Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp141-146)

Controller Design For Highly Nonlinear Systems

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Abstract: - The main purpose of this paper is to report on development of a new robust and high performance controller design platform for use with highly nonlinear systems without any restrictions on non-linearity type, non-linearity arrangement, the number of nonlinear terms, the order of the system, and the number of inputs and/or the outputs. The design platform is based on describing function approach to analysis, diagnose, and design of highly nonlinear systems. The presented describing function approach to design of nonlinear control systems is fully systematic, computer-aided, and it minimizes the subjective judgment that needs to be employed by the designer. The approach is applied to robotic and aerospace example problems.

Key-Words: - Describing Functions; Fourier Integrals; Model Matching

1 Introduction

Generally, design activities heavily rely on experience and judgment of the designer [1]. In recent years, in order to lift this problem, various approaches have been proposed. The primary approaches are: (1) to build artificial intelligent or expert systems, (2) to develop theories based on the experience and skills of the designer using a cognitive approach, and (3) to develop algebraic and systematic techniques that would minimize the need for experience and judgment of a designer. One of the advantages of the third approach is that design technique may easily be automated on a digital computer. This research falls in the third approach in dealing with problem of the need for experience and judgment of the designer in designing effective nonlinear feedback control systems.

A robust controller design platform based on a unique describing function approach has been developing since its original introduction by Taylor [2]. This approach has been pursued by a number of research activities (e.g., [2]-[11]), and the corresponding results have enjoyed considerable success in arriving at robust nonlinear feedback systems.

A perspective on most popular available nonlinear controller design approaches is outlined in [7] which includes variable structure systems (VSS) technique, feedback linearization, geometric transformation, high-gain feedback, Liaponouv, quantitative feedback theory (QFT), optimal, adaptive, fuzzy, and neural networks (e.g., [12]-[19] and references therein).

The problem statement follows. Given a real system, which is most likely an amplitude dependent nonlinear system, how would one go by designing a controller that would not only assure robust closed-loop stability but it would also robustly satisfy a set of user-defined performance measures in a near optimum fashion. In this work, an answer is provided. The specific contributions of the presented work are in three fold: (1) development of a previously proposed describing function based controller design platform for use with such highly nonlinear systems as robotics and servomechanisms, (2) development of computer-aided design environments and software to implement the approach, and (3) verification of the approach and the associated software by computer applications to a number of highly nonlinear example problems.

2 Describing Functions

At the beginning, describing functions (DF) were primarily used for analysis of simple nonlinear systems, and the DF method was used to predict limit cycles in nonlinear systems. To some extent, DF method was also used to advance design of simple nonlinear systems. Some of the standard works in this context that are limited to single-input single-output systems are the texts given by [20] and [21]. Early DF works were limited to: (1) SISO systems, (2) systems with one separable nonlinear term, (3) non-time varying nonlinear terms, (4) dominance of the first harmonic, and (5) odd nonlinear systems. These restriction no longer exist [22].

The primary advantages of the describing function technique are: (1) correspondence with the SSL approach if SSL models exist; in other words, DF results for small signal excitations are identical to SSL technique, and therefore one has nothing to loose by using DF models, (2) the approach is applicable to a large class of nonlinear systems which may be representable in standard state-variable differential equation form, and (3) since DF approach takes into account amplitude sensitivity issues, designs would be robust.

There is a body of literature in the past two decades that successfully uses describing functions in order to design robust nonlinear feedback systems (e.g., [2]-[11], [23]). In this work, sinusoidal-input describing function (SIDF) models are used for the following reasons: (1) standard linear models (obtained on the basis of a small-signal or Taylor series) do not capture the amplitude dependency of the original plant, (2) other describing function models such as random-input describing function models are not able to characterize the dependency of the nonlinear plant on the expected range of frequencies of interest, (3) a set of describing function models, covering the expected range of amplitudes of interest, are an excellent basis for a robust design because dependency of the nonlinear plant on the amplitudes of excitation is an important issue in design of robust nonlinear closed-loop systems, (4) designs based on describing function models result in robust stable closed-loop systems without sacrificing performance, (5) unlike standard linear models, DF models are characterized only by one parameter which is the amplitude of excitation; hence, design is much simpler and restrictive than if the design were based on several parameters that are obtained by replacing each nonlinear term by a linear gain, and (6) small signal models may not exist for nonlinear plants with discontinuous or multi-valued nonlinear terms.

