_4 - \varphi) \\
 y_{1,5} &= \cos(q_2 - q_1) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + \ddot{q}_{r2}] + \sin(q_2 - q_1) [\dot{q}_{1r} (\dot{q}_0 + \dot{q}_1) - \dots \\
 &\quad \dot{q}_{r2} (2\dot{q}_0 + 2\dot{q}_1 + \dot{q}_2) - \dot{q}_{r0} (\dot{q}_0 + \dot{q}_1 + \dot{q}_2)] \\
 y_{1,6} &= \cos(q_3 - q_1) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3}] + \sin(q_3 - q_1) [\dot{q}_{1r} (\dot{q}_0 + \dot{q}_1) - \dots \\
 &\quad \dot{q}_{3r} (2\dot{q}_0 + 2\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - \dot{q}_{r0} (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3)] \\
 y_{1,7} &= \cos(q_4 - q_1) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4}] + \dots \\
 &\quad \sin(q_4 - q_1) [\dot{q}_{1r} (\dot{q}_0 + \dot{q}_1) - \dot{q}_{4r} (2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &\quad \dot{q}_2 + \dot{q}_3 + \dot{q}_4) - \dot{q}_{r0} (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4)] \\
 y_{1,8} &= \cos(q_3 - q_2) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r3}] + (2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &\quad 2\dot{q}_2 + \dot{q}_3) \sin(q_3 - q_2) [\dot{q}_{2r} - \dot{q}_{3r}] \\
 y_{1,9} &= \cos(q_4 - q_2) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4}] + (2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &\quad 2\dot{q}_2 + \dot{q}_3 + \dot{q}_4) \sin(q_4 - q_2) [\dot{q}_{2r} - \dot{q}_{4r}] \\
 y_{1,10} &= \cos(q_4 - q_3) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4}] + (2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &\quad 2\dot{q}_2 + 2\dot{q}_3 + \dot{q}_4) \sin(q_4 - q_3) [\dot{q}_{4r} - \dot{q}_{3r}]
 \end{aligned}$$

