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GENERALIZED EIGENFUNCTION METHOD FOR DIELECTRIC ANTENNAS

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Abstract: Dielectrically loaded antennas meet growing interest in wireless and satellite communication due to small dimensions, controllable properties and perfect protection from damages even in the case of explosions. Real construction can be computed using time domain finite difference methods, but those methods meet big difficulties in dynamic problems. The generalized eigenfunction method applied earlier to diffraction problems and in laser theory can help to find within the reasonably limited amount of calculation not only static but dynamic characteristics of antennas. The importance of the dynamic analysis is easily seen from the presented figures, which show big changes in antenna parameters in the case of transmission (or reception) of short pulses. For high bit rate communication systems the detailed analysis of such changes is of vital importance.

Key-Words:- antennas , transmission , communication , Dielectric , protection , energy , resonant frequencies

1. INTRODUCTION

The generalized eigenfunction method was developed for diffraction problems [1]. The possibility of its application for laser theory was reported in [2] and [3]. It was proposed in [4] to use this method for the antenna design. One of its possible applications is the design and analysis of dielectrically loaded antennas. This type of antennas meets growing interest due to small dimensions, controllable properties and perfect protection from damages even in the case of explosions. Real construction can be computed using time domain finite difference methods, or frequency methods like entire domain Galerkin technique. Being difficult enough in principle, those methods meet additional difficulties in dynamic problems. Practically always the consideration is limited by stationary cases. We reported earlier the preliminary results of the dynamic analysis for the pattern of dielectric antennas [4]. Within the generalized eigenfunction method we have developed the technique of calculation for dynamic cases. Without big difficulties of the time domain methods this technique helps to analyze the
dynamic behavior of the antenna parameters.

GENERALIZED EIGENFUNCTION METHOD

The idea of the generalized eigenfunction method can be explained on the example of the ε-method [1]. We begin with a simple scalar problem of dielectrics in a closed resonator with ideal walls. Find a function u, which satisfies in side the volume v+ the equation

$$\nabla^2 u + \varepsilon k^2 u = f, \qquad k^2 = \frac{\omega^2}{c^2}$$

outside v+(in v-)- the equation

$$\nabla^2 u + k^2 u = f, \qquad (1)$$

and the boundary conditions on the dielectric surface Sε

$$u^+{}_{|S_\varepsilon} = u^-{}_{|S_\varepsilon}; \quad \partial u^+ / \partial N_{|S_\varepsilon} = \partial u^- / \partial N_{|S_\varepsilon}$$

and on the external boundary

$$u_{|S} = 0$$

We'll seek the solution of the problem as a series

$$u = u_0 + \sum_n A_n \qquad (2)$$

where u_n are the eigenvalues of the following auxiliary problem

$$\nabla^2 u_n + k^2 \varepsilon_n u_n = 0 \ \ in \ \ v^+; \ u^+_{n|S_\varepsilon} = u^-_{n|S_\varepsilon}; \ \partial u^+ / \partial N_{|S_\varepsilon} = \partial u^- / \partial N_{|S_\varepsilon} \quad (3)$$

$$\nabla^2 u_n + k^2 u_n = 0 \ \ in \ \ v^-; \ u_{n|S} = 0$$

Eigenvalues of the homogeneous problem are ε_n .

The eigenfunctions u_n is orthogonal in the sense of

$$\int_{v+} u_n u_m dv = 0 \quad if\ n \neq m,$$

This can be easily shown [1], using Green's formula

$$\int_v (U\nabla^2 V - V\nabla^2 U)dv =$$

$$\oint_S (U \partial V/\partial N - V \partial U/\partial N)dS$$

and the boundary conditions (3).

