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Application of Basic Fluid Mechanic Relations to Estimate the Influence of Surface Roughness on the Aerodynamics of Centrifugal Compressor Impellers

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Abstract: - To discuss the impact of surface roughness on the efficiency of centrifugal compressor impellers, in this paper a theoretical examination of several parameters influencing the aerodynamic behavior is presented. The work is based on the available literature upon the influence of surface roughness on the aerodynamics of fluids. Outgoing from a theoretical approach which is established on the basic fluid mechanic relations an algorithm was created which allows to compute the prospective efficiency deficit of radial impellers in dependence on the specific technical roughness. With the help of the numerical code the impact of several parameters on the efficiency of a radial impeller due to surface roughness was evaluated. The results are discussed in comparison to a hydraulically smooth surface of the impellers. Thus only additional losses due to surface roughness are focused on.

Key-Words: - Surface roughness, efficiency deficit, centrifugal compressor impeller

1 Introduction

Milling of radial impellers is a progressive manufacturing procedure, which has several advantages. But normally the surface quality of a milled impeller is inferior. To achieve the same surface roughness at both sides of the blades and also at the hub and at the shroud surface the required effort is very high. For manufacturing costs optimization it is essential to machine the impeller surfaces as little as possible and at the same time as much as necessary to achieve the required aerodynamic conditions. The designers of radial impellers always would ask for the best surface quality but sometimes this is not able to be produced for reasonable costs.

From this point of view the question rises, what quality of the impeller surface is the optimum considering both, aerodynamic and economical aspects. Whilst the efficiency is the number that mainly affects the price for which a compressor can be sold, the surface roughness strongly affects the manufacturing costs. To optimize the manufacturing process of radial impellers the efficiency and the surface roughness have to be set in dependence. Then the influence of further variables on this dependency can be investigated. To do so, a tool is required which is able to compute the losses of a radial impeller which arise from a specific surface roughness in comparison to an impeller with a hydraulically smooth surface.

To evaluate the efficiency deficit due to a certain surface roughness two main approaches are possible: For the first one the losses due to the surface roughness must be known for a specific operating point of the impeller at hand. From the data of the actual operating point which is designated by a certain Reynolds number it is feasible for the same machine to calculate the losses due to surface roughness for any other operating point and its categorizing Reynolds number. There exist some quite similar approaches from several authors [1-6] which give an empirical relationship between the efficiency of a specified operating point and a test condition.

For the second approach, other authors used a one-dimensional attempt to estimate the friction losses in the flow channels of a compressor impeller. In the paper of Schröder [7] for example the fluid wetted inner walls of the impeller flow channels were considered as flat plates and the loss of impulse of the pouring fluid was calculated using dissipation coefficients.

To introduce an enhanced method for the calculation of radial impeller friction losses for a certain wall roughness the exact velocity distributions of the boundary layers at the confining walls must be known. As this is not possible for such complicated flow at this time the present paper is addressed to the derivation of an elementary method which uses basic fluid mechanic relations. This kind of method may not be absolutely accurate but it reproduces the existent tendencies in a correct way and gives deep insight into the basic procedures.

2 Estimation of wall friction losses

2.1 Dissipation energy of a fluid

To estimate the losses which are enforced by the surface roughness of shrouded radial compressor impellers a one dimensional approach is used. According to Traupel [8] the dissipation energy of a fluid which flows along a wall can be calculated by:

$$d\dot{W} = \tau \cdot w \cdot dA \tag{2}$$

Introducing the wall sheer stress which is given by

$$\tau = c_{d,c} \frac{\rho}{2} w^2 \tag{3}$$

the dissipation energy can be written as

$$d\dot{W} = c_{d,c} \cdot \frac{\rho}{2} \cdot w^3 \cdot dA \tag{4}$$

The fluid mass which is flowing through one impeller passage is given by

$$\dot{m} = \frac{\pi(r_S{}^2 - r_H{}^2)}{z''} \rho_0 c_{n0} \tag{5}$$

Dividing the dissipation energy by the mass flow, it can be rewritten as the specific friction losses of an impeller:

$$dj = \frac{c_{d,c} \cdot z''}{2\pi(r_S{}^2 - r_H{}^2)c_{n0}} \frac{\rho}{\rho_0} w^3 dA \tag{6}$$

