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SUBSONIC GAS-PARTICLE TWO-PHASE FLOW IN PIPES

Mofreh H. Hamed
Mechanical Power Engineering Department,
Faculty of Engineering, Menoufiya University,
Shebin El-kom, Egypt

Abstract - A mathematical model for subsonic two-phase flow in pipes is presented. The model takes into account the momentum and heat transfer between the gas and the particle phases. The wall surface roughness and the coupling effect are also considered. In addition effects of loading coefficient and particle diameter on the subsonic and choked flow behaviour of gas-solid flow are investigated. The present study covers three separate cases firstly the case of no heat transfer (adiabatic flow), secondly the case of heat transfer (heating flow) and the third one is the case of heat transfer (cooling flow). The three cases are studied for equal gas and particle temperatures and also in the case of hot particle and cold gas. Results of present predictions are compared with the published data and a reasonable degree of agreement is obtained. The validation has proved that the present model adequately predicts the basic flow parameters in many aspects of the flow of gas-solid mixtures at low and high speeds.

Keywords: Gas-particle suspension, pipe, heat transfer, friction, loading coefficient, roughness, coupling parameters.

1 Introduction

Predictions of particle transport in gas flow are important in studying the particle behaviour in dust collectors, combustors and heat exchanger conductor devices. The injection of powder into metals is becoming increasingly important for steel refining processes.

There have been many experimental and theoretical studies on two-phase flow through pipes [1-10]. Thakurta et al. [1] have used direct simulation to compute numerically the thermophoertic deposition rate of small particles of turbulent channel flow. Young et al. [2] have investigated numerically the thermophoresis phenomenon taking into account the particle radiation. The radiation effects by both gas and particles on particle transport due to thermophoresis have been studied. A steady state one dimensional flow model at a very low velocity has been presented by Irene et al. [3-4]. Numerical simulation of gas-solid two-phase flow in a two dimensional channel was given by [5-7]. In these studies, some effects such as coupling effect and velocity lag are absent. The modeling of gas-particles flows in horizontal channels with different wall roughnesses was made by Sommerfeld et el. [8] and Eskin, [9]. While Modeling dilute gas-particles flows in constant area lance with heating and friction was made by Han et al. [10].

Based on this review, a classification of dispersed two-phase flows with regard to the importance of interaction mechanisms was provided by [11]. A two-phase system may be regarded as dilute for small and moderate volume fraction, α_p (up to 10^{-3}). Therefore, in this regime the influence of particle phase on the fluid may be neglected. On the other hand, for higher values of volume fraction, the influence of particle phase on the fluid flow, which is often referred to as two-way coupling, needs to be accounted for.

It is concluded that the problem of gas-particle flow through pipes has been studied by many authors. However, many details remain unknown. Therefore, the aim of this study is to investigate the effects of thermal radiation, heat transfer from or to the wall, and coupling between gas and particle phases on the subsonic and choked flow of gas-solid in pipes. In addition, the effects of some parameters such as, loading coefficient and wall roughness are also investigated.

2 Mathematical Model

To formulate the model, a quasi-one dimensional situation has been considered.

2.1 Basic assumptions

In order to study the subsonic and choked gas-solids flow in pipes, a simplified mathematical model is introduced. The essential physical features of the problem are retained. The following assumptions are considered:

a- The flow is one-dimensional and steady.
b- The particles are spherical in shape.
c- The radiative properties of gas and particles are gray.
d- The heat transfer between the duct wall and particles is negligible.
e- The particle density is constant, (ρ_p = constant).

2.2 Governing equations

Based on the assumptions mentioned above, and considering a central surface shown in Fig. (1), the governing equations for both continuous phase (carrier gas) and discrete phase (steel particles), coupling parameters and complementary equations can be derived according to the basic laws of fluid mechanics as reported in [4 ,12] as follows,

2.2.1Continuous phase

Continuity equation

The continuity equation for the continuous phase can be written as,

$$\frac{1}{A}\frac{\partial}{\partial x}(\alpha_c\,\rho_c U\,A) = S_{mass} \qquad (1)$$

where, S_{mass} is the mass coupling parameter

Momentum equation

The momentum equation for the continuous phase can be expressed as,

$$\frac{1}{A}\frac{\partial}{\partial x}(\alpha_c\rho_c U^2 A) = -\alpha_c\frac{\partial P}{\partial x} + \alpha_c\,\rho_c\,g - \frac{1}{R_h}\tau_w + S_{mass}.V + S_{mom_p} \qquad (2)$$

where, S_{mom_p} is the momentum coupling parameter.