Each term in the series (2) satisfies the equation (1) in v- .If we want the series to satisfy (1) in v+ we must substitute it in (1) and find A_n.

$$\nabla^2 u_0 + k^2 u_0 \varepsilon +$$

$$\sum A_n(-k^2\varepsilon_n + k^2\varepsilon)u_n = f$$

but

$$\nabla^2 u_0 + k^2 u_0 = f,$$

hence

$$\sum_n k^2(\varepsilon - \varepsilon_n)A_n u_n = k^2(1-\varepsilon)u_0; \tag{4}$$

$$\sum_n (\varepsilon - \varepsilon_n)A_n u_n = (1-\varepsilon)u_0;$$

Multiplying (4) by u_m, we integrate over v+ and use the orthogonality of u_m, and u_n. Then

$$A_n = \frac{1-\varepsilon}{\varepsilon - \varepsilon_n}\frac{\int_{v+} u_0 u_n dv}{\int_{v+} u_n^2 dv} \tag{5}$$

To express A_n directly trough f, one can use the expression

$$\int_{v+} u_0 u_n dv = \frac{1}{k^2(1-\varepsilon_n)}\int_v u_n f dv \tag{6}$$

which can be obtained from obvious equations

$$u_0\nabla^2 u_n - u_n\nabla^2 u_0 =$$

$$-u_n f + k^2(1-\varepsilon_0)u_0 u_n \quad in\ v^+ \tag{7}$$

$$u_0\nabla^2 u_n - u_n\nabla^2 u_0 = -u_n f \quad in\ v^-$$

again with the aid of Green's theorem and the boundary conditions [1].

From (6) follows

$$A_n = \frac{1}{(\varepsilon - \varepsilon_n)}\frac{1-\varepsilon}{k^2(1-\varepsilon_n)}\frac{\int_v u_0 f dv}{\int_{v+} u_n^2 dv} \tag{8}$$

Both formulas (5) and (8) give us the formal solution of the problem.

In the $\varepsilon -$ method the field outside the body can not be represented as a series in terms of u_n as in this region the system of eigenfunctions u_n is not complete. We must add u_0 to the series. Inside the body v+ it is possible to represent u as

$$u = \sum_n B_n u_n$$

where B_n obviously can be expressed trough A_n .

$$u = \sum_n C_n u_n;$$

$$C = \frac{\int_{v+} u_0 u_n dv}{\int_{v+} u_n^2 dv} = \frac{1}{k^2(1-\varepsilon_n)}\frac{\int_v u_n f dv}{\int_{v+} u_n^2 dv}$$

hence

$$u = u_0 + \sum_n A_n u_n = \sum_n (C_n + A_n)u_n;$$

$$B_n = C_n + A_n = [\frac{1}{k^2(1-\varepsilon_n)}$$

$$+\frac{1}{\varepsilon - \varepsilon_n}\frac{1-\varepsilon}{k^2(1-\varepsilon_n)}]\frac{\int_v u_n f dv}{\int_{v+} u_n^2 dv}$$

that is in v+ (9)

$$u = \sum_n B_n u_n$$

$$B_n = \frac{1}{k^2(1-\varepsilon_n)}]\frac{\int_v u_n f dv}{\int_{v+} u_n^2 dv}$$

Resonance frequencies may be defined from the equation

$$\varepsilon - \varepsilon_n(k) = 0; k = \omega/c$$

The homogeneous problem (3) does not include the dielectrical permittivity of the body ε . It means that if ε is, for example, complex(losses or gain in dielectrics), the solution of the homogeneous problem is not complicated and its eigenvalues ε_n is real .

If the resonator has some losses, not connected with the complex ε, than ε_n become complex. But even in this case there exists only the discrete set of eigenvalues ε_n . They correspond to various types of oscillations at given frequency, which compensate the external losses by the energy generated in the active dielectrics.

DYNAMIC PROBLEM

We can not use (9) directly in the dynamic theory, as it represents the solution of a stationary problem while we have to solve a nonstationary problem. We can consider the equation (1)

$\nabla^2 u + k^2 u = f$ in v- $\qquad \nabla^2 u + \varepsilon k^2 u = f$ in v+

as the Fourier transform of the equations

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = f(t) \quad and$$
$$\nabla^2 u - \frac{\varepsilon}{c^2}\frac{\partial^2 u}{\partial t^2} = f(t) \tag{10}$$

The boundary conditions may be considered similarly. Then to find the solution of (10) is possible using the inverse Fourier transform of the solution

$$u = \sum_n B_n(k) u_n(k, x, y, z); \quad k = \omega/c$$

with B_n defined by (9). We'll limit our consideration with the case of narrow band processes $f(t) = F(t)\exp(i\omega t)$, where f(t)- an analytical signal, F(t)– a slowly varying function of time, ω –a frequency, close to one of the frequencies, satisfying the equation

$$\varepsilon - \varepsilon_n(k) = 0; \quad k = \omega/c \tag{11}$$

In this case the spectrum F(ω) of the F(t) will be narrow.