$$\begin{aligned}
 y_{1,10} &= \cos(q_4 - q_3) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4}] + (2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &\quad 2\dot{q}_2 + 2\dot{q}_3 + \dot{q}_4) \sin(q_4 - q_3) [\dot{q}_{4r} - \dot{q}_{3r}] \\
 y_{1,11} &= \ddot{q}_{r0} \cos(-q_{torso} - (\pi/2) - \varphi) + \dots \\
 &\quad \sin(-q_{torso} - (\pi/2) - \varphi) \dot{q}_0 (\dot{q}_{0r} + \dot{q}_{torso}) \\
 y_{1,12} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r,torso}] \cos(-q_{torso} - (\pi/2) - q_1) + \dots \\
 &\quad \sin(-q_{torso} - (\pi/2) - q_1) [\dot{q}_{r,torso} (2\dot{q}_0 + 2\dot{q}_1 + \dot{q}_2 + \dot{q}_{torso}) + (\dot{q}_0 + \dot{q}_1) \dot{q}_{r1} - \dots \\
 &\quad \dot{q}_{r0} (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_{torso})] \\
 y_{1,13} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r,torso}] \cos(q_{torso} + (\pi/2) + q_2) - \dots \\
 &\quad (\dot{q}_{2r} + \dot{q}_{r,torso}) \sin(q_{torso} + (\pi/2) + q_2) [2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dot{q}_{torso}] \\
 y_{1,14} &= \ddot{q}_{r0} \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dot{q}_0 (\dot{q}_{r0} + \dots \\
 &\quad \dot{q}_{r,fswing}) \sin(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{1,15} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4} + \dots \\
 &\quad \ddot{q}_{r,fswing}] \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dots \\
 &\quad [(2\dot{q}_0 + 2\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4 + \dot{q}_{fswing}) \dot{q}_{r,fswing} + \dot{q}_{r1} (\dot{q}_0 + \dot{q}_1) - (\dot{q}_0 + \dot{q}_1 + \dots \\
 &\quad \dot{q}_2 + \dot{q}_3 + \dot{q}_4 + \dot{q}_{fswing}) \dot{q}_{r0}] \sin(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{1,16} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4} + \dots \\
 &\quad \ddot{q}_{r,fswing}] \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &\quad 2\dot{q}_2 + \dot{q}_3 + \dot{q}_4 + \dot{q}_{fswing}) (\dot{q}_{r2} + \dot{q}_{r,fswing}) \sin(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{1,17} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + \ddot{q}_{r4} + \dots \\
 &\quad \ddot{q}_{r,fswing}] \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + 2\dot{q}_3 + \dot{q}_4 + \dots \\
 &\quad \dot{q}_{fswing}) (\dot{q}_{r3} + \dot{q}_{r,fswing}) \sin(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{1,18} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + 2\ddot{q}_{r4} + \dots \\
 &\quad \ddot{q}_{r,fswing}] \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + 2\dot{q}_3 + \dots \\
 &\quad 2\dot{q}_4 + \dot{q}_{fswing}) (\dot{q}_{r4} + \dot{q}_{r,fswing}) \sin(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{1,19} &= \ddot{q}_{r0} + \ddot{q}_{r1}, y_{1,20} = \ddot{q}_{r2}, y_{1,21} = \ddot{q}_{r3}, y_{1,22} = \ddot{q}_{r4}, y_{1,23} = \ddot{q}_{r,fswing}, y_{1,24} = \ddot{q}_{r,torso} \\
 ----- \\
 y_{2,1} &= 0 \\
 y_{2,1} &= \cos(q_2 - \varphi) \ddot{q}_{r0} + [2\dot{q}_{0r} + \dot{q}_{1r}] \dot{q}_0 \sin(q_2 - \varphi) + c_1^* \cos(\pi - q_2 + \lambda) \\
 y_{2,3} &= \cos(q_3 - \varphi) \ddot{q}_{r0} + (\dot{q}_{0r} - \dot{q}_{3r}) \dot{q}_0 \sin(q_3 - \varphi), y_{2,4} = \cos(q_4 - \varphi) \ddot{q}_{r0} + \dots \\
 &\quad (\dot{q}_{0r} - \dot{q}_{4r}) \dot{q}_0 \sin(q_4 - \varphi), y_{2,5} = \cos(q_2 - q_1) [\dot{q}_{r0} + \dot{q}_{r1}] + [\dot{q}_{r0} + 2\dot{q}_{1r}] \dot{q}_0 + \dots \\
 &\quad \dot{q}_1 \sin(q_2 - q_1), y_{2,6} = \cos(q_3 - q_1) [\dot{q}_{r0} + \dot{q}_{r1}] + [\dot{q}_{1r} - \dot{q}_{3r}] (\dot{q}_0 + \dot{q}_1) \sin(q_3 - q_1) \\
 y_{2,7} &= \cos(q_4 - q_1) [\dot{q}_{r0} + \dot{q}_{r1}] + (\dot{q}_0 + \dot{q}_1) [\dot{q}_{1r} - \dot{q}_{4r}] \sin(q_4 - q_1) \\
 y_{2,8} &= \cos(q_3 - q_2) [2\dot{q}_{r0} + 2\dot{q}_{r1} + 2\dot{q}_{r2} + \dot{q}_{r3}] + [(2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dots \\
 &\quad \dot{q}_3) (\dot{q}_{2r} - \dot{q}_{3r}) - (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3) (\dot{q}_{0r} + \dot{q}_{1r} + \dot{q}_{2r})] \sin(q_3 - q_2) \\
 y_{2,9} &= \cos(q_4 - q_2) [2\dot{q}_{r0} + 2\dot{q}_{r1} + 2\dot{q}_{r2} + \dot{q}_{r3} + \dot{q}_{r4}] + [(2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dots \\
 &\quad \dot{q}_3 + \dot{q}_4) (\dot{q}_{2r} - \dot{q}_{4r}) - (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4) (\dot{q}_{0r} + \dot{q}_{1r} + \dot{q}_{2r})] \sin(q_4 - q_2) \\
 y_{2,10} &= \cos(q_4 - q_3) [2\dot{q}_{r0} + 2\dot{q}_{r1} + 2\dot{q}_{r2} + 2\dot{q}_{r3} + \dot{q}_{r4}] + [(2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dots \\
 &\quad 2\dot{q}_3 + \dot{q}_4) (\dot{q}_{3r} - \dot{q}_{4r})] \sin(q_4 - q_3), y_{2,11} = [\dot{q}_{r0} + \dot{q}_{r1}] \cos(-q_{torso} - (\pi/2) - \varphi) + \dots \\
 &\quad \dot{q}_0 (\dot{q}_{0r} + \dot{q}_{torso}) \sin(-q_{torso} - (\pi/2) - \varphi) \\
 y_{2,12} &= [\dot{q}_{r0} + \dot{q}_{r1}] \cos(q_{torso} - (\pi/2) - q_1) + (\dot{q}_0 + \dot{q}_1) \dot{q}_{r1} + \dots \\
 &\quad \dot{q}_{r,torso}) \sin(q_{torso} - (\pi/2) - q_1)
 \end{aligned}$$