Energy equation

The differential form of the total energy equation for the continuous phase is,

$$\frac{1}{A}\frac{\partial}{\partial x}[\alpha_c\rho_c UA(h_c + \frac{U^2}{2})] = -\frac{q'_w}{R_h} - \frac{1}{A}\frac{\partial}{\partial x}(Aq'_c) + \alpha_c\rho_c gU - \frac{P}{A}\frac{\partial}{\partial x}(\alpha_p VA) + S_{mass}.(h_s + \frac{V^2}{2}) + S_{energy_p} \qquad (3)$$

where, S_{energy_p} is the energy coupling parameter and q_c' is the effective heat transfer through both phases across surface 1. In most applications, the heat transfer through both surfaces 1 and 2 is small compared with the enthalpy flux. While, q_w' is the heat transfer per unit length from or to the wall.

2.2.2 Discrete phase

The particle velocity equation

The equation of motion for a particle in a gas is,

$$\frac{dV^2}{dx} = \frac{2.C_D}{\tau_v}(U - V) + 2g - 4f_s\frac{V^2}{D_p} \qquad (4)$$

where, $\left(\tau_v = \left(\rho_p D_p^2/18.\mu_c\right)\right)$ is defined as the velocity response time.

The particle temperature equation

The equation for particle temperature, assuming the temperature is uniform throughout the particle and including the radiative heat transfer is,

$$V.\frac{dT_p}{dx} = \frac{Nu}{2.\tau_T}(T_c - T_p) - \frac{6.\sigma.\varepsilon}{D_p.C_S.\rho_p}(T_p^4 - T_w^4) \qquad (5)$$

where, $\left(\tau_T = \left(\rho_p D_p^2 C_S/12.k_c\right)\right)$ is defined as the thermal response time.

The particle Nusselt number Nu, can be calculated as reported in [10 and 12] by,

$$Nu = 2.0 + 0.6Re_p^{1/2}Pr^{1/3} \quad , 1 \le Re_p \le 70000$$
$$Nu = 2.0 \quad , \quad Re_p < 1 \qquad (6)$$

where, Re_p is defined as,

$$Re_p = \left(|U - V|\rho_c D_p/\mu_c\right)$$

2.2.3 Coupling parameters

An important concept in the analysis of two-phase flow is to consider the coupling parameters effect between two phases. If the flow of one phase affects the other while there is no reverse effect, the flow is said to be one-way coupling. When a mutual effect exists between the two phases, the flow is called two-way coupling.

Momentum coupling

The momentum coupling source term due to the reverse effect of particles is,

$$S_{mom_p} = 3\,n\,\pi\,C_D\,\mu_c\,D_p\,(V - U)$$

The number density n is related to the volume flow rate and the particle mass flow rate respectively by ($n' = A\,V\,n$) and ($M'_P = n'\,m_P$).

Then, momentum coupling source term is expressed as,

$$S_{mom_p} = \frac{Z_L M'_c C_D (V - U)}{\tau_v.V.A} \qquad (7)$$

where, Z_L is defined as the ratio between the particles mass flow rate and continuous phase mass flow rate, ($M'_P / M'_C = (\alpha_P AV\rho_P)/(\alpha_C AU\rho_C)$)

Energy coupling

The energy coupling source term for the total energy equation evolves from convective heat transfer and the work done due to particle drag. The coupling source term due to convective heat transfer from the particles to the gas phase in the control volume is,

$$S_{energy_{p,conv.}} = \frac{Nu.c_p\,M'_c\,Z_L}{3\,\tau_v\,Pr.V.A}(T_p - T_c)$$

The work done due to particle drag is,

$$S_{energy_{p,drag}} = S_{mom_p}.V$$

Then energy coupling is the sum of the two terms,

$$S_{energy_p} = S_{energy_{p,conv.}} + S_{energy_{p,drag}} \qquad (8)$$

2.2.4 Complementary equations

Equations (1) to (5) form a set of differential equations. In order to solve this set of equations several complementary equations, definitions and empirical correlations, are required.

