Abstract: - Through analyzing the product structure tree, the complete supply chain network (SCN) including materials supplying, products manufacturing and distribution is established. The directions of all the flows in the original SCN are narrowed down into one single way by dividing up all the possible circulation of both manufacturing and transportation. Each node of the SCN is simplified according to its different functions, outputs and raw material sources. According to the layer of the product structure tree, all the divided nodes are combined so that each route includes all the raw material sources of the finished product. The most suitable plants location under the time constraints are worked out through the application of generalized permanent labeling algorithm on the combined SCN. Finally, by tracing backward to the original SCN according to the most suitable plants location and routes selection, the real optimal plants location and routes selection can be found out.

Key-Words: - Supply chain management, Plant location, Route selection, Network transformation

1 Introduction

Nowadays, living in a complex market environment whose situations including fierce competition, quickly changed consumer's taste, shorter and shorter product's life cycle, all the enterprises are forced to meet the consumer's demand as soon as possible with lower cost and shorter production cycle. To achieve these goals, enterprises are no longer only absorbed in improving their interior production procedures, but extend the feeler of management to other enterprises in the SCN. That is to say, enterprises want to control, operate and plan the total supply chain correlated with its products entirely. With the popularization of the supply chain and the development of the information technology, supply chain management (SCM) have been greatly spread and applied. How to plan and design the SCN from the view of the whole product, such as the suppliers selection, plants location, distribution network designing, becomes the focal issue in the study of the SCM [1]. According to the classification by Cohen and Mallik [2], the problem studied in this paper refers to the plant location of the SCN model.

To the plant location problem, there are two kinds of solutions mainly. One is using various kinds of optimization algorithms to find out the most suitable plant combination. And the other is to confirm the plant combination, assess the tactics by using computer to simulate the situation of the actual supply chain. All those kinds of optimization algorithms can be divided into two categories roughly as following. The first one includes those various kinds of mathematics analytic methods based on the linear programming or mixed-integer programming, which contribute to find out the best solution of the maximal or minimal object functions under some assumptions and limitations. Cortinhal [3], Tjendera [4] have done great contribution researches to this. And Lagrangean heuristic algorithm, which brought forward by Correia [5], is commonly used in large-scale multi-facility location problems, such as Yuri [6]. Besides the aforementioned methods, simulation, which used by Jung [7], et al., is another one to confirm the plant location and policy evaluation.

But the following question generally exists in these methods. Firstly, the plant location belongs to NP-hard problem usually, and the algorithm complexity presents the index increase with the range. Secondly, most solutions do not consider the product structure tree. Thirdly, many researchers divide the supply chain into different sub-network, and calculate each sub-network as the optimum solving of the total supply chain. But the concatenation of each sub-network optimum solving might not the best solving of the entire supply chain. Last, the time factor is seldom considered. But actually, the time factor should be studied as an important goal in the new market environment [8].

From all above-mentioned researches, it lacks a simple, general and feasible method to solve the plant location in the SCN. In that, Chen tries to transform the original supply chain and applies the shortest path algorithm, the maximal flow algorithm to solve the plant location [9]. The most defect of Chen's research does not consider the time restriction among supply chain nodes. So the plant location and routes
selection of the SCN merged with the product structure tree under the time window is studied emphatically.

2 Model and methodology

2.1 Description, assumptions and notations

For any supply chain with a final product, there is a dendrite product containing all the materials/modules used in the supply chain to produce the final product, and showing the hierarchical composition relationships between these materials/modules, as the example shown in Fig.1. The raw material $P_01$ and $P_02$ are used to produce the semi-final product $P_03$. And the raw material $P_{04}$ and the $P_{03}$ are used to make the final product $P_0$. In addition, the letter in the parentheses after each component in the product tree indicates the quantity of that component needed to make one above level component.

![Fig.1. Product structure tree of $P_0$](image)

SCN, shown in Fig.2, is the network of facilities that performs the functions of procurement of material, transformation of material to intermediate and finished products, and distribution of finished products to customers [10]. In this network, there are four modality nodes: supplier (S), manufacturer (M), distributor (D) and retailer (R).