$$\begin{aligned}
 y_{2,13} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \dots \\
 &\ddot{q}_{r,torso}] \cos(q_{torso} + (\pi/2) + q_2) - [(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_{torso})(\dot{q}_{0r} + \dot{q}_{1r}) - \dots \\
 &(2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dot{q}_{torso})(\dot{q}_{2r} + \dot{q}_{r,torso})] \sin(q_{torso} + (\pi/2) + q_2) \\
 y_{2,14} &= \ddot{q}_{r0} \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dot{q}_0(\dot{q}_{0r} + \dots \\
 &\dot{q}_{r,fswing}) \sin(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{2,15} &= (\ddot{q}_{r0} + \ddot{q}_{r1}) \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (\dot{q}_0 + \dot{q}_1)(\dot{q}_{1r} + \dots \\
 &\dot{q}_{r,fswing}) \sin(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{2,16} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4} + \dots \\
 &\ddot{q}_{r,fswing}] \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 &[(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r0} + \dot{q}_{r1} + \dot{q}_{r2}) - (2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dots \\
 &\dot{q}_3 + \dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r2} + \dot{q}_{r,fswing})] \sin(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{2,17} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + \ddot{q}_{r4} + \dots \\
 &\ddot{q}_{r,fswing}] \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dots \\
 &2\dot{q}_3 + \dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r3} + \dot{q}_{r,fswing}) \sin(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{2,18} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + 2\ddot{q}_{r4} + \dots \\
 &\ddot{q}_{r,fswing}] \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + \dots \\
 &2\dot{q}_3 + 2\dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r4} + \dot{q}_{r,fswing}) \sin(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{2,19} &= 0, y_{2,20} = \ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}, y_{2,21} = \ddot{q}_{r3}, y_{2,22} = \ddot{q}_{r4}, \\
 y_{2,23} &= \ddot{q}_{r,fswing}, y_{2,24} = \ddot{q}_{r,torso}
 \end{aligned}$$

$$\begin{aligned}
 y_{3,1} &= 0, y_{3,2} = 0, y_{3,3} = \cos(q_3 - \varphi) \ddot{q}_{r0} + (2\dot{q}_{0r} + \dot{q}_{1r} + \dots \\
 &\dot{q}_{2r}) \dot{q}_0 \sin(q_3 - \varphi) + (\dot{q}_{0r} + \dot{q}_{1r} + \dot{q}_{2r} + \dot{q}_{3r})(\dot{q}_0 + \dot{q}_1) \sin(q_3 - q_1) + \dots \\
 &(\dot{q}_{0r} + \dot{q}_{1r} + \dot{q}_{2r} + \dot{q}_{3r})(\dot{q}_0 + \dot{q}_1 + \dot{q}_2) \sin(q_3 - q_2) - c_1^* \cos(q_3 - \lambda) \\
 y_{3,4} &= \cos(q_4 - \varphi) \ddot{q}_{r0} + (\dot{q}_{0r} - \dot{q}_{4r}) \dot{q}_0 \sin(q_4 - \varphi), y_{3,5} = 0 \\
 y_{3,6} &= \cos(q_3 - q_1) [\ddot{q}_{r0} + \ddot{q}_{r1}] + [\dot{q}_{1r} - \dot{q}_{3r}](\dot{q}_0 + \dot{q}_1) \sin(q_3 - q_1) \\
 y_{3,7} &= \cos(q_4 - q_1) [\ddot{q}_{r0} + \ddot{q}_{r1}] + (\dot{q}_0 + \dot{q}_1) [\dot{q}_{1r} - \dot{q}_{4r}] \sin(q_4 - q_1) \\
 y_{3,8} &= \cos(q_3 - q_2) [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}] + (\dot{q}_0 + \dot{q}_1 + \dot{q}_2)(\dot{q}_{2r} - \dot{q}_{3r}) \sin(q_3 - q_2) \\
 y_{3,9} &= \cos(q_4 - q_2) [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}] + (\dot{q}_0 + \dot{q}_1 + \dot{q}_2)(\dot{q}_{2r} - \dot{q}_{4r}) \sin(q_4 - q_2) \\
 y_{3,10} &= \cos(q_4 - q_3) [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + \ddot{q}_{r4}] + [(2\dot{q}_0 + 2\dot{q}_1 + \dots \\
 &2\dot{q}_2 + 2\dot{q}_3 + \dot{q}_4)(\dot{q}_{3r} - \dot{q}_{4r}) - (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4)(\dot{q}_{0r} + \dot{q}_{1r} + \dots \\
 &\dot{q}_{2r} + \dot{q}_{3r})] \sin(q_4 - q_3), y_{3,11} = y_{3,12} = y_{3,13} = 0 \\
 y_{3,14} &= \ddot{q}_{r0} \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dot{q}_0(\dot{q}_{0r} + \dots \\
 &\dot{q}_{r,fswing}) \sin(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}), y_{3,15} = (\dot{q}_{r0} + \dots \\
 &\dot{q}_{r1}) \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (\dot{q}_0 + \dot{q}_1)(\dot{q}_{1r} + \dots \\
 &\dot{q}_{r,fswing}) \sin(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}), y_{3,16} = [\ddot{q}_{r0} + \ddot{q}_{r1} + \dots \\
 &\ddot{q}_{r2}] \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) - [(\dot{q}_0 + \dot{q}_1 + \dot{q}_2)(\dot{q}_{r2} + \dots \\
 &\dot{q}_{r,fswing})] \sin(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}), y_{3,17} = [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + \dots \\
 &2\ddot{q}_{r2} + 2\ddot{q}_{r3} + \ddot{q}_{r4} + \ddot{q}_{r,fswing}] \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 &[(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r0} + \dot{q}_{r1} + \dot{q}_{r2} + \dot{q}_{r3}) - (2\dot{q}_0 + \dots \\
 &2\dot{q}_1 + 2\dot{q}_2 + 2\dot{q}_3 + \dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r3} + \dots \\
 &\dot{q}_{r,fswing})] \sin(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing})
 \end{aligned}$$