1- Generally, the total friction is defined as the sum of the gas and the particle friction coefficients, which is expressed as [10],

$$f = f_c + \frac{\lambda}{1-\lambda}f_s \qquad (9)$$

Continuous phase friction can be represented with the Darcy-Weisbach friction factor and as reported in [13] by,

$$f_c = 0.25.\left[0.434\,\ell n\frac{RR}{3.7} + \frac{5.74}{Re^{0.9}}\right]^{-2}$$

where, RR is the ratio between the roughness heights to pipe diameter. While, Re is the continuous phase Reynolds number and defined by,

$$Re = \left(U\,\rho_c\,d/\mu_c\right)$$

In addition, the friction factor between particles and the wall of the pipe as reported in [10] is,

$$f_s = 1.0503.Fr_p^{-1.831}$$

Where, Fr_p is the particle Froude number and written as,

$$Fr_p = \left(V / (g.d)^{0.5} \right)$$

2- Since the volume fraction of the dispersed phase and the continuous phase is unity, the continuous phase volume fraction is:

$$\alpha_c = (1 - \alpha_p) \qquad (10)$$

3- The drag factor C_D can be taken as reported in [1, 5, 13 and 14].

4- The equation of state for the gas phase is written as;

$$P = \rho_c R_c T_c \qquad (11)$$

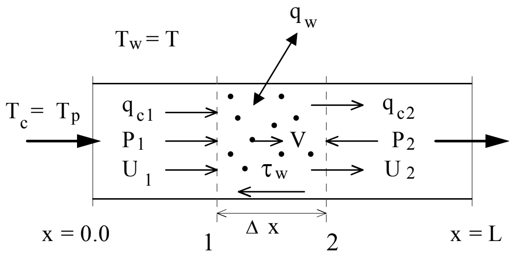


Fig. 1 Duct geometry and control volume.

2.2 Initial and boundary conditions

In the present numerical analysis, calculations have been carried out for a gas-particle flow composed of air and steel particles.

2.2.1 Initial conditions

The initial values of gas density, gas volume fraction and particle velocity are as follows:

$$\rho_{co} = \frac{P_o}{R_c T_{co}}, \quad \alpha_{co} = \frac{1}{1 + \frac{\lambda}{1-\lambda} . \frac{\rho_{co} . U_o}{\rho_p . V_o}}, \quad S_{Vo} = V_o / U_o \qquad (12)$$

The two cases studied are described as in Tables 1 and 2.

Table 1 Equal gas and particle temperatures, **Case-1**

Temperature	Adiabatic-1	Cooling-2	Heating-3
Gas	293 K	1000 K	293 K
Particle	293 K	1000 K	293 K
Wall	293 K	293 K	1000 K

Table 2 Hot particles and cold gas, **Case-2**

Temperature	Adiabatic-1	Cooling-2	Heating-3
Gas	293 K	293 K	293 K
Particle	500 K	500 K	500 K
Wall	293 K	293 K	1000 K

Within this temperature range, the properties of air such as, specific heat and Prandtl number are approximately constant. While the other properties of air are taken as a function of air temperature as reported in [15].

2.2.2 Boundary conditions

The heat transfer per unit length from the wall to the gas is given as a function of pipe wall temperature and
is expressed as in [4 , 13] by,

$$q'_w = Stn . c_p (T_w - T_c) \frac{4A}{d_h} . \rho_c U \qquad (13)$$

where, Stn is the Stanton number defined as in [4 , 11] by,

$$Stn = \left(h_{pipe} / \left(\rho_c \, U \, c_{p_c} \right) \right)$$

The convective heat transfer coefficient, h_{pipe}, is simply calculated from

$$h_{pipe} = \left(Nu_{pipe} . k_c / d \right) \qquad (14)$$

where, Nu_{pipe} is the Nusselt number, defined as in [4]. When the pipe wall is insulated such that the operating condition of the flow is adiabatic, $q'_w = 0.0$. Otherwise it is specified as a function of the pipe wall temperature using Eqn. (14).