Assuming a polytropic change of state, the pressure ratio Π of an impeller can be determined by

$$\frac{\rho}{\rho_0} = (\frac{p}{p_0})^{1/n} = \Pi^{1/n} \tag{7}$$

and the fluid wetted surface of one impeller channel is

$$dA = \frac{2b}{\sin\beta} ds + \frac{4\pi r(1-e)}{z''} ds \tag{8}$$

Imposing equ.'s (7) and (8) into equ. (6) the losses of a centrifugal impeller can be calculated by:

$$j = \frac{c_{d,c}}{\pi(r_S{}^2 - r_H{}^2)c_{n0}} \int_0^{s_2} \Pi^{1/n} w^3 [\frac{bz''}{\sin\beta} + 2\pi r(1-e)] ds \tag{9}$$

The isentropic efficiency of a compression process can be expressed as:

$$\eta_s = \frac{\Delta h_s}{\Delta h} \tag{10}$$

Introducing the constant rothalpy ($h_{t,rel}{}^* =$const.), the polytropic enthalpy rise can be written as a function of relative velocities and impeller speeds at the entrance (index 1) and at the outlet of the impeller (index 2). The isentropic efficiency then can be computed by:

$$\eta_s = 1 - \frac{2j}{w_1{}^2 - w_2{}^2 + u_2{}^2 - u_1{}^2} \tag{11}$$

The difference between the efficiency of an impeller which has a smooth surface and an impeller with a rough surface is:

$$\Delta\eta_s = \eta_{s,smooth} - \eta_{s,rough} \tag{12}$$

Setting the isentropic efficiency for the smooth surface to $\eta_{s,\,smooth} = 1$ leads to the following equation:

$$\Delta\eta_s = \frac{2j}{w_1{}^2 - w_2{}^2 + u_2{}^2 - u_1{}^2} \tag{13}$$

The loss coefficient of the impeller $c_{d,c}$ (equ. (9)) is an empirical value, which considers the fluid friction at the walls of the impeller flow channel. The amount of $c_{d,c}$ is always bigger than that for a flat plate and the shape of the channel should also be considered. After Ito [9] $c_{d,c}$ can be calculated by

$$c_{d,c} = (c_f + 0.0015)(1.1 + 4\frac{b_2}{d_2}) \tag{14}$$

The quantity c_f is the skin friction coefficient. Following Schlichting [10] the skin-friction coefficient for flat plates and smooth pipes is given by

$$c_f = \lambda/4. \tag{15}$$

λ is the friction factor coming from experiments, whose dependency on the Reynolds number and on the surface roughness is shown in the well known diagram created by Moody [11]. The level of surface roughness has been classified into three characteristic categories of sand roughness by Prandtl [12]:

The first category is the hydraulically smooth surface. There are some irregularities at the surface, but these are fully covered by the laminar layer, which develops when a fluid flows along a surface. For this category there is no influence of the roughness on the pipe friction factor:

$$0 \le k_s^+ \le 5 \quad \Rightarrow \lambda = \lambda\,(Re) \tag{16}$$

The second category is the transition region. Here some of the roughness is covered by the laminar layer and some peaks extend into the turbulent layer or even into the main stream. Then the pipe friction factor is a function of the roughness and the Reynolds number:

$$5 < k_s^+ < 70 \quad \Rightarrow \lambda = \lambda\,(Re, \frac{k_s}{d_h}) \tag{17}$$

In the last category all the roughness peaks reach out of the laminar layer into the turbulent layer or into the main stream. The pipe friction factor is only a function of the roughness:

$$70 \le k_s^+ \quad \Rightarrow \lambda = \lambda\,(\frac{k_s}{d_h}) \tag{18}$$

To calculate the pipe friction factor λ of a hydraulically smooth surface ($k_s^+ \le 5$) the formula by Nikuradse [14,15] has to be used:

$$\frac{1}{\sqrt{\lambda}} = 2\log^{10}(Re\sqrt{\lambda}) - 0.8 \tag{19}$$

In the transition region ($5 < k_s^+ < 70$) the implicit formula from Colebrook [13] can be utilized:

$$\frac{1}{\sqrt{\lambda}} = -2\log^{10}\left[\frac{2.51}{Re\sqrt{\lambda}} + \frac{k_s}{3.715 \cdot d_h}\right] \tag{20}$$

For a rough surface ($70 \le k_s^+$) λ can be calculated from:

$$\frac{1}{\sqrt{\lambda}} = 2 \cdot \log^{10} \cdot (3.715 \cdot \frac{d_h}{k_s}) \tag{21}$$

The goal of the work presented here, is to calculate the efficiency drop due to a specific surface roughness. According to Speidel [16] it can be stated that if the roughness remains within the laminar layer no additional losses will be produced, but only the losses due to the boundary layer itself. Therefore it is necessary first to calculate the thickness of the boundary layer and then to check, whether the roughness stays within this layer or not. In case it does, one has to use equation (19) to calculate λ and in case it does not, λ has to be computed by equation (20) or equation (21).