SIDF models may be obtained by a procedure similar to that used in limit cycle analysis; in this approach each nonlinearity term is replaced by a quasi-linear term, and a set of nonlinear algebraic equations, that correspond to harmonic balance, are solved to determine the parameters of the quasi-linear term. This method assumes that input to each nonlinear term is nearly sinusoidal. Such assumption may be removed if the SIDF models are obtained by direct simulation and evaluation of Fourier integrals [24].

3 Controller Design

The design platform based on a describing function platform considers three different cases [2]. In the first case, the nonlinear system may be characterized by one operating regime, and the designed controllers are said to be single-range (SRLCD – single-range linear controller design) [1],[7]-[[9],[11]. In the second case, the nonlinear system may be characterized by two different operating regimes, and in this case the designed controllers are said to be dual-range (DRLCD – dual-range linear controller design) [4],[5]. Finally, in the third case, the nonlinear system is characterized by many operating regimes (more than two) and the designed controllers are said to be multi-range which could be either linear or nonlinear (MRNCD - multi-range nonlinear controller design) [3],[10].

3.1 SRLCD

The SRLCD procedure is comprised of five primary steps [7],[8],[9],[11]. Those steps are: (1) specification of the desired reference linear model, (2) obtain of describing function model of the plant at nominal regime, (3) identification of the linear model of the nominal describing function model of the previous step, (4) design of a controller based on a linear technique, and (5) verification of design.

In Step 1, the user must specify the model of the process that he wants to mimic. Different approaches are proposed in [25] to fulfill the objective of this step. The simplest approach is to define a second-order transfer function that possesses the desired natural frequency and damping ratio.

In step 2, two sets of a priori information must be available. Those are, the mathematical model of the system in state variable differential equation form, and the knowledge of operating regime of interest. Note that unlike operating points, operating regimes are characterized by the range of expected amplitudes and frequencies of the excitation signal. Then, the Fourier based approach of [22] is utilized to obtain the pseudo frequency response data or the describing function models.

In step 3, the nominal pseudo frequency response data is set aside for system identification purposes. The procedure must take into account several key points that was originally discussed in [26]; also, the MATLAB invfreqs command takes these important issues into consideration. The user may either use the software presented in [26] or the above mentioned MATLAB command. In this research, the MATLAB command is used.

In step 4, a controller may be designed using a factorization approach [8], [9]; in this work an H_∞ approach is adopted.

Finally, in step 5, the design is verified.

3.2 DRLCD

The DRLCD procedure is comprised of six primary steps [4],[5]. Those steps are: (1) identification of a reference linear model whose static and dynamic behavior matches the desired closed-loop system performance specifications, (2) obtain of DF models of the nonlinear plant, (3) selection of two describing function models whose gain plot bounds those of the others in the class along with linear system identification, (4) determination of the set of *all* linear controllers that simultaneously stabilize these linear systems [27], (5) *s*earch of the set identified in the previous step set for the minimum-sensitivity linear controller, and (6) validation of the design via digital simulation.

3.3 MRNCD

The MRNCD synthesis method is composed of 11 steps [3],[10]. These steps are described below. (1) select a set of values for amplitude levels and frequencies of the excitation signal, which fall into operating regimes of interest, (2) obtain the input-output frequency models of the nonlinear plant, (3) select one of the input-output models of the previous step as the nominal model for which a linear controller is to be designed, (4) design a linear PID controller for the nominal frequency model [1], (5) place the model of the designed controller of the previous step in series with the nonlinear plant, (6) generate the input-output frequency model of the open loop system of the previous step, (7) select a set of values for the amplitude level of excitation signal of the controller, which corresponds to the set of values for the amplitude level of the excitation signal of the nonlinear plant, (8) design a set of linear controllers to achieve minimum sensitivity, (9) synthesize a series of nonlinear functions by applying describing function inversion, (10) construct a model of the nonlinear controller incorporating the synthesized nonlinear gain functions of the previous step, and (11) verify design; for this purpose, the closed-loop feedback system, which is comprised of the nonlinear controller and the nonlinear plant, is simulated.

4 Demonstration Example Problems

The design platform is demonstrated via 3 example problems.

4.1 Example 1

The first example problem is of the sort encountered in aerospace [8],[9],[28],[29], and it demonstrates the application of the SRLCD controller design approach outlined above to a problem of the sort encountered in pressure control of a combustion chamber of a liquid propellant engine. A schematic drawing of the engine is depicted in figure 1; see [28] for a description of engine operation. The nonlinear and dynamic computer model of the liquid propellant engine is utilized to design a controller for the combustion chamber. As was mentioned, for the linear design part, an H_∞ approach is used; the results are compared with one other approach that uses a factorization approach [30]. The verification results are shown in figure 2.