2.3 Solution procedure

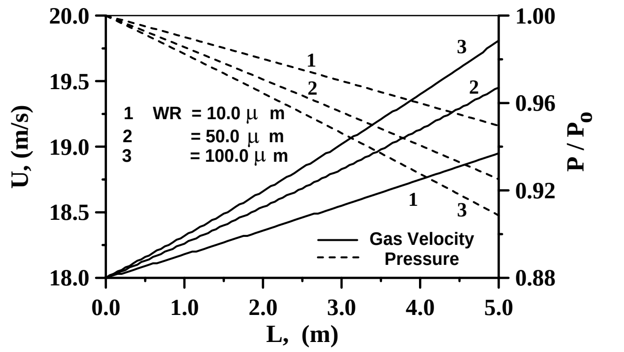
The solution for the system of equations, Eqns. (1-5) with the help of coupling equations (7–8) and complementary equations (9-11) are solved numerically by using an iterative approach. This approach is a marching solution in which the inlet conditions are specified and the computer program calculates the flow properties in the duct cell by cell. In the case of subsonic flow, the iteration is continued until the pressure or velocity no longer changes with continued iterations and the procedure is repeated until the end of the duct. While for a specified choked duct length the initial velocity has to be iteratively found such that a sonic condition is achieved at the duct exit.

3 Results

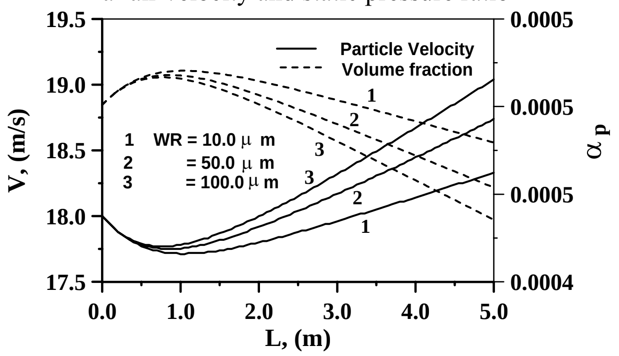
The developed model has been coded in a versatile program to facilitate the study of many aspects of the flow of gas-solid mixtures at low speed. Although the theoretical results have covered a wide range of gas flow rates and solid loadings, only a sample of typical results have been presented here. The continuous phase flow through a duct of 6 cm diameter is air of initial pressure and velocity of 1.1 bar, and 18 m/sec respectively. While the discrete phase flow is steel particles of density 2500 kg/m^3 and is charged with the flow of air at different loading coefficients. The steel particles has different diameters and have also different temperatures but with the same gas velocity. In each case, the computed variations of gas velocity (U), gas pressure (P), particle velocity (V), particle volume fraction (α_p) gas temperature (T_c), and particle temperature (T_p) along the length of the pipe are shown in Figures (2-6).

Figure (2) shows the behavior of the gas-particles flow in the case of adiabatic flow conditions along the pipe axis with different wall roughnesses. It is obvious that the wall roughness has a strong effect on the gas and particle behaviour in the case of adiabatic conditions. It can be also seen that increasing the wall roughness, will increase the slip velocity coefficient. This is caused mainly by an increase in the momentum loss for the particle phase.

Results of Fig. (3) depict flow in case-1, where the inlet gas and particles are at the same temperature and velocity. For adiabatic flow, the figure show that, the pressure decreases almost linearly along the pipe axis but with higher rate in the case of heating flow as shown in Fig. (3.a). It can be seen also that the gas and particles velocities and temperatures are approximately constant, Fig. (3.b and c). Also, the gas and particles velocities are increased along the axis of the pipe for the heating flow as shown in Fig. (3). This is due to the heat transferred from the wall to the gas and consequently to the particles. It is also seen that the volume fraction of the particles is decreased rapidly in the inlet zone of the pipe which is referred to the acceleration of the particles and the fact that mass flow rate of particles is constant. Furthermore, the particle velocity increases with a small rate along the rest of the pipe length accompanied with a slowl decrease in particle volume fraction. On contrary of heating flow is the case of cooling flow as shown in Fig. (3).



