

# Unitary Matrix Frequency Modulated OFDM for Power Line Communications over Impulsive Noise Channels

Chang-Jun Ahn<sup>†</sup>, Hiroshi Harada, Satoshi Takahashi<sup>‡</sup>  
National Institute of Information and Communications Technology (NICT)<sup>†</sup>  
3-4 Hikarino-oka, Yokosuka, 239-0847, Japan

Faculty of Information Science, Hiroshima City University<sup>‡</sup>  
3-4-1 Ozuka-Higashi, Asa-Minami, Hiroshima, 731-3194 Japan

**Abstract.** Power line channel is the time-frequency variant channel with impulsive noise. Therefore, power line communication makes performance degradation due to time-frequency variant and impulsive noise. To overcome performance degradation, we consider the unitary signal constellation scheme and investigate the performance improvement of unitary matrix frequency modulated orthogonal frequency-division multiplexing(OFDM) for power line communication(UMFM-PLC/OFDM) over a PLC channel, and evaluate the BER performance. The proposed UMFM-PLC/OFDM system outperforms compared with the conventional PLC/OFDM.

**Key words:** Unitary signal constellation, impulsive noise, PLC, OFDM

## 1 Introduction

Power line communication (PLC) systems have many advantages and are assumed to be one of prospective solutions for short distance or in-home communication networks [1]-[3]. PLC takes the advantage of use in everyplace at home without additional network line. However, power line channel is the time-and-frequency variant and exhibits remarkable difference between locations, according to its network topology, the types of wire lines. Moreover, many electrical appliances frequently cause man-made electromagnetic noise on power line channels. Such man-made noise has the impulsive characteristics. These are technically critical problems to realize the power line communications with high rate and high reliability [4].

In such channels, impulsive noise and inter-symbol-interference (ISI) due to the frequency selective channel cause an unacceptable degradation of the error performance. OFDM is an efficient scheme to mitigate the effect of multi-path channel, since it eliminates ISI by inserting guard interval longer than the delay spread of the channel [5],[6]. Therefore, OFDM is generally known as an effective technique for high data rate services over the PLC channel.

Recently the combination of PLC/OFDM and space-time processing are employed to overcome the impulsive noise and multipath effect [7],[8]. These combination schemes exhibit better system perfor-

mance than the conventional PLC/OFDM in channel corrupted by impulsive noise. However, these combination systems are required the multi-wire power line cable for achieving a space-diversity. Therefore, the usage of space-time processing for PLC/OFDM is very limited for in-home communications network.

To overcome the above-mentioned problems, in this paper, we consider the unitary signal constellation scheme. Unitary signal constellation scheme has been proposed to perform space-time diversity in wireless communications system. Marzetta and Hochwald proposed and investigated unitary space-time modulation(USTM) as a mean of achieving capacity [9],[10]. These unitary signal constellations may be viewed as a multiple antenna extension. A unitary signal constellation is a matrix, whose columns are transmitted from multiple antenna elements and mutually orthogonal to each other. Such constellations have been designed and shown to perform well for uncoded transmission [11]. In this paper, we consider the diagonal code as an unitary signal constellation [12]. For a diagonal code, the components except for diagonal components in unitary signal constellation are 0. Since this code achieves a diversity with only diagonal components of unitary signal constellation, in this paper, we consider the unitary signal constellation with only diagonal component non-zero for simulation. Note that we do not claim the optimality of the unitary signal constellation, but rather, we argue that the unitary

signal constellation with its flexible scalability, and high performance in this paper.

In the PLC channel, the channel response at a particular subcarrier frequency is not supposed to be totally different from its neighboring frequencies, and hence, they must have correlation which depends on the coherence bandwidth of the channel  $B_c$ . When we assign the diagonal components of the diagonal code as an unitary signal constellation on neighboring frequencies. In this case, the diagonal components do not achieve the frequency diversity. However, we split the diagonal components over the coherence bandwidth, the detected signal can achieve the frequency diversity. In this paper, we propose the diagonal components of unitary signal constellation with splitting over the coherence bandwidth, and evaluate the system performance for PLC/OFDM over impulsive noise channels. The system model is described in Section 2. In Section 3, we show the simulation results. Finally, the conclusion is given in Section 4.

