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Flow Pressure Analysis of Pipe Networks with Linear Theory Method

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Abstract: In the paper authors present flow – pressure analysis of transmission systems, linearization of non – linear equation with LTM method. The method is used on the test case of pipe networks with three nodes. The analyses of the influence of changing pipe diameter on flow – pressure characteristics of pipe network is made as well as possibilities of determining defects in designing new current pipe networks.

Key - Words: Fluid mechanics, pipe transmission system, pressure losses, linear theory method

1 Introduction

In the past, the piping networks were modelled with the classical method [1]. The designer defined the dimensions of the systems based on experience and recommendations and then investigated by calculation to ensure that the designed system did not exceed the permitted decrease of hydraulic parameters. If all the constraints were not satisfied, new calculations with new dimensions or a new network design had to be done [2].Designing and dimensioning with respect to flow velocity was carried out on speed of fluxon the basis of [3]:

- equation for fluid velocity,
- use of tables, and
- use of nomograms

Development of computer science promoted the computer modelling of pipe networks. The whole system is represented as a non-linear target function by a set of non-linear equations of hydraulic constraints. By minimizing the non-linear function, the optimal design and dimensions of the pipe network are determined [4].

Knowledge on pressure losses of a flow system is crucial for the final decision on network dimensions. Incorrect choice of pipe dimensions leads to too high or too low pressure drops and consequently increased maintenance costs caused by disturbances in operation.

2 Calculation of pressure drop in pipe networks

With the transport of fluid through pipe networks we are facing incorrect flowrate – pressure functioning because of the wrong choice of elements and devices, which is a result of not knowing the dependence on pressure, flow, pressure drop and capacity of pumps [5]. In those cases it is necessary to carry out the analysis of flowrate – pressure conditions in a pipe network to determine the errors, which have been made in planning and rebuilding of a pipe network [6].

The laws of flow through the pipe networks are defined with Darcy – Weisbach equation (1), which determines the pressure drop in the incompressible fluid flow from node i to node j as a function of the pipe diameter, pipe roughness, flow, physical properties of fluid, local friction etc.:

$$\begin{aligned}\Delta p &= p_i - p_j = \\ &0.81 \cdot \frac{\rho \cdot q_{v\,ij}^{2}}{D^4} \cdot \left(\frac{\lambda \cdot L}{D} + \sum \zeta \right) = k_{ij} \cdot q_{v\,ij}^{2}\end{aligned} \tag{1}$$

For determination of friction factor in pipes which belong to hydraulic smooth and hydraulic rough, implicit Prandtl – Colebrook equation is used:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2.51}{\mathrm{Re}\cdot\sqrt{\lambda}} + \frac{\mathrm{k}}{3.71\cdot \mathrm{D}}\right) \tag{2}$$

The equation is valid for region:

$$\mathrm{Re} > 200 \cdot \frac{d}{\sqrt{\lambda}\cdot k} \tag{3}$$

When planning the mathematical model (- set of linear and nonlinear hydraulic equations) describing the flowrate – pressure conditions in the pipe network – we must consider [7]:

- continuity of pipe network flow,
- continuity of nodal flow - I. Kirchhoff law ,
- conservation of energy through a closed loop - II. Kirchhoff law,
- pressure drops due to fluid flow through the pipeline.

3 Linear (LTM) method

The most often the LTM method is used to solve the mathematical model due to its reliability and relatively high speed [8].

Set of nonlinear equations, which are included in mathematical model, is solved by iteration with previous linearization of Darcy – Weisbach equation by pressure:

$$q_{vi} = \left|p_i - p_j\right|_{k+1} \cdot \sqrt{\frac{1}{K_{ij}\cdot\left|\ p_i - p_j\right|_{k+1}}} \tag{4}$$

where:
i = 1,2,3 …,N
j = 1,2,3, ...,N
k = 1,2,3, …M.