$$\begin{aligned}
 y_{3,18} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + 2\ddot{q}_{r4} + \ddot{q}_{r,fswing}] \dots \\
 &\cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + 2\dot{q}_3 + 2\dot{q}_4 + \dots \\
 &\dot{q}_{f,swing})(\dot{q}_{r4} + \dot{q}_{r,fswing}) \sin(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}), y_{3,19} = y_{3,20} = 0, \\
 y_{3,21} &= \ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3}, y_{3,22} = \ddot{q}_{r4}, y_{3,23} = \ddot{q}_{r,fswing}, y_{3,24} = 0
 \end{aligned}$$

$$\begin{aligned}
 y_{4,1} &= y_{4,2} = y_{4,3} = 0, y_{4,4} = \cos(q_4 - \varphi) \ddot{q}_{r0} + (\dot{q}_{1r} + \dot{q}_{2r} + \dots \\
 &\dot{q}_{3r}) \dot{q}_0 \sin(q_4 - \varphi) + (\dot{q}_{0r} + 2\dot{q}_{1r} + \dot{q}_{2r} + \dot{q}_{3r})(\dot{q}_0 + \dots \\
 &\dot{q}_1) \sin(q_4 - q_1) + (\dot{q}_{0r} + \dot{q}_{1r} + 2\dot{q}_{2r} + \dot{q}_{3r})(\dot{q}_0 + \dot{q}_1 + \dots \\
 &\dot{q}_2) \sin(q_4 - q_2) + (\dot{q}_{0r} + \dot{q}_{1r} + \dot{q}_{2r} + 2\dot{q}_{3r})(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dots \\
 &\dot{q}_3) \sin(q_4 - q_3) - c_1^* \cos(q_4 - \lambda), y_{4,5} = 0, y_{4,6} = 0, y_{4,7} = \cos(q_4 - q_1) [\ddot{q}_{r0} + \ddot{q}_{r1}], y_{4,8} = 0 \\
 y_{4,9} &= \cos(q_4 - q_2) [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}], y_{4,10} = \cos(q_4 - q_3) [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3}], \\
 y_{4,11} &= y_{4,12} = y_{4,13} = 0, y_{4,14} = \ddot{q}_{r0} \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dot{q}_0(\dot{q}_{0r} + \dots \\
 &\dot{q}_{r,fswing}) \sin(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}), y_{4,15} = (\dot{q}_{r0} + \dots \\
 &\dot{q}_{r1}) \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) - (\dot{q}_0 + \dot{q}_1)(\dot{q}_{1r} + \dots \\
 &\dot{q}_{r,fswing}) \sin(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{4,16} &= [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}] \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) - [(\dot{q}_0 + \dots \\
 &\dot{q}_1 + \dot{q}_2)(\dot{q}_{r2} + \dot{q}_{r,fswing})] \sin(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{4,17} &= [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3}] \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) - \dots \\
 &(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3)(\dot{q}_{r3} + \dot{q}_{r,fswing}) \sin(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{4,18} &= [2\ddot{q}_{r0} + 2\ddot{q}_{r1} + 2\ddot{q}_{r2} + 2\ddot{q}_{r3} + 2\ddot{q}_{r4} + \dots \\
 &\ddot{q}_{r,fswing}] \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) + [(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dots \\
 &\dot{q}_4 + \dot{q}_{f,swing})(\dot{q}_{r0} + \dot{q}_{r1} + \dot{q}_{r2} + \dot{q}_{r3}) - (\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4)(\dot{q}_{r4} + \dots \\
 &(2\dot{q}_0 + 2\dot{q}_1 + 2\dot{q}_2 + 2\dot{q}_3 + 2\dot{q}_4 + \dots \\
 &\dot{q}_{f,swing})(\dot{q}_{r,fswing})] \sin(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{4,19} &= y_{4,20} = y_{4,21} = 0, y_{4,22} = \ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4}, \dots \\
 y_{4,23} &= \ddot{q}_{r,fswing}, y_{4,24} = 0
 \end{aligned}$$