Then the spectrum u must be narrow as well and we will use that. In the Fourier transform

$$u(t) = \frac{1}{2\pi}\sum_n \int_{-\infty}^{\infty} B_n u_n(k) e^{i\Omega t} d\Omega$$

$u_n(k)$ is a slowly varying function of k, so the integral can be represented approximately as

$$u(t) \cong \sum_n u_n(\omega/c)\frac{1}{2\pi}\int_{-\infty}^{\infty} B_n(\Omega) e^{i\Omega t} d\Omega$$
$$\cong \sum_n u_n(\omega/c) B_n(t) e^{i\omega t}$$

if the spectrum of B_n has a sharp peak at Ω = ω, where ω is one of the resonant frequencies from (11). B_n (t) is defined as the inverse Fourier transform of (9). For the simplicity let the sources of excitation f in (9) are limited by the volume v+. Then (9) becomes

$$B_n = \frac{1}{\varepsilon - \varepsilon_n(k)}\frac{1}{k^2}\int_{v+} U_n f dv,$$

where U_n – the normalized function u_n with

$$\int_{v+} U_n^2 dv = 1$$

The factor

$$\frac{1}{\varepsilon - \varepsilon_n(k)}\frac{1}{k^2}$$

as a function of ω or k has the only singularity at $\varepsilon - \varepsilon_n(k) = 0$. We denote this meaning of ω as ω_p In the complex plane this singularity is a simple pole and the function has no singularities at all and is a slowly varying function of k:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} B_n(\omega - \omega_p) e^{i\omega t} d\omega \cong$$
$$\frac{\omega - \omega_p}{\varepsilon - \varepsilon_n(k)}\frac{1}{k^2}\int_{v+} U_n F dv \tag{12}$$

where F- a slowly varying function of time, mentioned before in (11). From the other side, the left part of (12) may be transformed to

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} B_n(\omega - \omega_p) e^{i\omega t} d\omega$$
$$= \frac{1}{2k}\left(\frac{d}{dt} - i\omega_p\right)\int_{-\infty}^{\infty} B_n e^{i\omega t} d\omega \tag{13}$$
$$\cong e^{i\omega t}\left[\frac{dB_n}{dt} + i(\omega - \omega_p) B_n\right]$$

Finally, the equation for B_n is as follows

$$\frac{dB_n}{dt} + i(\omega - \omega_p) B_n =$$
$$\frac{1}{\varepsilon - \varepsilon_n(k)}\frac{i(\omega - \omega_p)}{k^2}\int_{v+} U_n(k) f dv, \tag{14}$$

If B_n is constant, then $dB_n/dt = 0$ and we come back to (9).

APLICATION TO THE CASE OF A LONG CYLINDRICAL ANTENNA

To illustrate the application of the generalized eigenfunction method we consider the case of a long cylindrical dielectric antenna with the excitation from a long and thin metal rod inside the dielectric. The stationary case is not very interesting, as the eigenfunctions are well known - they are Bessel functions inside and Hankel functions outside the dielectric [1]. Dynamic problem is more complicated but very important. It is known from the theory of resonators that the stationary mode of oscillation requires some time for its stabilization inside the dielectrics [2]. This time even for small antennas is several nanoseconds. It becomes very important if we come to communication with the information transmission rate gigabits per second, as in this case the radiation is switched on and off every nanosecond and neither antenna pattern nor the input impedance have enough time for stabilization. The dynamic analysis using traditional methods is time consuming because of the integration over radioactive modes. The generalized eigenfunction method helps to simplify the numerical analysis due to the discrete character of the spectrum of eigenvalues [1]. For the calculations the rectangular pulse excitation was considered with variable duration of pulses and intervals between them. Our aim was to understand how the antenna pattern behaves in the first moments after the pulse excites the metal rod inside the dielectric. Obviously the stable mode of oscillation requires some time for coming to a stationary case. We can estimate this time as a time of the pulse propagation through the dielectric multiplied by some integer, but we do not know how many passes through the resonator are necessary for the mode stabilization. It is especially interesting to see the form of the antenna pattern in dynamics and the dynamic behavior of the antenna impedance. To solve these problems the coefficients in front of the various eigenfunctions were found numerically from (14). Dominant terms were taken into consideration first, than other eigenfunctions were included and the convergence of the series was estimated.