## 2 System model

### 2.1 Channel model

In PLC/OFDM systems, the channel response of frequency domain at the  $k$ -th subcarrier can be expressed as

$$\mathbb{H}(k) = \sum_{l=0}^{L-1} h_l(k) e^{j2\pi kl/K} = \mathcal{H}(k)^H \alpha(k) \quad (1)$$

where  $\mathcal{H} = [h_0, h_1, \dots, h_{L-1}]^H$  is  $L$ -sized vector containing the time responses,  $L$  is the number of channel paths, and  $\alpha(k)$  is FFT coefficient. Moreover, in this paper, we introduce Middleton's Class A noise model [13] into a statistical model of impulsive noise environment. This model is widely applicable by adjusting parameters, and provides fine closeness to experimental values. Middleton's noise model is composed of sum of Gaussian noise and impulsive noise. The Class A model is defined that the bandwidth of the noise is narrower than the bandwidth of the receiving system, i.e., the noise pulses do not produce transients in the front end of the receiver. According to the Class A noise model, the PDF (Probability Density Function) of the noise amplitude  $z$  is as follows,

$$p_a(z) = \sum_{m=0}^{\infty} \frac{e^{-A} A^m}{m!} \cdot \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{z^2}{2\sigma_m^2}\right) \quad (2)$$

where  $\sigma_m^2 = \sigma^2 \cdot \frac{(m+A)+\Gamma}{1+\Gamma}$ ,  $A$  is the impulsive index,  $\Gamma = \sigma_G^2/\sigma_I^2$  is the GIR (Gaussian-to-impulsive noise power ratio) with Gaussian noise power  $\sigma_G^2$  and impulsive noise power  $\sigma_I^2$ , and  $\sigma^2 = \sigma_G^2 + \sigma_I^2$  is the total noise power. The noise amplitude  $z$  followed by

Eq.(2) always includes the background Gaussian noise with power  $\sigma_G^2$ . On the other hand, sources of impulsive noise are distributed with Poisson distribution  $(e^{-A} A^m)/m!$ . One impulsive noise source generates noise which is characterized by the Gaussian PDF with variance  $\sigma_I^2/A$ . The parameter  $A$  is defined as the average number of impulses on the receiver in unit duration times the mean duration of them. The larger  $A$ , the impulsive noise will be more continuous, and then the Class A noise is led to be more likely to the Gaussian noise. Conversely, the smaller  $A$ , the Class A noise will be more impulsive. In particular, if  $A$  is nearly equal to 10, the statistical feature of the Class A noise is almost similar to that of the Gaussian noise. As the Gaussian noise power  $\sigma_G^2$  is comparatively larger in the total noise power  $\sigma^2$ , that is,  $\Gamma$  is larger, the Class A noise will approach to the Gaussian noise. Conversely, the smaller  $\Gamma$ , the Class A noise will be more impulsive.

### 2.2 A Class of Unitary Space-Time Signal Constellations

Unitary space-time signal is a matrix, whose rows are transmitted from the transmitted elements and mutually orthogonal to each other in wireless communication systems. Let  $L \geq 2$  denotes the size of a unitary signal constellation. We define  $\theta_L = \frac{2\pi}{L}$ . For any given integers  $\eta_1, \eta_2, \eta_3 \in \mathbb{Z}$ , we define the following unitary signal constellation of size  $L$

$$\nu = \nu(\eta_1, \eta_2, \eta_3) = \{\xi(l\eta_1, l\eta_2, l\eta_3) \mid l \in \mathbb{Z}_L\} \quad (3)$$

where  $\mathbb{Z}_L = (0, 1, \dots, L-1)$ , and,  $\xi(l\eta_1, l\eta_2, l\eta_3)$  is given by

$$\begin{aligned} \xi(l\eta_1, l\eta_2, l\eta_3) &= \begin{pmatrix} e^{j\theta_L} & 0 \\ 0 & e^{j\eta_1\theta_L} \end{pmatrix}^l \\ &\cdot \begin{pmatrix} \cos(\eta_2\theta_L) & \sin(\eta_2\theta_L) \\ -\sin(\eta_2\theta_L) & \cos(\eta_2\theta_L) \end{pmatrix}^l \\ &\cdot \begin{pmatrix} e^{i\eta_3\theta_L} & 0 \\ 0 & e^{-j\eta_3\theta_L} \end{pmatrix}^l. \end{aligned} \quad (4)$$

For any given constellation size  $L \geq 2$ , we will find a unitary signal constellation from the following class

$$\Omega_L \equiv \{\nu(\eta_1, \eta_2, \eta_3) \mid \eta_1, \eta_2, \eta_3 \in \mathbb{Z}_L\} \quad (5)$$

such that the unitary signal constellation has the largest diversity product in the constellation class as Eq. (5). The above unitary signal constellation is built from the parametric form of  $2 \times 2$  unitary matrices. We therefore call the signal constellation as Eq. (4) *parametric code*. It is seen that when  $\eta_2 = \eta_3 = 0$  is imposed in the constellation class as Eq. (5), the parametric code as Eq. (4) is exactly the *diagonal code* in the case  $M = 2$ . There have been several classes of

$2 \times 2$  unitary space-time constellation which were proposed in the previously. A diagonal code cyclic group code for a general  $M$  was introduced. A main difference between the diagonal code and the parametric code is that the diagonal code is in general a non-group signal constellation.