$$\begin{aligned}
 y_{5,1} &= y_{5,2} = y_{5,3} = y_{5,4} = y_{5,5} = y_{5,6} = y_{5,7} = y_{5,8} = y_{5,9} = y_{5,10} = y_{5,11} = y_{5,12} = y_{5,13} = 0 \\
 y_{5,14} &= \ddot{q}_{r0} \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dot{q}_0(\dot{q}_{1r} + \dot{q}_{r2} + \dot{q}_{r3} + \dots \\
 &\dot{q}_{r4}) \sin(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) + c_1^* \cos(\varphi - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{5,15} &= (\ddot{q}_{r0} + \ddot{q}_{r1}) \cos(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) + (\dot{q}_0 + \dot{q}_1)(\dot{q}_{r0} + \dots \\
 &\dot{q}_{r2} + \dot{q}_{r3} + \dot{q}_{r4}) \sin(q_1 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{5,16} &= [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}] \cos(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) + [(\dot{q}_0 + \dots \\
 &\dot{q}_1 + \dot{q}_2)(\dot{q}_{r0} + \dot{q}_{r1} + \dot{q}_{r3} + \dot{q}_{r4})] \sin(q_2 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{5,17} &= [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3}] \cos(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 &(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3)(\dot{q}_{r0} + \dot{q}_{r1} + \dot{q}_{r2} + \dot{q}_{r4}) \sin(q_3 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{5,18} &= [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4}] \cos(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) + \dots \\
 &(\dot{q}_0 + \dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4)(\dot{q}_{r0} + \dot{q}_{r1} + \dot{q}_{r2} + \dot{q}_{r3}) \sin(q_4 - (\pi/2) + q_{fswing} - \beta_{fswing}) \\
 y_{5,19} &= y_{5,20} = y_{5,21} = y_{5,22} = 0, y_{5,23} = \ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r3} + \ddot{q}_{r4} + \ddot{q}_{r,fswing}, y_{5,24} = 0
 \end{aligned}$$

$$\begin{aligned}
 y_{6,1} = y_{6,2} = y_{6,3} = y_{6,4} = y_{6,5} = y_{6,6} = y_{6,7} = y_{6,8} = y_{6,9} = y_{6,10} = 0 \\
 y_{6,11} = \ddot{q}_0 \cos(q_{torso} - (\pi/2) - \phi) - \dot{q}_0(\dot{q}_{r1} + \dot{q}_{r2}) \sin(q_{torso} - (\pi/2) - \phi) + \dots \\
 \dot{c}_1^* \cos(\pi - q_{torso} - \lambda), y_{6,12} = [\ddot{q}_{r0} + \ddot{q}_{r1}] \cos(q_{torso} - (\pi/2) - q_1) - \dots \\
 (\dot{q}_0 + \dot{q}_1)(\dot{q}_{r1} + \dot{q}_{r2}) \sin(q_{torso} - (\pi/2) - q_1), y_{6,13} = [\ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2}] \cos(q_{torso} + (\pi/2) + q_2) + \dots \\
 [(\dot{q}_0 + \dot{q}_1 + \dot{q}_2)(\dot{q}_{r1} + \dot{q}_{r2})] \sin(q_{torso} + (\pi/2) + q_2), y_{6,14} = y_{6,15} = y_{6,16} = y_{6,17} = y_{6,18} = y_{6,19} = \dots \\
 y_{6,20} = y_{6,21} = y_{6,22} = y_{6,23} = 0, y_{6,24} = \ddot{q}_{r0} + \ddot{q}_{r1} + \ddot{q}_{r2} + \ddot{q}_{r,torso}
 \end{aligned}$$

With aid of the needed breakpoints in gait cycle and subsequently trajectory planning of the robot in combined moods which contain three phases, two level ground (horizontal plane) and either declined or descending surfaces by using mathematical interpolation, especially VANDERMONDE Matrix strategy and designing software for simulation of the robot in single or combined manners and using the stated adaptive control process, the user's desired parameters can be obtained easily. More details are in submitted references [1].