EIGENVALUES AND EIGENFUNCTIONS PROBLEM

To find the eigenvalues and corresponding eigenfinctions for the case of a long dielectric cylinder, a standard separation of variables was used so the functions were chosen as Bessel functions inside the dielectric

$$u_n^+ = \sum_m c_{mn} J_m (kr\sqrt{\varepsilon_n}) \cos m\varphi \qquad (15)$$

and Hankel functions outside the dielectric.

$$u_n^- = \sum_m d_{mn} H_m^{(2)}(kr) \cos m\varphi \qquad (16)$$

$k=\omega/c$ is a real wave number corresponding to the frequency of the source, as in the ε- method eigenvalues are the resonant values of the dielectric permittivity, not of the frequency.

Complex eigenvalues ε n were found from the condition at the boundary $r=a$ of the dielectric [1,4]

$$\frac{ka\sqrt{\varepsilon_n} J_m'(ka\sqrt{\varepsilon_n})}{J_m(ka\sqrt{\varepsilon_n})} = \frac{kaH_m^{(2)\prime}(ka)}{H_m^{(2)}(ka)} \qquad (17)$$

Of course, for this simple problem the abovementioned conditions are well known, and the exact eigenvalues and eigenfunctions can be easily found. For more complicated cases this part of the problem can be the most difficult one.

The approximate eigenvalues and eigenfunction can be calculated through variation of the functional [1]

$$\varepsilon(u) = \frac{\int_{v^+ + v^-} (\nabla u)^2 dV - k^2 \int_{v^-} u^2 dV}{k^2 \int_{v^+} u^2 dV} \qquad (18)$$

Functions u, corresponding to a stationary value of the functional are the eigenfunctions, and the value of the functional $\varepsilon(u)$ gives the eigenvalue. The functional can be analyzed by the Ritz method in the first approximation resulting in the set of approximate eigenvalues εn and corresponding approximate eigenfunctions as combinations of cylindrical waves.

APPLICATION TO THE CASE OF A SHORT CYLINDRICAL ANTENNA

The method developed in the previous sections was applied for the analysis of a cylindrical dielectric antenna with limited length. The problem becomes a three dimensional one and cannot be solved strictly analytically. Numerical methods like Ritz or Galerkin techniques meet difficulties connected with the continuous spectrum of radiating modes. Generalized eigenfunction method can help to simplify calculations due to the discrete spectrum of the modes.

In this section we present an application of the above mentioned method to a numerical calculation of the dynamic behavior of a short dielectric antenna. We consider a vertical dielectric cylinder with radius b and height d . A metal rod is placed along the central axis of the cylinder with the dielectric permittivity ε. The cylinder is placed on a horizontal conductive plane. Considering the dielectric resonator as a radial waveguide and the metal rod as a source of excitation we can come to a scalar wave potential problem [1,6]. For a dielectric with a high dielectric permittivity we can consider eigenfunctions propagating close to the radial direction as dominant ones [1]. The full problem was solved numerically in three steps. First approximate eigenvalues were found using the Ritz method from (18), then eigenfunctions were defined using boundary conditions. Next the entire domain Galerkin technique [5,6] was used for numerical calculation of the field at the boundary of the dielectric. Finally the fields in the far zone were defined using Kerchief integral.