2.2 Turbulent boundary layers

A sketch of the near wall flow along a flat plate is presented in figure 1 in a qualitative manner. Starting with a laminar boundary layer at the beginning of a flat plate the flow turns to turbulent at the transition point developing a laminar sub-layer close to the wall.

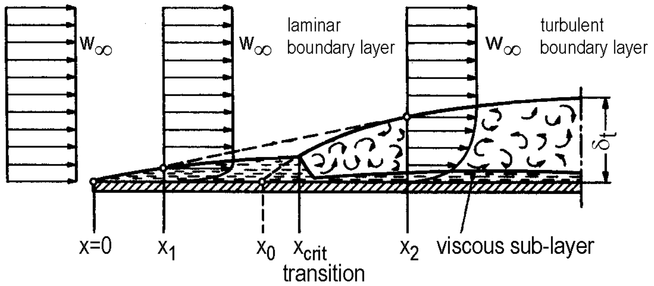

Figure 1: Definition sketch of a boundary layer

Following Schlichting [10] or the modified theory of laminar boundary layer flow over flat plates by Sohrab [17] the thickness of the laminar boundary layer can be calculated by:

$$\delta_{lam}(L) = \frac{5L}{\sqrt{Re_l}} \tag{22}$$

As the pressure difference along the channel walls cannot be neglected, at this point one has to use the more general formula given by Wagner [18]:

$$\delta_{lam}(L) = \left(\frac{32 \cdot \nu_1 \cdot L}{w \cdot p}\right)^{0.5} \tag{23}$$

which needs a special definition of the Reynolds number according to Traupel [8]:

$$Re_p = \frac{w \cdot p^* \cdot L}{\nu_1} \qquad ; \qquad \nu = \frac{\nu_1}{p^*} \tag{24}$$

After a certain distance the boundary layer becomes turbulent. The critical Reynolds number for transition is in the range of :

$$3.2 \cdot 10^5 \le Re_{crit} < 3.5 \cdot 10^5 \tag{25}$$

Following Klapdor [19], the thickness of the viscous sub-layer for a flat plate can be derived to:

$$\delta_{visc,plate} = \left[\frac{1}{0.0225}\right]^{\xi/(\xi-1)} \cdot \frac{\delta_t}{(32\,Re_{plate})^{3\xi/8(\xi-1)}} \tag{26}$$

whereas δ_t is the thickness of the turbulent layer:

$$\delta_t(L) = \left(\frac{0.1125}{4N}\right)^{0.8} \cdot \frac{L}{Re_L^{\ 0.2}} \tag{27}$$

and N after Wagner [18]:

$$N = \frac{\xi}{(\xi+1)\cdot(\xi+2)} \tag{28}$$

Also after Klapdor [19] for a fully developed flow in a pipe the viscous sub-layer can be calculated by:

$$\delta_{visc,pipe} = \left[\frac{M^{3/4} \cdot 2^{(4-\xi)/4\xi}}{0.0225}\right]^{\frac{\xi}{\xi-1}} \cdot \frac{d_h}{Re_{pipe}^{\ 3\xi/4(\xi-1)}} \tag{29}$$

with

$$M = \frac{2\xi^2}{(\xi+1)\cdot(2\xi+1)} \tag{30}$$

For moderate Reynolds numbers and hydraulically smooth surfaces in the literature the exponent of the boundary layer power law ξ often is chosen to ξ = 7 for simplification, but in the present contribution ξ is designated according to Nunner [20] by:

$$\xi = \frac{1}{\sqrt{\lambda}} \tag{31}$$

The thickness of the laminar boundary layer and the laminar sub-layer are decisive for developing of additional losses due to the surface roughness. When the roughness is smaller than the thickness of these laminar layers no additional impulse losses due to the surface roughness will be counted for. This situation is comparable to the flow along a hydraulically smooth surface.