For sequentially connected pipe sections with three nodes and with inflow in first and outflow in third node can write down:

$$\begin{aligned} -q_{v1,2} + q_{v1} &= 0 \\ q_{v1,2} - q_{v2,3} &= 0 \\ q_{v2,3} - q_{v3} &= 0 \end{aligned} \tag{5}$$

According to linear Darcy – Weisbach equation (4) in the set of equations (5) we get the following system:

$$\begin{aligned} -(p_1 - p_2)\cdot k_{12} + \qquad\qquad & q_{v1} = 0 \\ (p_1 - p_2)\cdot k_{2,1} - (p_2 - p_3)\cdot k_{2,3} &= 0 \\ (p_2 - p_3)\cdot k_{3,2} - \qquad\qquad & q_{v3} = 0 \end{aligned} \tag{6}$$

where:

$$k_{vi} = \sqrt{\frac{1}{K_{ij}\cdot\left|\ p_i - p_j\right|_k}} \qquad k = 1,2,\ldots,M \qquad i = 1,2,\ldots,N \qquad j = 1,2,\ldots,N \tag{7}$$

In the further course of solving the system of equations (6) we use the matrix:

$$\begin{bmatrix} -k_{12} & k_{12} & o \\ k_{21} & -k_{21} - k_{23} & k_{23} \\ 0 & k_{32} & -k_{32} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} -q_{v1} \\ 0 \\ q_{v3} \end{bmatrix} \tag{8}$$

Where the non – diagonal parts of the matrix are

$k_{ij} = k_{ji}$, $i \neq j$

i = 1,2,…,N
j=1,2,…N

and the diagonal ones are:

$k_{ii} = -\sum k_{ij}$

i = 1,2,…,N
j=1,2,…N

In order to carry out a hydraulic analysis of a pipe network at least one pressure has to be known, either the pressure of input or output fluid. In the case where one of these pressures is known, the set of equation (8) can take the following form:

$$\begin{bmatrix} 1 & 0 & 0 \\ k_{21} & -k_{21}-k_{23} & k_{23} \\ 0 & k_{32} & -k_{32} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ 0 \\ q_{v3} \end{bmatrix} \qquad (9)$$

The solution procedure of the set of linear equations (9) is implicit in that way that in the first iteral step one assumes the initial pressure in the node. When solving all the next equations the initial pressures are being corrected. The number iterations - M depends on minimum relative error (10) and the accuracy of assumed pressures as described in the first step. Because we generalize the algorithm the values of the initial pressures are chosen so – that they differ in values by hundred or more Pa:

$$\varepsilon = \frac{\left| p_i^{(k+1)} - p_i^{(k)} \right|}{p_i^{(k)}} \qquad (10)$$

$$k = 1,2,3,\ldots,M$$

4 Analysis of changing diameter influences

Flow – pressure analysis of simple pipe network on figure 1 is done by using linear (LTM) method. The data needed are given in table 1 and 2.

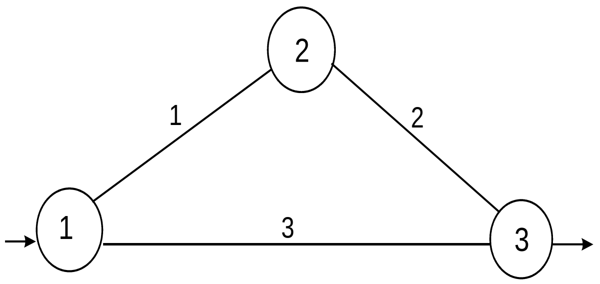

Figure 1: Simple pipe network with three nodes and three pipes

Pipe network has inflow in node 1 and outflow in node 3. Node 2 is just an idle node. In the flow – pressure analysis third pipe diameter was being changed at constant first and second pipes diameters. Also the changes of flow – pressure conditions were analysed by using the computer program which was made at on the Faculty of Chemistry and Chemical Engineering of the University of Maribor.