4 Simulation Results

As stated above, implementation from the designed software, simulations for four cases, in three phases of trajectory planning, are represented. The used robot's specifications are listed in table (1). As can be seen from the table, the robot's inertial and geometrical values have been chosen symmetric and equal whereas one can select the different properties. Here, just stick diagrams for four slopes: $-12^\circ, -8^\circ, 5^\circ, 10^\circ$ involved in the second phase, are displayed in figures (5) to (8). Using the ZMP formulation [1], the obtained diagrams are calculated relative to fixed coordinate systems which are assumed at the bottom of the support feet in each gait.

5 Conclusions

After trajectory planning of seven link biped robot on combined manner and designing software to simulation of the robot in MATLAB/SIMULINK environment and extracting the kinematics and dynamic parameters, then using the adaptive control process for the known systems with the above cited relations, the robot required joint's torques to trace predefined path over the trajectory have been obtained. Note that in the simulations the robot walk with nominal gait conditions and Moving ZMP where the fixed type and uninominal walking process can be found in given references [1].

References:

[1] Mousavi P.N., "Adaptive Control of a 5DOF Biped Robot Moving on a Declined Surface", Master's thesis, Guilan University, 2005.
 [2] John J. G., "Introduction to Robotics: Mechanics and Control", Addison-Wesley, 1989.
 [3] Sloting J.J. and Li W., "Applied Nonlinear Control", Prentice-Hall, 1991.
 [4] Najafi F., Bagheri A., and Farrokhi R., "Adaptive Control Algorithm for Single - Support Phase of Motion of Motion of a New Biped Robot", Proceeding of MMAR 2003, Poland, P.P. 1003-1010.

Table 1 - The simulated robot specifications

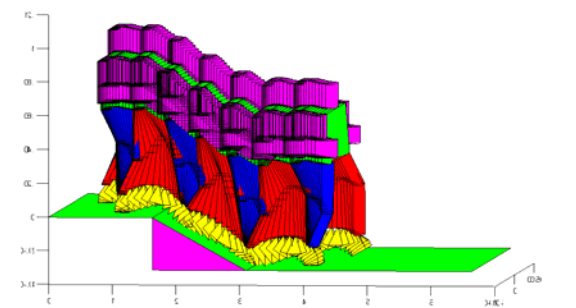
$l_{Sh.}$	$l_{Ti.}$	$l_{To.}$	l_{an}	l_{ab}	l_{af}
0.3m	0.3m	0.3m	0.1m	0.1m	0.13m

$m_{sh.}$	$m_{th.}$	$m_{to.}$	$m_{fo.}$	D_s	T_c
5.7kg	10kg	43kg	3.3kg	0.5m	0.9s

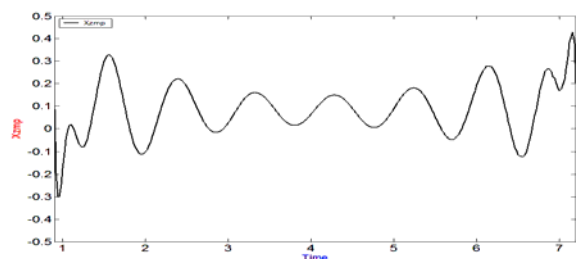
T_d	T_m	H_{ao}	L_{ao}	x_{ed}	x_{sd}
0.18s	0.4s	0.16m	0.4m	0.23m	0.23m

g_{gs}	g_{gf}	H_{min}	H_{max}	I_{shank}	I_{right}
0	0	0.60m	0.62m	0.02kg	0.08kg

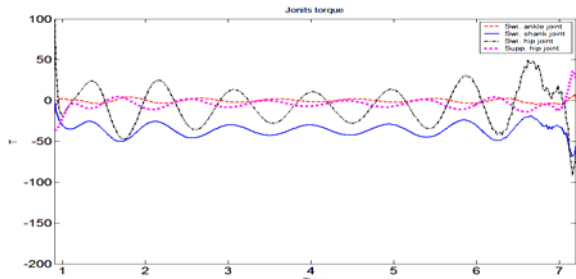
I_{torso}	I_{foot}
1.4kg - m ²	0.01kg - m ²



