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Vectorial Analysis of the Tapered Dielectric Waveguides

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Abstract- Different types of tapered waveguides are analyzed by full vectorial method. Mode expansion and mode orthogonality are also used. Full vectorial without approximation in the wave equation makes the analysis more accurate and closer to the real problem.

Key-Words: Tapered dielectric waveguide, finite difference

1-Introduction

In some of the optical components such as couplers and switches, waveguides with different dimensions (cross section) have to be connected directly. This increases the loss and the reflection (return wave) at the interconnection point. In recent yours several solutions are proposed to reduce the effect of this problem, where using a tapered waveguide at the inter connection is a well known method.

In order to analyze the tapered waveguide, beam propagation method (BPM) is used conventionally. However, this method with full vectorial analysis is not accurate due to large approximations [1]. Several other methods are proposed in references [2-6].

In this paper, different types of tapered waveguides are introduced, and then finite difference method is applied to the non approximate wave equation. Analysis is done fully vectorial in 3-dimension by resolving the wave equation into orthogonal modes.

2-Tapered waveguides

We have investigated four types of tapered waveguides shown in fig1. In fig 1 (a) and (b), the widths of the waveguides are varying in z direction, where their heights (thickness) are constant.

In the waveguides shown in Fig 1 (c) and (d), both of the widths and thicknesses of the waveguides are varying in z direction. It is clear that the variations of the width and/or thickness have to follow a fixed relation. This relation can be linear as in fig 1 (a) and (c) or nonlinear (exponential) as in Fig 1 (b) and (d).

In Fig 2, the connection of two waveguides with different widths are shown. In Fig 2 (a), the connection is done directly without any tapered waveguide. In this case the loss is large and the reflected wave is significant. However, in fig 2 (b), a tapered waveguide is used to connect two different waveguides.

In this case the loss and reflected wave are lower. In fact, appropriate dimension

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of the tapered waveguide and a proper variation of dimension reduce the effect of discontinuity in wave propagation greatly.

3-Analysis of tapered waveguides

One approach to analyze the tapered waveguide is dividing the waveguide into m section each having the length Δz in z direction as shown in fig.3. The proper amount of Δz depends on the rate of variation of the tapered waveguide, i. e. the amount of $(d_1\text{-}d_2)$ and *l*.

In order to find the propagation constant β and field distribution of each propagating modes in all directions, finite difference method is used to solve the 2-dimensional wave equation (1). [8].

$$\nabla^2 H + n^2 k^2 H + \frac{1}{n^2}\nabla n^2 \times (\nabla \times H) = 0 \qquad (1)$$

In this equation n is the matrix of refraction coefficient at the points in the cross section of the waveguide.

Considering a very low variation of n versus Z, equation (1) can be written as:

$$\begin{aligned} &\nabla^2 H_x + n^2 k^2 H_x - \frac{1}{n^2}\frac{\delta n^2}{\delta y}\left(\frac{\delta H_x}{\delta y} - \frac{\delta H_y}{\delta x}\right) = 0 \\ &\nabla^2 H_y + n^2 k^2 H_y - \frac{1}{n^2}\frac{\delta n^2}{\delta x}\left(\frac{\delta H_y}{\delta x} - \frac{\delta H_x}{\delta y}\right) = 0 \end{aligned} \qquad (2)$$

We introduce u and w to write H_x and H_y in the following forms:

$$\begin{aligned} H_x &= u.\exp(-j\beta z) \\ H_y &= w.\exp(-j\beta z) \end{aligned} \qquad (3)$$

Where u and w are not functions of z. Now application of finite difference method to both equations given in (2) and using eq. (3), results the following equations:

$$\begin{aligned} &\frac{1}{\Delta x^2}u(i+1,j) + \Big(\frac{2}{\Delta x^2} - \frac{2}{\Delta y^2} + n^2k^2 \\ &+ \frac{1}{\Delta y^2}\frac{n(i,j+1)-n(i,j)}{n(i,j)}\Big)u(i,j) + \frac{1}{\Delta x^2}u(i-1,j) \\ &+ \Big(\frac{1}{\Delta y^2} - \frac{1}{\Delta y^2}\frac{n(i,j+1)-n(i,j)}{n(i,j)}\Big)u(i,j+1) \\ &+ \frac{1}{\Delta y^2}u(i,j-1) + \frac{1}{\Delta x\Delta y}\frac{n(i,j+1)-n(i,j)}{n(i,j)}w(i+1,j) \\ &+ \frac{1}{\Delta x\Delta y}\frac{n(i,j+1)-n(i,j)}{n(i,j)}w(i,j) = \beta^2 u(i,j) \end{aligned} \qquad (4)$$