### 2.3 UFMF-PLC/OFDM

Here, we employ the diagonal code for achieving a diversity gain in the PLC channel. The data stream is divided into bit sequences that consist of  $R \cdot M$  bits, where  $R$  and  $M$  denote information bits per parallel symbol to be transmitted, and the number of diagonal element. Each  $R \cdot M$  bit sequence is mapped into the constellation  $\nu(l)$  ( $l \in \mathbb{Z}_L$ ) selected from  $L = 2^{RM}$ . The constellation of unitary matrix for UFMF can be written as

$$\text{diag}\{\nu(l)\} = [e^{jl\theta_L}, \dots, e^{jl\theta_L}], \quad (l \in \mathbb{Z}_L) \quad (6)$$

where  $\nu(l)$  is  $M \times M$  unitary matrix,  $\text{diag}\{\cdot\}$  is the diagonal operator, respectively. For example, in the case of  $M = 2$  and  $R = 1$ , which is equal to BPSK modulation. In this case, one of  $2 \times 2$  unitary matrix  $\nu(l)$  is assigned.

In the PLC channel as Eq. (1), the channel response at a particular subcarrier frequency is not supposed to be totally different from its neighboring frequencies, and hence, they must have correlation which depends on the coherence bandwidth of the channel  $B_c$ . When we assign the diagonal components of the unitary signal constellation on neighboring frequencies. In this case, the diagonal components do not achieve the frequency diversity. However, we split the diagonal components over the coherence bandwidth, the detected signal can achieve the frequency diversity. In this paper, we employ the diagonal code and split diagonal components of the selected code over the coherence bandwidth. Hereafter, we call this processing as an unitary matrix frequency modulation for PLC/OFDM (UMFM-PLC/OFDM). In the UFMF-PLC/OFDM systems, the diagonal components of the selected unitary signal constellation are splitting over the coherence bandwidth and are transmitted to the receiver. In this case, the received signal  $\mathbb{Y}(k)$  of the  $k$ -th subcarrier at receiver side is given by

$$\mathbb{Y}(k) = \mathbb{H}(k)\mathbb{X}(k) + \mathbb{N}(k) \quad k = 1, \dots, K \quad (7)$$

where  $\mathbb{X}(k)$  is the splitted diagonal component of unitary signal constellation over the coherence bandwidth, and  $\mathbb{N}$  is an impulsive noise. After de-splitting of the received signals and channel estimation, the frequency domain signals  $\mathbb{Y}$  are divided into  $M$  bits. Here, we consider the same structure of unitary signal constellation for  $M \times M$  as Eq. (6). Each  $M$  bit of

frequency domain signals is demodulated by ML estimator. The ML decision rule of the signal model as Eq. (1) is given by

$$\hat{\nu} = \arg \min \sum_{k=1}^K \left| \mathbb{Y}(k) - \mathbb{H}(k) \right. \\ \left. \cdot \sum_{l=1}^L \text{diag}(\nu)_{\text{mod}(k,M)+1}(l) \right|^2 \quad (8)$$

where  $\sum_{l=1}^L \text{diag}(\nu)_{\text{mod}(k,M)+1}(l)$  is the diagonal component of unitary signal constellation. The neighboring signals of  $\sum_{l=1}^L \text{diag}(\nu)_{\text{mod}(k,M)+1}(l)$  without splitting must have correlated channel responses, however, the split signals over the coherence bandwidth achieve totally different channel responses. It means that UFMF-PLC/OFDM systems can achieve a frequency diversity gain.