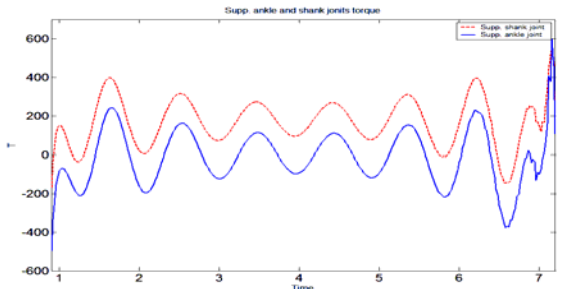
(a)



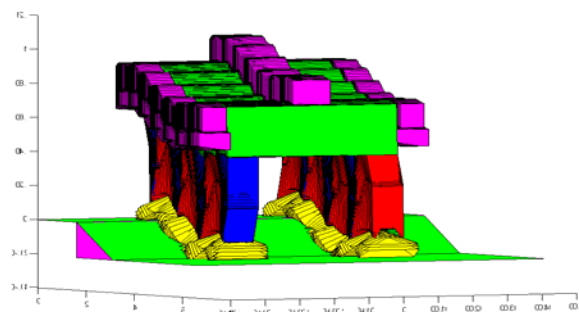
(b)



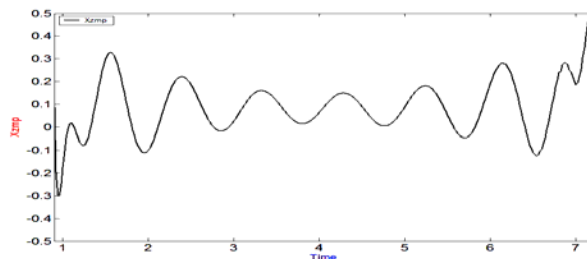
(c)



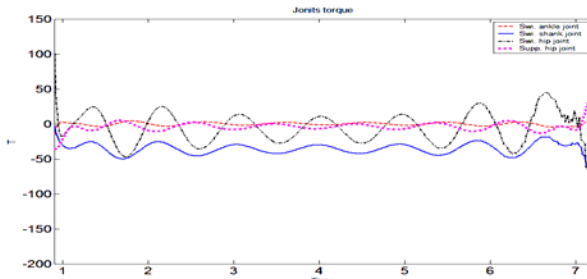
(d)



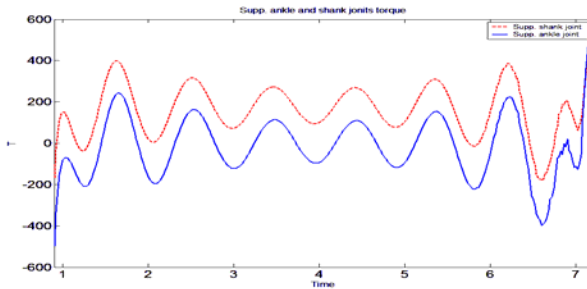
(a)



(b)



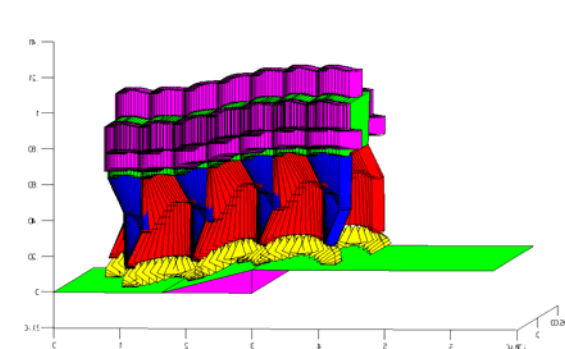
(c)



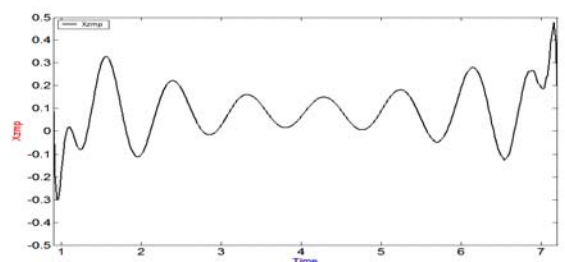
(d)

Fig. 5- (a) Stick Diagram on $\lambda = -12^\circ$, (b) The Moving ZMP on seven steps, (c) **Joint's torques:** __: Swing shank joint, --: Swing ankle joint, ...: Supp. hip joint, .-: Swing hip joint, (d) **Joint's torques:** __: Supp. ankle joint, --: Supp. shank joint