$$\begin{aligned} &\Big(\frac{1}{\Delta x^2} - \frac{1}{\Delta x^2}\frac{n(i+1,j)-n(i,j)}{n(i,j)}\Big)w(i+1,j) + \\ &\Big(\frac{2}{\Delta x^2} - \frac{2}{\Delta y^2} + n^2k^2 + \frac{1}{\Delta x^2}\frac{n(i+1,j)-n(i,j)}{n(i,j)}\Big)w(i,j) \\ &+ \frac{1}{\Delta x^2}w(i-1,j) + \frac{1}{\Delta y^2}w(i,j+1) \\ &+ \frac{1}{\Delta y^2}w(i,j-1) + \frac{1}{\Delta x\Delta y}\frac{n(i+1,j)-n(i,j)}{n(i,j)}u(i+1,j) \\ &+ \frac{1}{\Delta x\Delta y}\frac{n(i+1,j)-n(i,j)}{n(i,j)}u(i,j) = \beta^2 w(i,j) \end{aligned} \qquad (5)$$

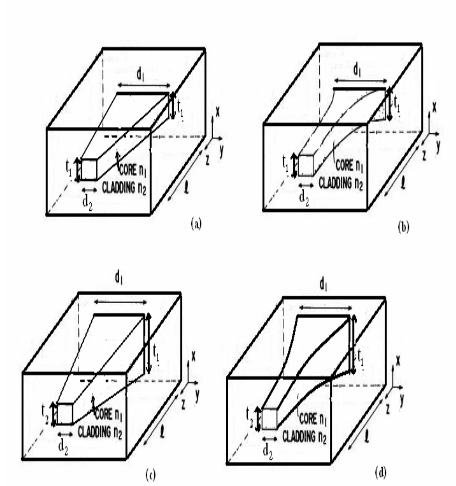

Fig 1: Four types of tapered waveguides

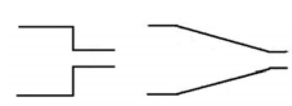

Fig2: Connection of two waveguides with different widths

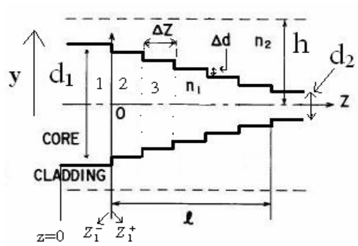

Fig3: The step approximation of a tapered waveguide

It is clear that, we have to solve the wave equation in the difference form, (eqs 4 and 5) at all the nodes of the whole waveguide. The resulting system of equations has 3 unknowns β, u, and w. Applying some mathematical manipulations we obtain the following system of eq.

$$\begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix}_{N_x N_y, N_x N_y} \begin{bmatrix} u \\ w \end{bmatrix}_{N_x N_y, 1} = \beta^2 \begin{bmatrix} u \\ w \end{bmatrix}_{N_x N_y, 1} \tag{6}$$

where, N_x and N_y are the numbers of nodes in x and y directions respectively. Matrix B contains the coefficients of vectors u and w as given in eqs (4) and (5). Solving the system of eq. (6) gives the eign values & eigen vectors, i. e. H_x, H_y and β in the matrix form. To include the boundary conditions, we consider the boundary of the cladding region in a place beyond that the amplitude of each mode is almost zero. On the other hand, the number of equations in system (6) depends on the number of mesh points in the cross section of the waveguide. The large number of mesh points makes the result more accurate, but the solution to the problem becomes more complicated or sometimes impossible. To prevent this problem, we do inhomogeneous meshing. In this type of meshing, the mesh size is small where the wave has large variations as in the core, whereas in the places with low variations as in the cladding, the mesh size is large. Fig 4 shows this type of meshing in the core and cladding of a waveguide.

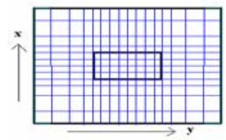

Fig4: Nonuniform meshing in the core and cladding of a waveguide.

Numerical solution to the Maxwell equation and using the matrices of H_x and H_y, give the other vectors of E and H as

$$E_z = \frac{1}{jw_0 n^2}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right), E_x = -\frac{1}{j\beta}\left(-jw_0\mu_0 H_y + \frac{\partial E_z}{\partial x}\right)$$
$$E_y = \frac{1}{j\beta}\left(-jw_0\mu_0 H_y - \frac{\partial E_z}{\partial y}\right), H_z = -\frac{1}{jw_0\mu_0}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \tag{7}$$

By eq. (7), the field distributions of all the propagating modes in the waveguide are determined. Now we can analyze the tapered waveguide in z direction. The exciting field into the tapered waveguide can be written as:

$$\begin{aligned} E_{in} &= E_x a_x + E_y a_y + E_z a_z \\ H_{in} &= H_x a_x + H_y a_y + H_z a_z \end{aligned} \tag{8}$$