3 Results

3.1 Design point of impeller

The foregoing formulas were implemented into a MATLAB® script which was used to investigate the influence of different parameters on the efficiency of centrifugal impellers. The computations were based on the design layout data of an impeller which had an outer diameter close to 300 mm. For the design point of this impeller the additional losses arising from surface roughness are referred to the efficiency of the impeller which was calculated for the same impeller data assuming a hydraulically smooth surface and then plotted versus the relative roughness R_r in figure 2 according to equ. 13.

First of all it is obvious that the efficiency deficit increases with increasing roughness of the impeller surfaces. For the considered range of roughness four different regions of the efficiency curve can be identified. Starting at the smooth surface (R_r=0) no additional losses are predicted. As mentioned before for very small R_r-values the peaks of the roughness are all covered by the laminar boundary layer or the by the viscous sub-layer and produce no additional losses. The critical value for the relative roughness $R_{r,crit}=2\cdot 10^{-4}$ which can be identified was also found before by other authors. From this point a noticeable efficiency deficit can be mentioned with increasing roughness.

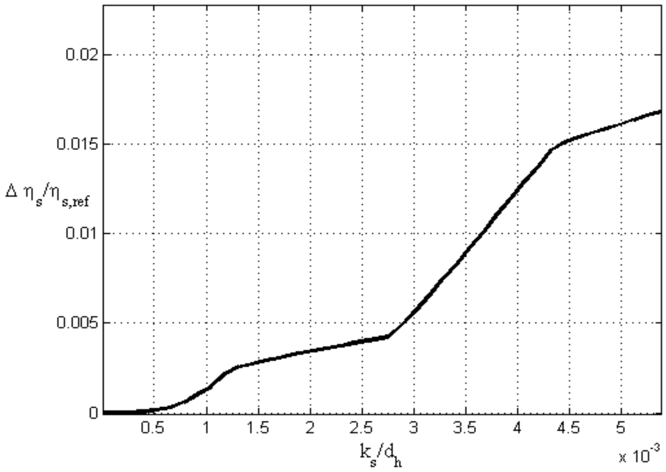

Figure 2: Additional losses versus the surface roughness for a shrouded radial compressor impeller

First a progressive rise of the losses with increasing roughness can be asserted. From a relative roughness $R_r>1.25\cdot 10^{-3}$ the efficiency decrease is nearly linear in three sections which have different gradients. This means, that there are some regions of surface roughness in which changes of the roughness will result in only slight changes of the impeller efficiency, but there are also regions in which slight changes of the surface quality will result in strong changes of the efficiency. The shape of the presented curve and especially the divers gradients are a consequence of the application of two boundary layer theories for the walls of the impeller flow channel: flat plate theory for the blade surfaces and the shroud disc and pipe theory for the hub disc.

The diverse gradients have an interesting effect on the statement which efficiency losses have to be expected for a certain surface roughness. In the roughness regions with a steep inclination it will be hard to say, what amount of efficiency loss exactly has to be expected due to the present surface roughness. But within the regions with a smaller gradient, the forecast accuracy for the expected losses will be much higher. Furthermore, in this region an excessive machining is only increasing costs but gives nearly no improvement of efficiency.

3.2 Parameter variation

In this section the impact of several parameters on the efficiency of a shrouded radial impeller due to surface roughness is evaluated. The parameters were in detail: intake pressure level; impeller pressure ratio; geometric size of impeller and the milling technique which is used for the manufacturing process.

3.2.1 Intake pressure level

The first quantity to investigate is the intake pressure level on which the impeller is working. Starting at atmospheric condition the intake pressure has been increased to the amount of 150 bar in several steps. In figure 3 the relative efficiency is shown versus the relative roughness for six intake conditions. The reference value $\eta_{s,ref}$ which has been chosen arbitrarily was the same for all calculations. By this method the impact of different pressure levels can be compared directly. With increasing intake pressure level the losses decrease for the hydraulically smooth case ($R_r \to 0$). This behavior can be easily explained, because the loss coefficient λ depends only on the Reynolds number in this region (equ. 19). As the Reynolds number is directly proportional to the pressure by the pressure dependence of the kinematic viscosity, the efficiency should be higher the higher the pressure is.