Standard boundary conditions- continuity of tangentional field components are satisfied on the periphery and on the upper face of the cylinder. Radiation conditions are supposed in the infinity, and the fields are limited on the axis of the cylinder. This standard set of boundary conditions is described in multiple books and papers (see, for example, [1,3,4,5,6] It is shown in [6] that because of the axial symmetry electromagnetic fields can be described in terms of a single scalar TM-type wave potential *u,* with the magnetic field *H* expressed through *u* in the form

$$H(r,z) = j\sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0}}\frac{\partial u}{\partial r}, \qquad (19)$$

where we accept standard cylindrical coordinate's r, z and φ, and standard definitions

$$\varepsilon_0 = 1/(36\pi * 10^9)F/m$$

$$\mu_0 = 4\pi * 10^{-7} H/m$$

and ε is the dielectric permittivity, which is equal to 1 outside the dielectric and more than 1 inside it.

The wave potential satisfy the wave equation

$$(\nabla^2 + \varepsilon k^2)u = 0 \qquad (20)$$

at r > 0 ,

Again ε =1 outside the dielectric and ε >1 inside it, and. k is the free space wave number (k=ω/c), ω is the angular frequency of excitation, c is the velocity of light in vacuum. Eigenfunctions have the form

$$u_q = u_q(z)\, u_q(r)$$

$$u_q(z) = C_1 \cos(k_i\sqrt{\varepsilon_n}\, z)\ at\ z < d$$

$$u_q(z) = C_2 \exp(jp_i\, z)\ at\ z > d \qquad (21)$$

$$u_q(r) = C_3 J_m(n_i\sqrt{\varepsilon_n}\, r)\cos m\varphi\ \ at\ r < b$$

$$u_q(r) = C_4\ H_m(q_i r)\cos m\varphi\ at\ r > b$$

where J and H are Bessel and Hankel functions, the constants of normalization $C_{1,2,3,4}$ can be found in[6], and index q is a aggregation of radial index n and angular indices m and i [1]The set of eigenfunction is received incorporating all the boundary conditions, and the eigenvalues ε_n are found together with the parameters q,p,n from the system of equations

$$k_i^2 + q_i^2 = k^2$$

$$p_i^2 + n_i^2 = k^2$$

$$k_i^2 + n_i^2 = \varepsilon_n k^2 \qquad (22)$$

$$-k_i \tan(k_i d) = jp_i \varepsilon_n$$

$$\frac{n_i J'_m(bn_i)}{J_m(bn_i)} = \frac{-q_i H'_m(bq_i)}{H_m(bq_i)}$$

In details eigenfunctions (21) and boundary conditions (22) are discussed in [3,4,5,6]. The wave potential and electromagnetic fields on the surface of the dielectric antenna are then presented in the form of a series in terms of eigenfunctions u_q.

$$U = \sum_q B_q u_q$$

Coefficients B_q. of the series are found from the differential equations [2,3,4]

$$\frac{dB_q}{dt} + j(\omega-\omega_q)B_q = \frac{(\omega-\omega_q)}{\varepsilon-\varepsilon_q}\frac{1}{k^2}\int_V f\, u_q dV \quad (23)$$

Where integration is made over dielectric volume only, *f* is the source function, ω_q is the angular frequency (complex), which satisfies the equation e_q (ω)- e=0. The full solution for the field on the dielectric surface was found numerically in the form of a series in term of basis functions using the entire domain Galerkin technique [6]. Then the fields in the wave zone are found using Kerchief integral over the surface of the dielectric (this method is used to find the antenna pattern in vertical plane. Antenna pattern in horizontal plane was found as a sum of the series in terms of eigenfunctions..For simplicity we consider the distribution of the excitation functions as a given distribution.

RESULTS OF NUMERICAL CALCULATIONS

The dynamic behavior of a short cylindrical dielectrically loaded antenna was analyzed numerically using the generalized eigenfunction method described above and in [1,2,3,4]. The following parameters were chosen. Dielectrical vertical cylinder has the radius b=1cm, height d=1cm (0<z<d), it placed on a horizontal conductive plane z=0. Dielectric permittivity ε =9. A metal rod inserted along the vertical axis of the cylinder The frequency of excitation 10 GHz (k=ω/c=200π/3). Distribution of current along the

rod is considered as a given function (uniform or sinusoidal). A horizontal shift of the rod ds is introduced to analyze its role in the transient period. For the antenna pattern in a vertical plane the distribution of the fields on the cylinder surfaces as a function of time after a step excitation was calculated in the form of a series in terms of eigenfunction and then the pattern was estimated using Kirhoff's principle for radioactive apertures. For the dynamic antenna pattern in the horizontal plane the fields outside dielectric were presented in terms of eigenfunctions .(we suppose that the source is inside the dielectric). The results are shown in the figures.