4.2 Example 2

The second example problem is of the sort encountered in robotics. The schematic diagram of this system is shown in figure 3, and the computer model of this system is given in [24] in terms of a FORTRAN subroutine. A dual-range linear controller is designed using the outlined approach. In order to examine the performance of this dual-range linear controller with the actual nonlinear system, the nonlinear closed-loop feedback system is simulated with step commands of various magnitudes. The results are depicted in figure 4, and it is evident that the dual-range linear controller has produced a nonlinear closed-loop feedback system which is fairly insensitive to the amplitude level of the excitation command.

4.3 Example 3

The example problem that was used for Example 2 is also used here. The designed multi-range nonlinear controller performance may be examined by studying figure 5. The system is fairly insensitive to the amplitude level of the excitation command.

5 Summary and Conclusions

A DF platform for systematic design of nonlinear feedback systems was discussed. The platform was demonstrated via three example problems of the sort encountered in aerospace and robotics. Satisfactory results were obtained.

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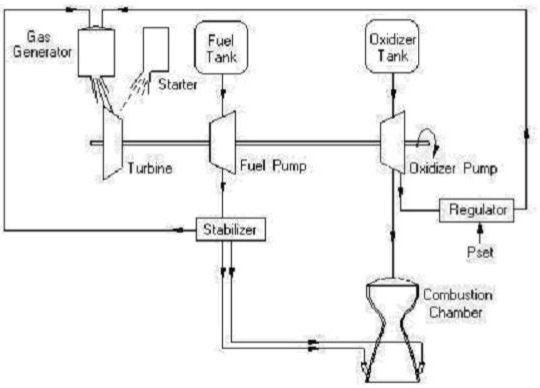

Figure 1 – Schematic of the liquid engine

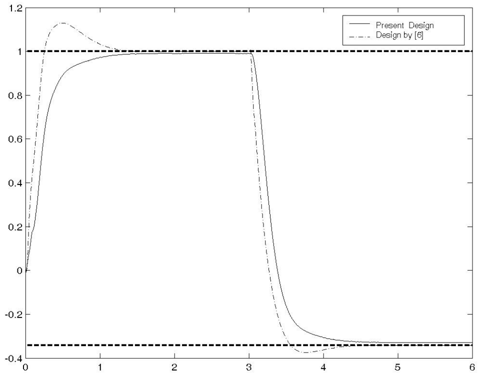

Figure 2 – Example 1 design verification and comparison

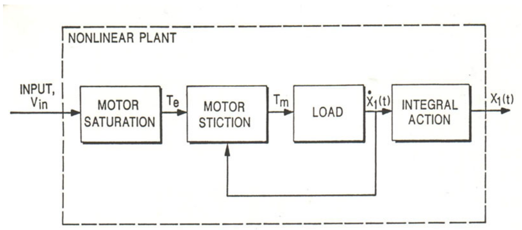

Figure 3 – Schematic of robot arm

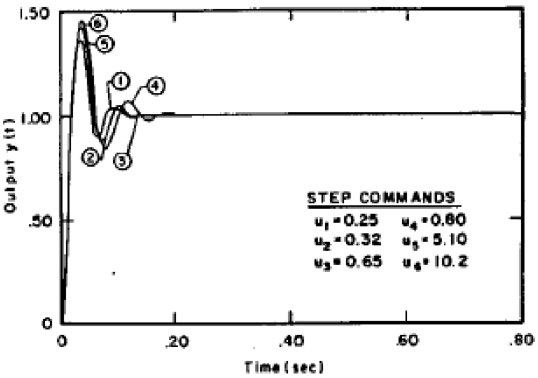

Figure 4 – Example 2 design verification [5]

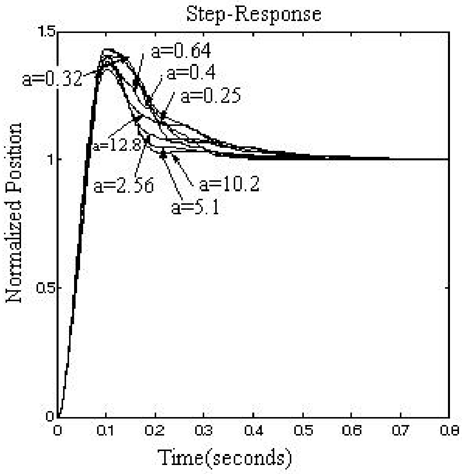

Figure 5 – Example 3 design verification