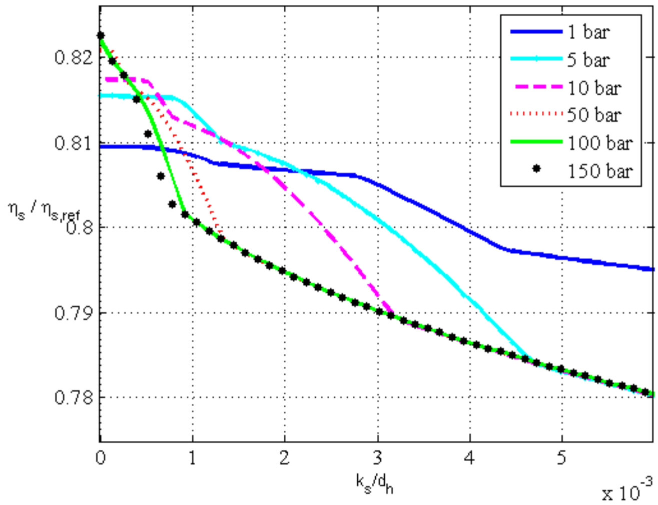

Figure 3: Influence of intake pressure level on the isentropic efficiency

For the hydraulically rough case the losses have the same amount for all pressure levels because the friction factor λ is only a function of the relative roughness in this region. In the region between the hydraulically smooth and the hydraulically rough case the friction factor is a function of the Reynolds number and the relative roughness (equ. 20). With increasing pressure level the influence of the roughness becomes stronger and the influence of the Reynolds number becomes weaker. This explains why the calculated efficiency curves meet the one for the hydraulically rough case earlier when the pressure level is increasing. Figure 3 shows that for impellers with an intake pressure level above the ambient pressure the surface roughness should be close to a hydraulic smooth surface. To avoid strong additional losses no inferior surface quality should be allowed. It holds, that the higher the intake pressure level, the smoother the surface should be, to avoid strong efficiency losses.

3.2.2 Impeller pressure ratio

The next parameter to examine is the pressure ratio of the impeller. For six different impeller pressure ratios the efficiency deficits have been calculated. Figure 4 shows the efficiency differences which again are referenced to the isentropic efficiency for the smooth surface versus the relative roughness. The first result, which can be taken from the figure is that the critical relative roughness $R_{r,crit}$, where the additional losses start to increase, does not depend on the pressure ratio of the impeller. The loss curves for all investigated pressure ratios are nearly congruent up to a relative roughness of $R_r = 2.8 \cdot 10^{-3}$ (exemption: $R_r \approx 1$). Second: for the hydraulically rough case the losses are higher the higher the pressure ratio is. The gradients of the curves become stronger with increasing pressure ratio.

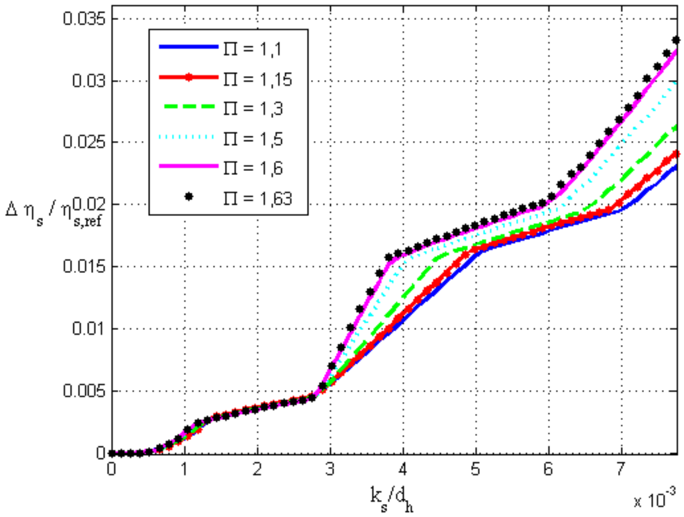

Figure 4: Impact of pressure ratio on the additional losses

3.2.3 Geometric size of the impeller

The impact of the hydraulic diameter d_h of the impeller on the isentropic efficiency is shown in figure 5.