Table 1: Pipe data

Pipe	L (m)	d (mm)	k (mm)
1	450	167,8	0,4
2	450	167,8	0,4
3	820	0,0	0,4
		51,2	
		94,4	
		125,0	
		167,8	
		204,0	
		231,9	
		254,4	
		284,3	
		309,7	

Table 2: Inflow and outflow data (+ inflow, - outflow)

node	$\pm q_v$ (m^3/s)
1	+ 0,085
2	0,0
3	-0,085

The results of third pipe diameter variation are given on figures 2 and 3.

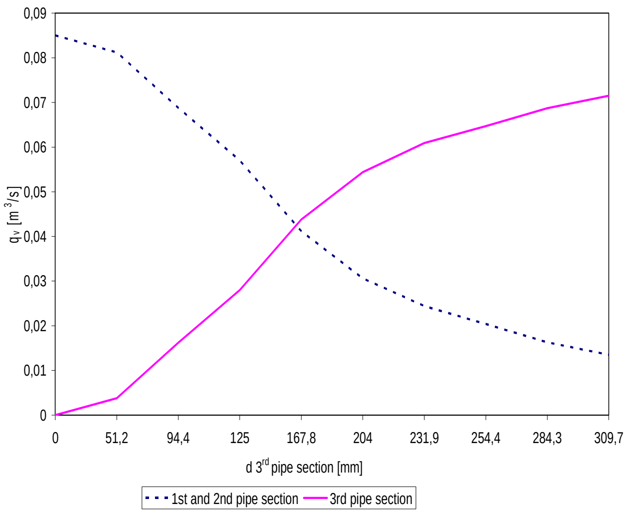

Figure 2: Volume flow rate vs. third pipe diameter

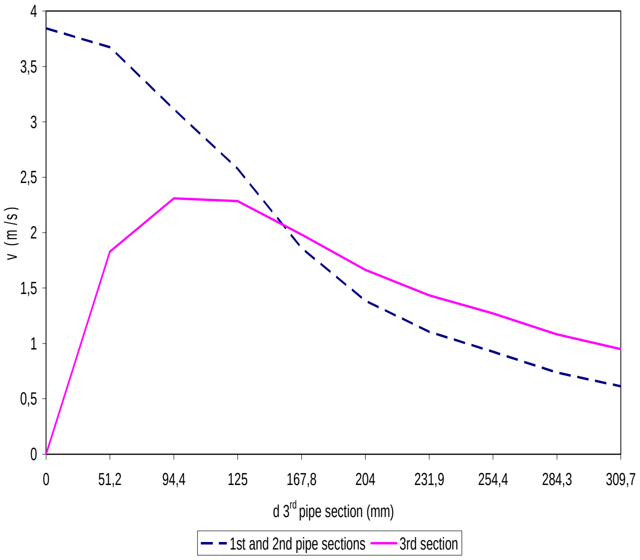

Figure 3: Fluid velocity vs.third pipe diameter

5 Conclusion

The results of fluid – pressure analysis show that by increasing the diameter of pipe 3 the volume flow in the pipe increases, whereas the flow in pipes 1 and 2 decreases. The fluid velocity in pipe 3 increases at the beginning, but then starts to decrease. The velocity in pipe section 1 and 2 decreases continuously with the increase diameter of 3rd pipe section. The drops of pressure by transpiration are in accordance with Darcy – Weisbach equation (1), because with the decrease of volume flow and increase of pipe diameter, the velocity and drops of pressure in 3rd pipe section decrease.

When choosing the optimal standard pipe diameter beside the functionality the economy is very important as well.With the right choice of standard pipe diameter one can save lot of unnecessary investment and lower the maintenance costs.

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Symbols

D	inner pipe diameter	mm
k	pipe roughness	mm
K	coefficient of Darcy – Weisbach equation	
L	pipe length	m
M	iteration number	/
p	pressure	kPa
Δp	pressure loss	kPa
Re	Reynolds number	/
v	fluid velocity	m/s
q_v	volume flow	m^3/s
ε	relative mistake	/
λ	friction coefficient	/
ζ	local losses coefficient	/
ρ	density	kg/m^3

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