![Fig.2. SCN of $P_0$](image)

In Fig.2, the $M_1$ receives the $P_{03}$, $P_{02}$ and $P_{04}$ supplied by the $S_1$, $S_2$ and $S_3$ to produce the $P_0$ and the $P_{03}$. And the $M_2$ receives the $P_{04}$ supplied by the $S_4$, $S_5$ and the $P_{03}$ supplied by the $M_1$ to produce the $P_0$. The $P_0$ manufactured by the $M_1$ and the $M_2$ are transported to the $D_1$ and the $D_2$ respectively. Last, the $D_1$ and the $D_2$ distribute the $P_0$ to the $R_1$ and the $R_2$). The letters beside each arc are the components produced or shipped from one entity to another, and according to the actual needs, the production or transportation cost, time restriction are added beside each arc.

However, in a SCN, not all entities are necessary to transport, produce, and distribute from the raw material to the final product. For instance, one possible way to produce and deliver the $P_0$ is $S_1$, $S_2$, $S_3$, $M_1$, $D_1$, $R_1$, as arrows shown in bold lines in Fig.2.

Another possible way, as shown in Figure 2 with bold dotted arrow lines, would go through $S_1$, $S_2$, $S_5$, $M_2$, $M_3$, $D_2$, and $R_1$. Nevertheless, as the example shown in Fig.1, all components, from the raw material $P_{01}$, $P_{02}$ and $P_{03}$ to the semi-finished product $P_{03}$, must be used in the process of manufacturing the final product $P_0$, therefore it is necessary for these components to appear in the production sub-networks for the corresponding SCN, such as the example sub-networks shown in Fig.2.

A node in a SCN may be a raw material supplier, a factory, or a distribution center. Hence, in this study, such a node is said to have either production function or distribution function or both [9]. If a node performs manufacturing work, which means transforming some materials, it will have received into another semi-finished or final product in a supply chain, it is said to have the production function, such as $M_1$ and $M_2$ in Fig.2. Moreover, a production node, which is a node with production function, is further classified into an external production node, an internal production node, or a production node with both functions. An external production node uses only what it receives as materials to produce items without using what it produces as materials. At last, a one-item production node is the one that ships out exactly one kind of item, such as $M_3$ in Fig.2. An internal production node is a production node, and moreover, it does not only use the components receiving as materials for production, but also takes the items producing again as materials to further produce other items. For example, node $M_1$ in Fig.2 is a good example of both external node (it imports the component $P_{01}$ and $P_{02}$ to produce the $P_{03}$ and transports the $P_{03}$ to the $M_3$) and internal production node (it imports the $P_{01}$ and $P_{02}$ to produce the $P_{03}$, and still more, the node $M_1$ again takes the $P_{03}$ as one raw material, along with the component $P_{04}$ that it imports, to make the final product $P_0$).

If a node ships out what it receives without manufacturing work, it is said to have the distribution function, such as $D_1$ and $D_2$ in Fig.2. For instance, the $D_1$ in Fig.2 is a node with distribution function only.

2.2 SCN transformation: decomposition and composition

Before applying any network flow algorithms, a SCN have to be transformed into another topology graph
to satisfy the regarding requirements of those algorithms. And the basic ideas of the SCN transformation are as following.

- Analyzing the product structure tree and setting up the original SCN;
- Simplifying each node of the SCN according the different nodal functions, outputs, and raw material sources;
- Combining the new supply chain node according to the layer of the product structure tree;
- Applying the optimization algorithm to the SCN after decomposition and composition and seeking out the most suitable plants and routes;
- Tracing backward to the original SCN to find the real optimal plant location and routes.

2.2.1 Product structure tree and SCN analyzing

As the description and assumption, different nodes are connected with directed arcs. Because the nodal type is not appointed at ahead, it is necessary to check out the circulation in the original SCN before analyzing the function of supply chain nodes. The transformation of product structure and SCN when decomposing supply chain circulations is divided into two parts: one is to check and transform the manufacturing process to the same entity. Just as manufactures, so this circulation can be viewed as the same node A, transit and return to the node B. All nodes in this kind circulation are also manufactures, so this kind circle can also be viewed as the manufacturing process to the same entity. Shown in Fig.5, the process route of $P_{03}$ is $M_1 \rightarrow M_2 \rightarrow M_1 \rightarrow M_3 \rightarrow M_1$. The start node of this process route is $M_1$, and the end node is $M_3$. Transforming this sub-network and revising the cost and time window on each route, the sub-network and the cost and time window after decomposition is presented in Fig.6 respectively.

Manufacturing process circulation--Start from the node A, transit and return to the node B. All nodes in this kind circulation are also manufactures, so this kind circle can also be viewed as the manufacturing process to the same entity. Shown in Fig.5, the process route of $P_{03}$ is $M_1 \rightarrow M_2 \rightarrow M_1 \rightarrow M_3 \rightarrow M_1$. The start node of this process route is $M_1$, and the end node is $M_3$. Transforming this sub-network and revising the cost and time window on each route, the sub-network and the cost and time window after decomposition is presented in Fig.4 respectively.