From Fig 3, it is clear that the input to the tapered waveguide is the input to the section 1 of the staircase model (fig.3). Expansions of the these waves into the modes are expressed as [7]

$$E_{in} = \sum_{\mu=1}^{M} a_\mu E_{1\mu} \quad , \quad H_{in} = \sum_{\mu=1}^{M} a_\mu H_{1\mu} \tag{9}$$

where $E_{1\mu}=E_{1\mu x}+E_{1\mu y}+E_{1\mu z}$ and $H_{1\mu}=H_{1\mu x}+H_{1\mu y}+H_{1\mu z}$ are the propagating modes in the first section of tapered waveguide of fig 3 and M is the number of the propagating modes in that section. At the point $z = z_1^-$ in the first section, we can write:

$$\begin{aligned} E(x,y,z_1^-) &= \sum_{\mu=1} a_\mu E_{1\mu} \exp(-j\beta z_1^-) \\ H(x,y,z_1^-) &= \sum_{\mu=1} a_\mu H_{1\mu} \exp(-j\beta z_1^-) \end{aligned} \quad (10)$$

It is possible to use mode orthogonality to find the field distribution at the point $z=z_1^+$. The electric and magnetic fields in the second section of the tapered waveguide can be written as:

$$\begin{aligned} E(x,y,z) &= \sum_{\mu=1} b_\mu E_{2\mu}(x,y,z) \\ H(x,y,z) &= \sum_{\mu=1} b_\mu H_{2\mu}(x,y,z) \end{aligned} \quad (11)$$

where $E_{2\mu} = E_{2\mu x} + E_{2\mu y} + E_{2\mu z}$ and $H_{2\mu} = H_{2\mu x} + H_{2\mu y} + H_{2\mu z}$ are the fields of propagating modes in the 2nd section of the tapered waveguide (Fig. 3). Now, b_μ can be determined as follows [7]:

$$b_\mu = \frac{\iint_A [E_{in} \times H_\mu^* + E_\mu^* \times H_{in}].k dxdy}{\iint_A [E_\mu \times H_\mu^* + E_\mu^* \times H_\mu].k dxdy} \quad (12)$$

Performing the same operation for all the sections in Fig. 3, we can determine the The wave propagation along the tapered waveguide. We have considered only two modes (1 and 2) of the waveguide in fig 3 as the input to first section of the tapered waveguide, thus:

$$\begin{aligned} E_{in1} &= E_{11x} + E_{11y} + E_{11z} \\ H_{in1} &= H_{11x} + H_{11y} + H_{11z} \\ E_{in2} &= E_{12x} + E_{12y} + E_{12z} \\ H_{in2} &= H_{12x} + H_{12y} + H_{12z} \end{aligned} \quad (13)$$

4-Numerical results:

Before analyzing the tapered waveguides, it is informative to analyze the connection of two waveguides without tapering shown in Fig 5. The dimension of the waveguide shown in Fig. 5(a) are: d_1= 8μm, d_2=3.2μm, $t_1=t_2$= 5μm, n_1=1.49 and n_2=1.46. The amount of power losses of the modes 1 and 2 obtained for the structure of fig. 5 (a) are given in table 1.

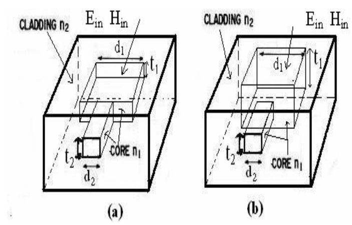

Fig.5: Interconnection of two waveguides without tapereing

In this table, P_{out} is the output power from the core and P_{in} is the input power to the core. The higher loss of the 2nd mode in table 1 can be explained as follows: the power of the first mode is more concentrated at the middle of the core, whereas for the 2nd mode, the power is concentrated at the sides of the core. During the propagation of mode 1 and mode 2 from section 1 to section 2, more power of the 2nd mode enters the cladding or reflected beak to section 1 than those of the l'st mode. This makes the 2'nd mode more lossy than the l'st one. Fig 6 shows the propagation of the l'st mode in the structure of fig 5 (a), where an abrupt change in the field distribution at the discontinuity point is observed.

Table 1: Power loss ($10*\log(P_{out}/P_{in})$) in fig 5(a) for the modes 1 and 2

input	Loss in Fig 5(a)
E_{in1},H_{in1}	1.2(db)
E_{in2} ,H_{in2}	31.8(db)

Table 2 is obtained from simulation of the first mode in the structure of fig 1 (a) and (b) in which, d_1=8μn, d_2=3.2μm, $t_1=t_2$=5μm n_1=1.49 and n_2=1.46. In fig 1(a) profile of the tapered waveguide is linear, whereas in fig 1(b), the profile has a variation of exp ($-12-385 \times 10^3$ z). Table 2 includes different lengths of tapered waveguide (L) and also different sizes of Δz and Δd. Generally, the losses in structures of fig 1 (tapered) are much less than those in the structures of fig 5 (without tapering).