Fig.1 shows the antenna pattern in horizontal plane for t=0.1 ns after the step excitation if the metal rod is placed on the dielectric cylinder axis. Fig.2 shows the amplitude of that antenna pattern as a function of time. Fig. 3 presents the antenna pattern in horizontal plane for t=0.1 ns if the metal rod is shifted 1mm from the axis. Fig. 4 corresponds to the time t=1 ns for the same shift. Figures 5 and 6 present the antenna pattern in vertical plane for the same case as in Figures 3 and 4. Fig 5 corresponds
to t=0.1 ns and Fig.6 –to t=1 ns

Without the horizontal shift (symmetrical excitation) horizontal antenna pattern is always symmetrical. Nevertheless, its amplitude shows oscillations up to the time about 10-15 passes of the reflected radiation through the dielectric. This time should be considered as a transient period, when the operation is not a steady-state one. A horizontal shift of the rod causes heavy deformations of the antenna pattern in the transient period. After the end of this period the pattern comes close to the symmetrical one. Changes of the antenna pattern in the vertical plane are not so dangerous for the operation of the antenna with the given configuration .

CONCLUSION

The results show the importance of the dynamic analysis both for antenna pattern and for the input impedance. An antenna working with a very high rate of information can be in the unstable operation for the time compared with the transmitted pulse duration. It can affect negatively the whole transmitting system. According to the reciprocity principle all the receiving antennas meet the same problems. The method presented in the paper is simple and universal. The generalized eigenfunction method can be applied to the dynamic analysis of any antenna , resonator, aperture, laser cavity or other radiating system. It helps to see clearly the configuration of the radiated field in space as a function of time. Using reasonably short calculations you can define the maximum possible rate of information which can be transmitted by your system through radiation without distortion. It becomes especially important if the information rate is so high that the frequency of radiation reaches the level where the dimensions of the radiator are more than the wavelength. For communication at 10 Mbit/s and higher instabilities in radiation pattern can be critical for the system operation, which makes the application of the dynamic analysis very important.

References

.[1].N. Voytovitch ,B. Kazenelenbaum , A. Sivov . Method of generalized eigenfunctions in the theory of diffraction. Nauka, Moscow, 1977, pp. 25-32

.[2].V. Petrunkin B. Lvov . "Use of generalized method of eigenmodes in the theory of lasers", *Tecn.Phys.*vol..42 1997, pp. 628-630.

[3]. Lvov B.V., Suleiman M.M. "Generalized Eigenfunction Method in Antenna Design". *IWTS'97*, May 1997, Malaysia, pp. 52-54

[4].B. Lvov, V. Petrunkin, "Dynamic Behavior of Dielectric Resonator Antennas", *ICECS 2001*, Malta 2-5 .09. 2001,pp. 341-344.

[5]. D. Kaklamani, N. Uzunoglu " Analysis of Dielectrically Loaded Resonators Using Entire - Domain Galerkin Technique", *IEEE Antenas and Propagation Magazine*, v.39,1997,pp30-54.

[6]. D. Kaklamani, C. Capsalis, N. Uzunoglu "Radiation from a Monopole Antenna Covered by a Finite Height Dielectric Cylinder", *Electromagnetics*,v.12,1992, pp.185-216.