Fig.4. Manufacturing process network and its data after transformation

Manufacturing process circulation--Start from the node A, transit and return to the node B. All nodes in this kind circulation are also manufactures, so this kind circle can also be viewed as the manufacturing process to the same entity. Shown in Fig.5, the process route of $P_{03}$ is $M_1 \rightarrow M_2 \rightarrow M_1 \rightarrow M_3 \rightarrow M_1$. The start node of this process route is $M_1$, and the end node is $M_3$. Transforming this sub-network and revising the cost and time window on each route, the sub-network and the cost and time window after decomposition is presented in Fig.6 respectively.

Transportation process circulation. In the actual SCN, the product transshipment among distributors to support retailers’ demands is existed generally. Shown in Fig.7, the products are transshipped from $D_2$ to $D_1$ to support the retailer $R_1$ when the products are out of stock in $D_1$, and so as to the retailer $R_2$. Decomposing these transportation nodes, which transships the products each other, and the transformed SCN is Fig.8.

Fig.5. Original manufacturing circulation network and its data (Type 2)

Fig.6. Decomposition of network

<table>
<thead>
<tr>
<th>Route</th>
<th>Cost</th>
<th>Time Window</th>
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<tr>
<td>$S_0 \rightarrow M_1$: $C_{S_0M_1}$, [$t_1$, $t_2$, $t_3$]</td>
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<td>$S_0 \rightarrow M_2$: $C_{S_0M_2}$, [$t_1$, $t_2$, $t_3$]</td>
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<tr>
<td>$M_1 \rightarrow M_2$: $C_{M_1M_2}$, [$t_1$, $t_2$, $t_3$]</td>
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<td>$M_2 \rightarrow M_3$: $C_{M_2M_3}$, [$t_1$, $t_2$, $t_3$]</td>
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<td>$M_3 \rightarrow M_4$: $C_{M_3M_4}$, [$t_1$, $t_2$, $t_3$]</td>
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<td>$M_4 \rightarrow M_5$: $C_{M_4M_5}$, [$t_1$, $t_2$, $t_3$]</td>
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Fig.7. Original manufacturing circulation network and its data (Type 2)
2.2.2 Node function decomposition

After analyzing and setting up the product structure tree and the SCN, then the primitive network structure needs to be changed into a new SCN according to the composition of the products and nodal function to make any node on this new network only bear the single function of single output. But to the production function node, the cost and time of obtaining raw materials from different sources are different. So when the production function node which has a lot of sources is of obtaining the same raw materials, this node should be spitted too. In other words, in the new SCN, each node is responsible for the single function. The distribution function node can sends the products to different nodes, and the production function node can only have one output, and to each component, this node can only be received from one supplier. And just as the distribution function node, the production function node sends the products to different nodes.

Among all SCN nodes, only the manufacturer node might possess two kinds of functions together. So while simplifying the nodal functions, all nodes which are involved the manufacture process only need to be checked. The method to generate the single function node is divided into three steps:

- Checking and isolating all nodal distribution functions;
- Isolating different outputs to the remainder production function nodes;
- Checking the production node after isolating different outputs and splitting the same raw materials with different sources.

After these three steps, because new nodes have replaced all old nodal functions and outputs, all these old nodes and their arcs are deleted and the nodes with the same inputs and outputs are amalgamated.

Examplng as the node $M_1$ in Fig.9, $P_0$ is produced by $P_{01}$ and $P_{06}$. $P_{03}$ is composed by $P_{01}$ and $P_{02}$. Seeing Fig.9, $M_1$ has three functions: (1) transports $P_{02}$ to $M_2$, (2) produces and transports $P_{06}$ to $M_3$, (3) transports $P_0$ to $D_1$. The Function 1 of $M_1$ belongs to the distribution function ($S_1$→$M_1$→$M_2$), so it needs to generate a new node to replace this function, as shown in Fig.10(a). The function 2 of $M_1$ is to produce $P_{03}$ ($S_1$+$S_2$→$M_1$→$M_3$), so it needs to generate another new node to replace this function, as shown in Fig.10(b). The function 3 of $M_1$ is to produce $P_0$, so it needs to generate the third node $M_{1,3}$ to replace this function, as shown in Fig.10(c). And because there are two different supplies ($S_1$ and $S_2$) to offer the same material $P_{04}$ to produce $P_{06}$, so the node $M_{1,3}$ needs to be split into two nodes ($M_{1,3}$ and $M_{1,3}$) on behalf of different supplies combination, as shown in Fig.10(d). Checking the new network, the inputs and outputs of
the node $M_{1,3}$ and $M_{1,1}$ are completely the same, so one node of both can be amalgamated and some related arcs are deleted.