Investigating the results represented in table 2, shows that the larger length of the tapered waveguide, the less is the loss, note that larger Δz or Δd (less number of sections in fig 3) increases the loss. From the results represented in table 2 we can see that the linear tapered waveguide has a lower loss than that of the exponential one.

Table 2 : Power loss ($10*\log(P_{out}/P_{in})$) in fig. 1(a) and fig. 1(b) for the mode 1

input	Waveguide length		Loss in fig1(a)	Loss in fig1(b)
E_{in1}, H_{in1}	L=2.5µ	Δz=0.1µ Δd=0.1µ	0.48(db)	0.55(db)
		Δz=0.2µ Δd=0.2µ	0.51(db)	--
	L=1.25µ	Δz=0.1µ Δd=0.1µ	0.54(db)	--

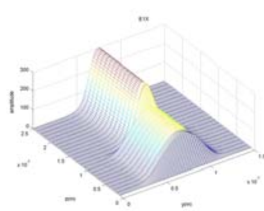

fig6: Propagation of the l'st mode in the structure of fig 5 (a)

Fig 7 shows the propagating of the l'st mode in the structure of fig 1(a), where there is no discontinuity in the field distribution at the interconnection point. This justifies reduction in the loss and reflection of fig 1. Fig 8 shows the propagation of the 2'nd mode in the tapered waveguide of fig 1(a).

Fig 5 (b), has dimensions: d_1=4.5µn, d_2=1.8µm, t_1=4.5 t_2=1.8µm n_1=1.49 and n_2=1.46. Comparing the results given in table 1 with those of table 3 shows that the loss in structure with discontinuity in 2-dimension (fig. 5 (b)) is more than the loss in structure with discontinuity in 1 dimension (fig 5 (a)) .

Table 3: Power loss ($10*\log(P_{out}/P_{in})$) in fig 5(b) for the modes 1 and 2

Input	Loss Fig5 (b)
E_{in1},H_{in1}	3.74(db)
E_{in2} ,H_{in2}	12.26(db)

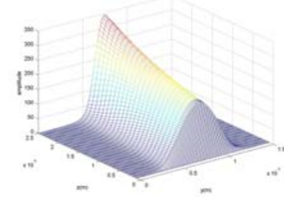

Fig 7: Propagation of the 1[st] mode in the structure of fig 1(a)

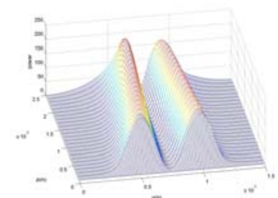

Fig 8: Propagation of the 2[nd] mode in the tapered waveguide of fig 1(a).

The results of simulation of the structures shown in fig 1 (c) and (d) are given in table 4. In these structures, d_1=1.46µn, d_2=1.8µm, t_1=4.5µm, t_2=1.8µm n_1=1.49 and n_2=1.46. the structure of fig. 1(c) is linearly tapered and that of the fig 1(b) has an exponential variation of exp (-

12-385 $\times 10^3$ z) for x and y. Comparing the results given in table 2 with those in table 4 shows that the taper with discontinuity in 2-dimension has much higher loss than those of the taper with 1-dimensional discontinuity.

Table 4 : Power loss ($10*\log(P_{out}/P_{in})$) in fig. 1(c) and fig. 1(d) for the modes 1 and 2

Input		Loss in fig1(c)	Loss in fig1(d)
E_{in1},H_{in1}	Δz=0.1µ Δd=0.1µ	3.1(db)	3.97(db)
E_{in2} ,H_{in2}	Δz=0.1µ Δd=0.1µ	11.8(db)	12.55(db)

6-Conclusion

In optical devices like MMI couplers, we are faced with junctions of dielectric waveguides of different cross sections. Tapering of the cross-sections reduce the reflection and loss.

In this paper a method is presented to analyze the tapered optical waveguide in 3-dimension by full vectorial approach. In this method, the complete (non approximate) equation of the wave propagation is used. In fact, replacing $\nabla.E = 0$ by $\nabla.D = 0$ makes the accuracy of the simulation higher than that of the conventional approximate case. However, many authors stated that $\nabla.E = 0$ is reasonable when the weak guidance approximation is applicable. Meanwhile, application of $\nabla.E = 0$ in devices with a large refractive index contrast (n_{core}-$n_{cladding}$), reduces the accuracy greatly, where this is not the case with $\nabla.D = 0$. In addition by doing a proper onohomogenuous meshing in the cross-section of the waveguide more accuracy in results is observed.

7-Reference

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