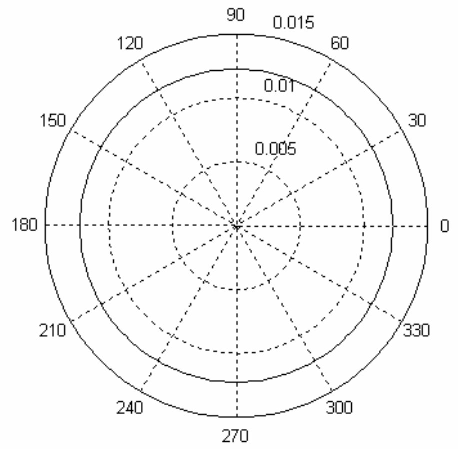

Fig. 1

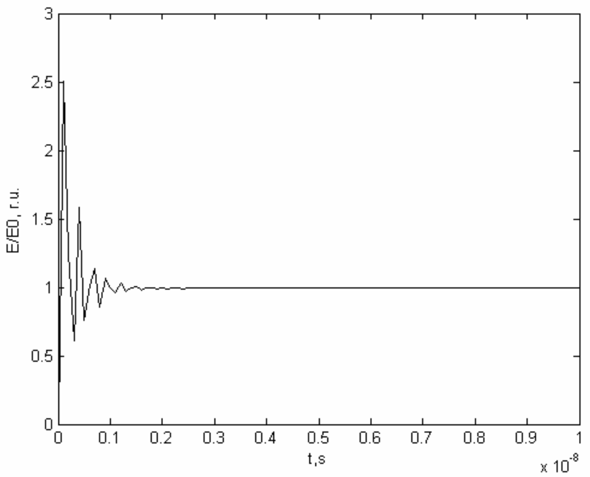

Fig.2

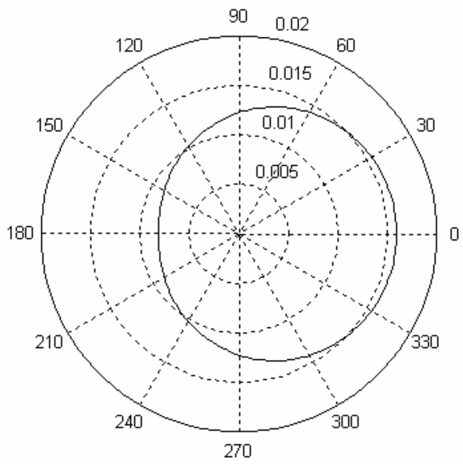

Fig.3

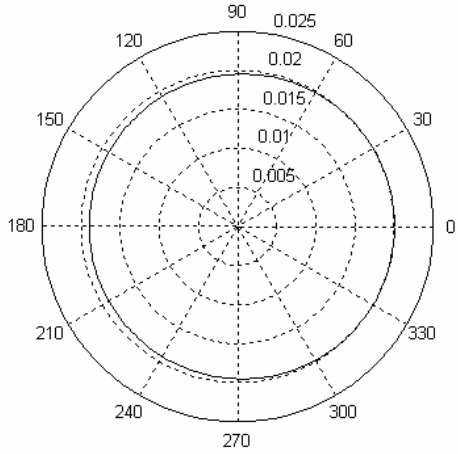

Fig. 4

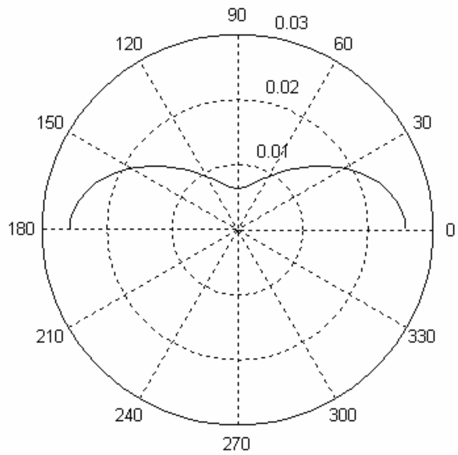

Fig.5

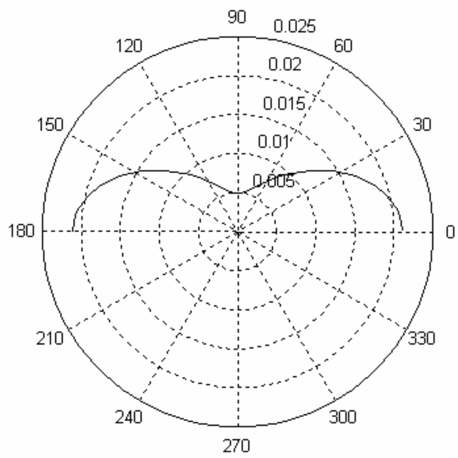

Fig.6