a- air velocity and static pressure ratio

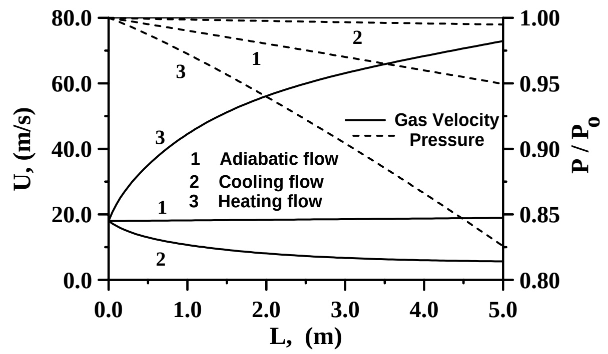


b- particle velocity and volume fraction

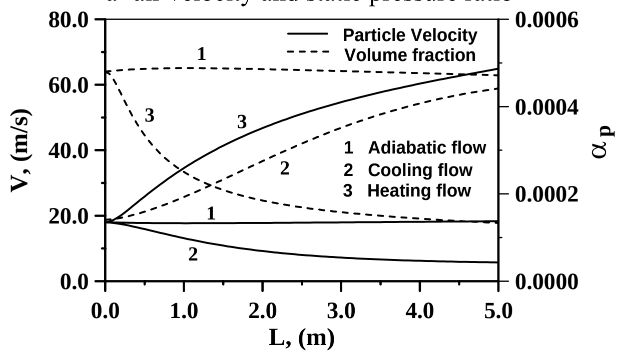
Fig. 2 Effect of pipe roughnesses on flow behaviour.

Plots of Fig. (4) present the variation of the outlet gas velocity, gas pressure drop, outlet particle velocity, in the three cases of no heat transfer (adiabatic conditions), heat transfer (cooling flow) and heat transfer (heating flow) along the pipe axis at different loading coefficients, Z_L respectively. It can be seen that in the case of adiabatic flow the loading coefficient Z_L, has approximately no effect on the flow of gas-particles behaviour. While in the case of heat transfer (cooling flow) it is noticed that the outlet gas velocity is increased as the loading coefficient was increased. On the other side, for heating flow the outlet gas velocity was decreased as the loading coefficient is increased.

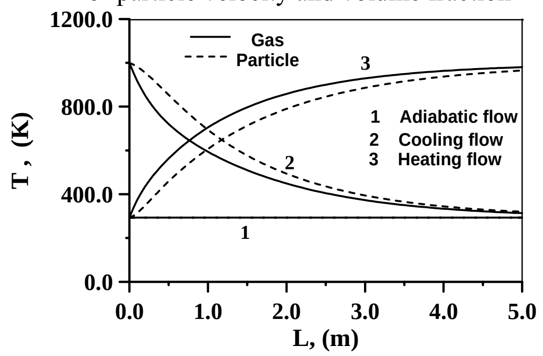
Results of hot particles suspended in the cold gas flow but with the same inlet velocity is shown in Fig. (5), Case-2. From this figure it can be noticed that in the case of adiabatic flow, the gas velocity is increased with higher rate in the inlet zone of the pipe and approximately constant along the rest of the pipe length as shown in Fig. (5.a). This is may be due to the heat transferred from the hot particles to the gas till the temperatures of both particles and gas becomes the same as shown in Fig. (5-c). While in the case of heat transfer (cooling flow) at the same conditions mentioned above the same behaviour is seen as in adiabatic flow as shown in Fig. (5). From this figure it is also seen that the gas and particles velocities are decreased along the rest of the pipe length as a result of heat transferred from the gas and particles to the wall as shown in Fig. (5). It is also seen that the volume fraction of the particles is decreased rapidly in the inlet zone of the pipe due to the acceleration of the particles and then increases to compensate the decrease in particles velocity along the rest of the pipe length. In the case of heat transfer (heating flow) the gas and particle velocities and temperatures are increased and the volume fraction of particles is decreased.