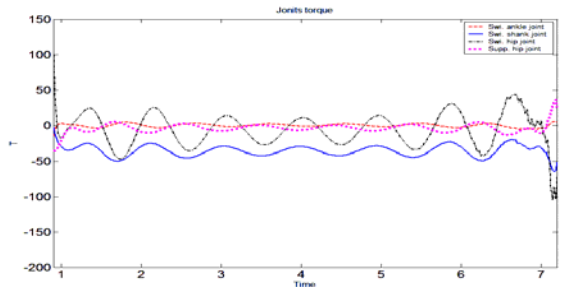
Fig. 6- (a) Stick Diagram on $\lambda = -8^\circ$, (b) The Moving ZMP on seven steps, (c) **Joint's torques:** __: Swing shank joint, --: Swing ankle joint, ...: Supp. hip joint, .-: Swing hip joint, (d) **Joint's torques:** __: Supp. ankle joint, --: Supp. shank joint



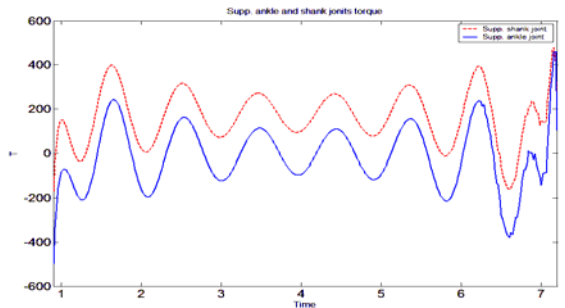
(a)



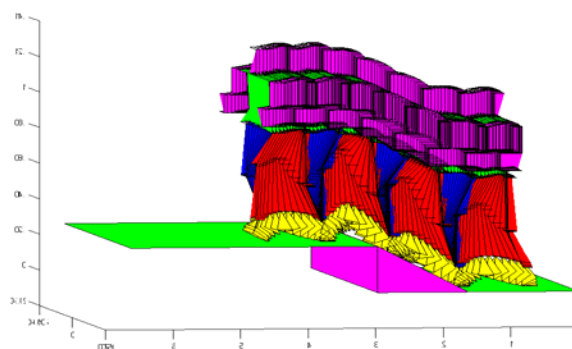
(b)



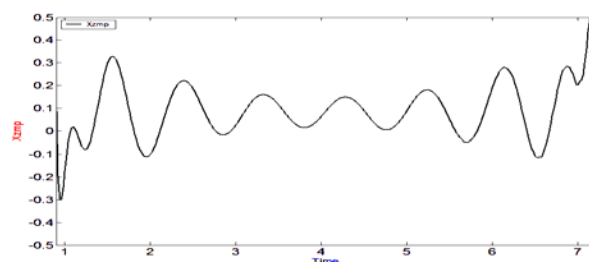
(c)



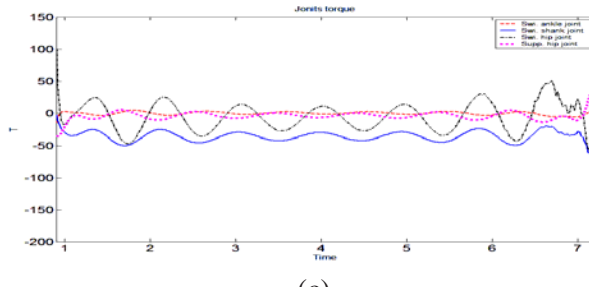
(d)



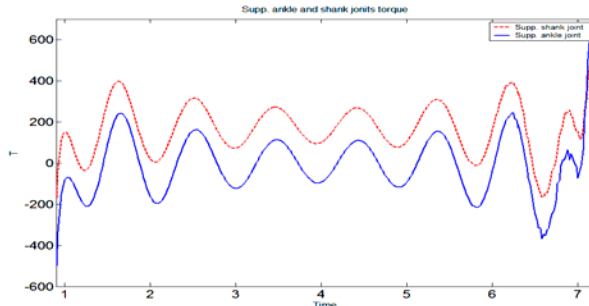
(a)



(b)



(c)



(d)

Fig. 7- (a) Stick Diagram on $\lambda=+5^\circ$, (b) The Moving ZMP on seven steps, (c) **Joint's torques:** __: Swing shank joint, --: Swing ankle joint, ...: Supp. hip joint, .-: Swing hip joint, (d) **Joint's torques:** __: Supp. ankle joint, --: Supp. shank joint

Fig. 8- (a) Stick Diagram on $\lambda=+10^\circ$, (b) The Moving ZMP on seven steps, (c) **Joint's torques:** __: Swing shank joint, --: Swing ankle joint, ...: Supp. hip joint, .-: Swing hip joint, (d) **Joint's torques:** __: Supp. ankle joint, --: Supp. shank joint