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<tr>
<td>$S \rightarrow M_{1,1}$</td>
<td>$C_{w_{i,j}} \left[ t_{i}(S,M_{j}), t_{f}(S,M_{j}) \right]$</td>
<td>$[\max(t_{i}(S,M_{j}), t_{f}(S,M_{j}))]$</td>
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<tr>
<td>$S \rightarrow M_{1,2}$</td>
<td>$C_{w_{i,j}} \left[ t_{i}(S,M_{j}), t_{f}(S,M_{j}) \right]$</td>
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<td>$C_{w_{i,j}} \left[ t_{i}(S,M_{j}), t_{f}(S,M_{j}) \right]$</td>
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<tr>
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Fig. 9. Original SCN and its data of $M_{f}$

2.2.3 SCN composition based on product structure tree

The SCN composition based on the product structure tree has two main steps: (1) the composition of production node with the same material, (2) the composition of production node without the same material. For a one-item production node-producing node with the same material, the tree has two main steps: (1) the composition of the node according to the product structure downstream from the root to the bottom. Only a route amalgamated out like this includes producing all compositions of the finished product. All manufacturer nodes changed in the SCN composition need to be deleted. Taking Fig. 11 as an example, $P_{0}$ is the final product and the transformed network after composition is shown in Fig. 12.

After the SCN composition, every nodal input contains all components for its outputs and each component is just offered by one supplier. The amalgamating process is to amalgamate the nodes of the same layer according to the product structure downstream from the root to the bottom. Only a route amalgamated out like this includes producing all compositions of the finished product. All manufacturer nodes changed in the SCN composition need to be deleted. Taking Fig. 11 as an example, $P_{0}$ is the final product and the transformed network after composition is shown in Fig. 12.

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<td>$M_{1,3} \rightarrow P_{0}$</td>
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<td>$M_{1,4} \rightarrow P_{0}$</td>
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<td>$[\max(t_{i}(S,M_{j}), t_{f}(S,M_{j}))]$</td>
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Fig. 11. Original network and its data of $M_{f}$

2.2.4 Applying the optimization algorithm to the transformed SCN

After all conversion operations above-mentioned in the SCN, the relevant algorithm can be applied to the
transformed network to find out the best node association. The optimization algorithm adopted in this research is the generalized permanent labeling algorithm (GPLA) of the single source shortest path with time window (SSP-TW) \[11\]. That is to say, the specific route of the minimum cost under the time constrains from the starting point to the terminal point should be searched out. And all nodes in this route are the optimal plant location in production and distribution.

2.2.5 Seeking the optimal plant location of original SCN
The shortest route found out after using the GPLA to the transformed network includes all nodes of the optimal plant location. Now, the real plant location should be confirmed from this route according to the amalgamated nodes, outputs and components. The corresponding method to find these is to take apart and return to the original SCN.

3 Discussion and conclusion
Unlike many studies done in the SCM field, this research takes a different approach by transforming the SCN with the product structure tree under the time restriction of the SCN, then makes use of the GPLA algorithm to develop supply chain algorithms that are good for SCM decision-makings. The most important results learnt from the research are summarized as follows.

First, these supply chain algorithms provide a simple yet practical fashion to build supply chain models that are useful for SCM without a large amount of money or professional knowledge. Second, in the academic research for a more general model to represent and discuss the SCM, the algorithms built here and the way to transform the SCN is a great candidate that serves as a good starting point. It is possible to add more new considerations into the representation and algorithms to improve its representative ability. Third, since these supply chain algorithms, along with the basic decomposition and composition of the SCN are designed closely connected to the product structure tree, it can be seen that there seems to be different sections or cuts existing in the supply chain after composition, as examples shown in previous chapters. Last, a manager in a centralized SCN can use this supply chain algorithm to generate initial results regarding output, cost or both factors for his SCM decisions. In a SCN that exists a dominate player, such as Acer in its personal computer supply chains, it is also a similar situation since the powerful player is able to choose his suppliers or customers to optimize the benefits for the whole supply chain.

Acknowledgements
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