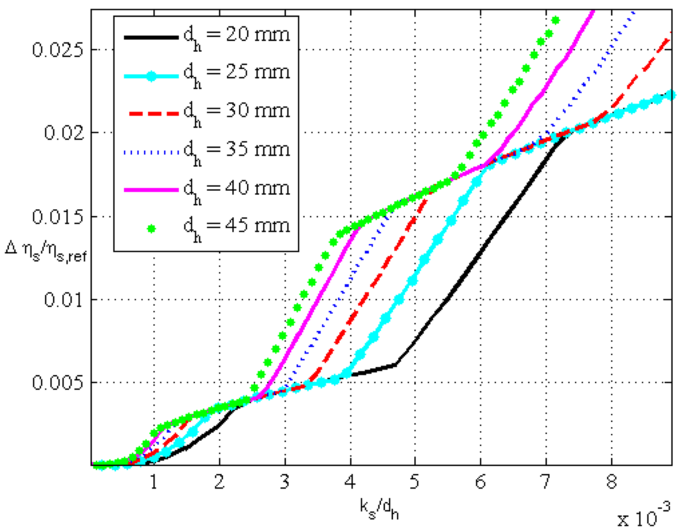

Figure 5: Influence of hydraulic diameter on the additional losses

For the calculations all impeller parameters were kept the same, merely the hydraulic diameter was changed from the original value to some bigger and some smaller amounts. With increasing hydraulic diameter the critical relative surface roughness $R_{r,crit}$ becomes smaller. That means: for impellers with a smaller diameter d_2 the losses start to increase at a smaller quantity of relative roughness. At the same time the region with high forecast exactness shifts to higher surface roughness values. Nevertheless, the width of this region is not affected by the size of the impeller.

4 Conclusions

In the present paper the influence of the surface roughness on the efficiency of shrouded radial compressor impellers is examined on a theoretical basis. The investigation was accomplished for subsonic flow. The obtained results may not be absolutely correct in quantity but by a parameter variation the right tendencies could be shown. By implementing the basic equations into a software code the following results were achieved:

1) A critical surface roughness can be obtained below which no additional losses are recognized.

2) The variation of the most important parameters provide some diagrams from which the expected additional losses due to the surface roughness can be estimated. The most important results are that the higher the intake pressure level, the more the losses depend on the surface roughness and that in the hydraulic rough region the compression ratio has also a strong influence on the impeller efficiency.

3) Especially for shrouded radial impellers which are manufactured by milling out of one piece of raw material it is important to define a certain milling strategy. The program may help to decide whether it is more cost efficient to manufacture higher in-line grooves or smaller crossways grooves. The losses which are determined for the achieved surface quality can also help to find a decision about the number of required finishing sequences.

As the present examination is strongly based on a theoretical background the achieved results need to be validated by some experiments. The influence of leakage and secondary flows should also be included in the theoretical investigation.

Nomenclature:

<u>Arabic letters</u>

A	m^2	area
b	m	channel width
$c_{d,c}$	-	loss coefficient for impeller
c_f	-	skin friction coefficient
c_{n0}	-	velocity normal to intake plane
d	m	diameter
d_h	m	hydraulic diameter of flow channel

e	-	blocking factor due to impeller blade
h	m^2/s^2	specific enthalpy
j	m^2/s^2	specific losses
k_s	m	sand roughness
$k_s^+ = k_s / \delta_{visc}$		non-dimensional sand roughness
L	m	length of a flat plate
$\dot{m}$	kg/s	mass flow rate
n	-	polytropic exponent
p^*	-	pressure ratio influencing the viscosity
r	m	radius
Re	-	Reynolds number
$R_r = k_s/d_h$		relative roughness
R_t	m	technical roughness
s	m	surface length
u	m/s	circumferential speed of impeller
$\dot{W}$	Nm/s	dissipation energy
w	m/s	relative velocity in the flow channel
$z"$	-	number of impeller blades

Greek letters

β	deg	relative flow angle
Δ	-	difference
δ_t	m	turbulent boundary layer
δ_{visc}	m	viscous sub-layer
η	-	impeller efficiency
λ	-	pipe friction factor
ν	m^2/s	kinematic viscosity
ν_1	m^2/s	reference viscosity at pressure of 1bar
ξ	-	exponent of boundary layer power law
Π	-	pressure ratio
ρ	kg/m^3	fluid density
τ	N/m^2	wall sheer stress

Subscripts

0	intake plane
1	impeller entry
2	impeller exit
crit	critical
des	design
H	hub
lam	laminar
ref	reference
s	isentropic
S	shroud
visc	viscous
∞	infinite

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