a- air velocity and static pressure ratio



b- particle velocity and volume fraction



c- air and particle temperatures

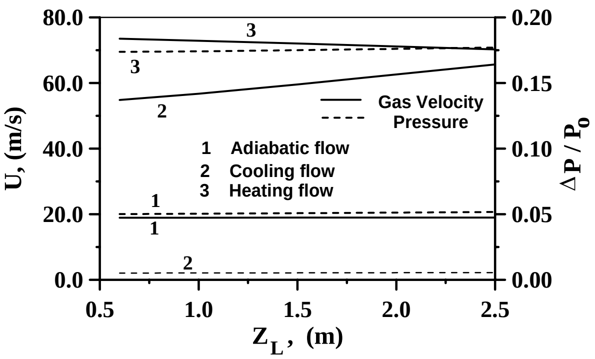
Fig. 3 Effect of heat transfer on flow behaviour, Case-1

Plots of Fig. (6) depict the air velocity in the case of choked flow of air-steel particles through a pipe of 7/8 inch at the same conditions mentioned in [16]. Also it is seen that for a given pipe length (choked length), the initial velocity has to be found by iteration such that the sonic condition is achieved at the pipe exit as shown in Fig. (6.a). To validate the present model, a comparison between the predicted flow velocity and published data reported in [1], at the choking conditions is shown in Fig. (6.b). The calculations were carried out at the same conditions reported in [16], (adiabatic flow). The comparison shows a satisfactory agreement between the present model and published data. The discrepancy between the present results and that of [16] may be due to the wall roughness and the heat radiation which have not been considered in [16].

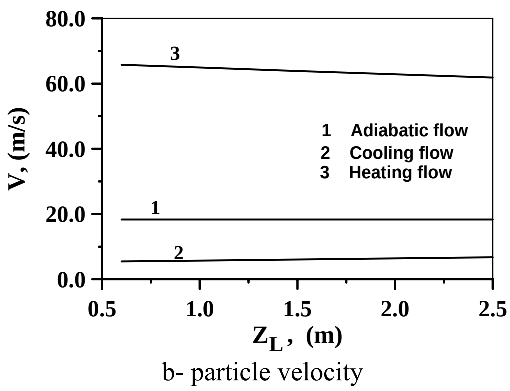
4 Conclusion

In the present study, a mathematical model describing the behaviour of gas-particle flow in pipes with heating and friction in the case of subsonic and choked flow has been presented. The present model has been thoroughly validated by comparison of the calculated results with data reported in the literature. From the obtained results the following conclusion can be drawn:

- The loading coefficient has no effect on exit parameters of adiabatic gas-particle flow while in the case of cooling and heating flow it has a considerable effect.
- The rate of increase of exit parameters of gas-particles flow in the case of cooling flow is greater than that in the case of heating and adiabatic flow.
- The pipe roughness has a strong effect on flow parameters.
- For a given duct length (choked length), the initial velocity has to be found by iteration such that the sonic condition is achieved at the pipe exit.

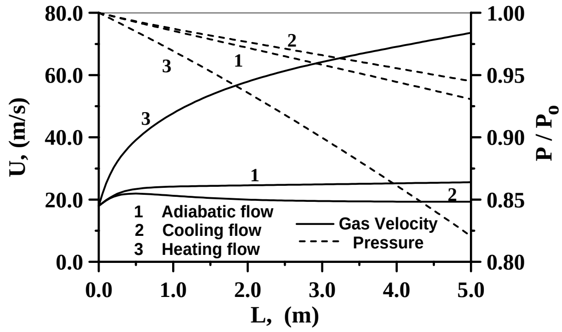


a- air velocity and static pressure ratio

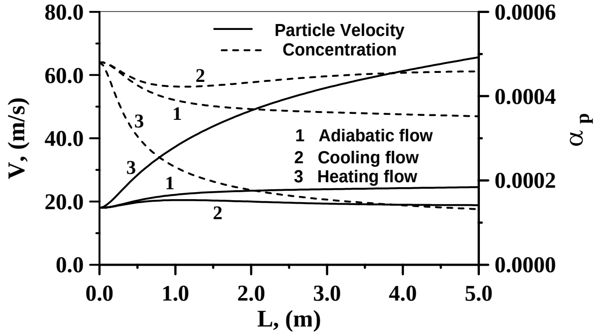


b- particle velocity

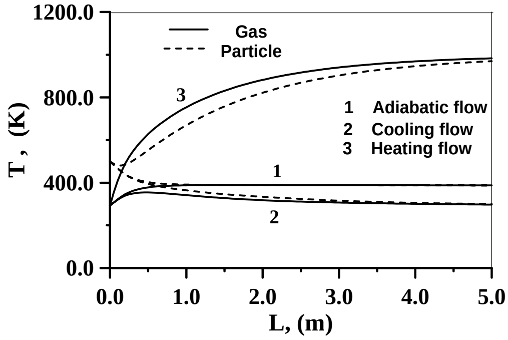
Fig. 4 Effect of loading coefficient on outlet flow conditions.



a- air velocity and static pressure ratio



b- particle velocity and volume fraction



c- air and particle temperatures

Fig. 5 Effect of heat transfer on flow behaviour, Case-2

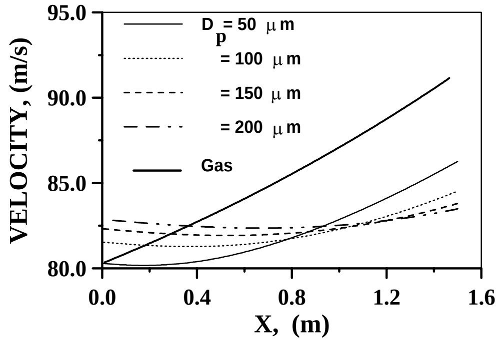


Fig. 6.a Effect of pipe length on initial conditions.

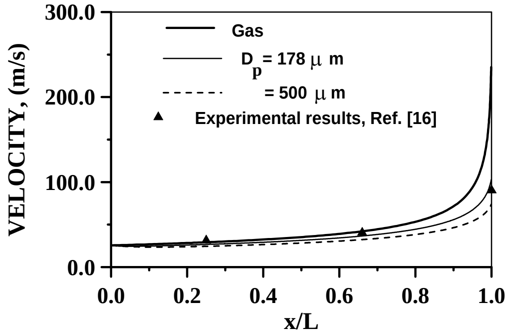


Fig. 6.b Comparison between present and published data.

NOMENCLATURE

A	area,	m^2
C_{pc}	specific heat of gas phase,	$J.kg^{-1}.K^{-1}$
C_s	particle specific heat,	$J.kg^{-1}.K^{-1}$
d	pipe diameter,	m
D_p	particle diameter,	m
f	friction coefficient,	-
C_D	particle-gas drag coefficient,	-
g	gravity acceleration,	$m.s^{-2}$
h	enthalpy,	$J.kg^{-1}$
k_c	gas thermal conductivity,	$W.m^{-1}K^{-1}$
L	pipe length,	m
M_c'	gas mass flow rate,	$kg.s^{-1}$
M_p'	particles mass flow rate,	$kg.s^{-1}$
Nu	Nusselt number,	-
N	no. of particles /volume particles.	m^{-3}
n'	particle volumetric rate,	s^{-1}
P	pressure,	$N.m^{-2}$
R_c	gas constant,	$J.kg^{-1}.K^{-1}$
R_h	hydraulic radius,	m
Re	Reynolds number,	-
S_V	slip velocity coefficient, (V/U),	-
Stn	Stanton number,	-
T	temperature,	K
U	gas velocity,	$m.s^{-1}$
V	particle velocity,	$m.s^{-1}$
WR	wall roughness height,	μm
X	axial length,	m
Z_L	loading coefficient,	-
Geek Letters		
α	volume fraction,	-
Δx	increment in distance,	m
ε	emissivity,	-
λ	coefficient used in Eq. (5), [ZL/(ZL+1)],	-
σ	Stephan - Boltzman constant,	$W.m^{-2}K^{-4}$
μ	air viscosity, $\mu_c = \mu_o\sqrt{T_o / T_c}$	$N.s.m^{-2}$
ρ	density,	$kg.m^{-3}$
τ_T	thermal response time,	s
τ_V	velocity response time,	s
τ_w	wall shear stress,	$N.m^{-2}$
Subscripts		
c	continuous phase	
co	inlet condition of continuous phase	
e	outlet	
p	particle	
s	particle surface	
vo	initial velocity slip coefficient	
w	wall	

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