### 3 Computer simulated results

In this section, the system performance of the proposed UFMF-PLC/OFDM system is compared with the conventional PLC/OFDM when the power line channel is corrupted by impulsive noise. Fig. 1 shows the simulation model of UFMF-PLC/OFDM for  $N_c = 128$  subcarriers over the impulsive noise and multi-path PLC channel. In the transmitter, data stream is serial-to-parallel(S/P) transformed, and the diagonal components of unitary signal constellation are splitted over the coherence bandwidth. The OFDM time signal is generated by an IFFT and is transmitted over the frequency selective and time variant PLC channel after the cyclic extension has been inserted. The transmitted signals are subject to 2-path quasi-static channel. In this model,  $L = 2$  path as an independent identically distributed (i.i.d.) complex random variable according to Middleton's Class A noise model. This case causes a severe frequency selective channel. Maximum delay spread is  $0.5\mu$ . In our simulations, we consider two different impulsive noise scenarios [8]. The first one corresponds to a power line channel that

Table 1: Simulation parameters.

Data Modulation	BPSK
Demodulation	Coherent ML detection
Data rate	4 Msymbol/s
OFDM Symbol duration	65 $\mu$ sec
Number of carriers	128
Channel	2 path quasi-static
Maximum path delay	0.5 $\mu$ sec
Noise model	Additional white Class A

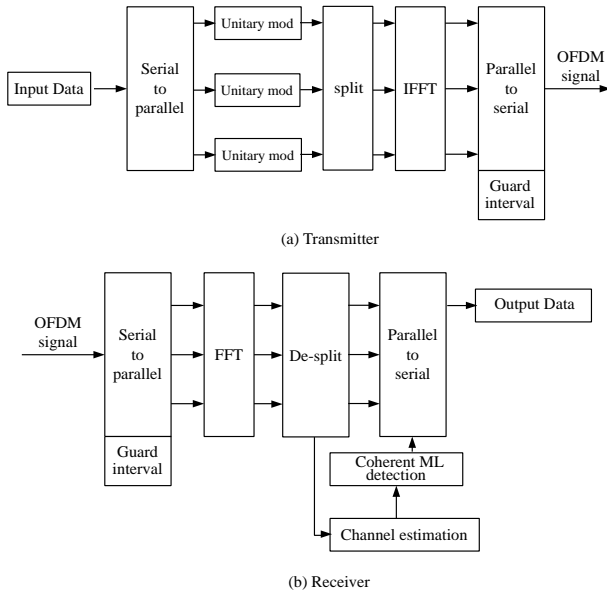


Figure 1: Proposed UFM-PLC/OFDM system.

is heavily disturbed by impulsive noise because the inter-arrival times between strong impulses are very short. The second one corresponds to a power line channel that is weakly disturbed by impulsive noise because the inter-arrival times between impulses are very long. We simulate this situation by considering the complete summation in Eq. (2). In the receiver, the received signals are S/P converted and  $N_c$  parallel sequences are passed to a FFT operator, which converts the signal back to the frequency domain. This frequency domain signals are de-split and coherently demodulated by using ML detection. The simulation parameters are listed in Table 1.

Fig. 2 shows the BER performance of UFM-PLC/OFDM and the conventional PLC/OFDM system for power line channels heavily distributed by impulsive noise. From Fig. 2, it is clear that the UFM-PLC/OFDM system outperforms the conventional PLC/OFDM over the entire range of energy per bit-to-noise spectral density ratio ( $E_b/N_0$ ) values. At BER of  $10^{-3}$ , the proposed UFM-PLC/OFDM system performs better than the conventional PLC/OFDM by 6 dB. This is because UFM-PLC/OFDM systems can achieve a diversity with splitting the diagonal components over the coherence bandwidth for a frequency diversity.

Fig. 3 shows the BER performance of UFM-PLC/OFDM and the conventional PLC/OFDM system for power line channels weakly distributed by impulsive noise. BERs of the proposed UFM-PLC/OFDM and the conventional PLC/OFDM with weak noise achieve better than those of with strong noise. At BER of  $10^{-3}$ , the proposed UFM-

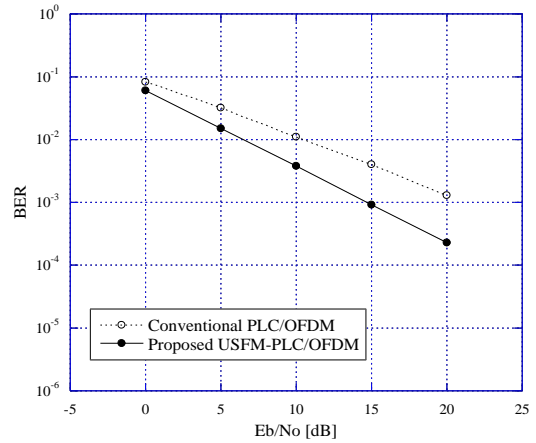


Figure 2: BER performance of UFM-PLC/OFDM and the conventional PLC/OFDM system for power line channels heavily distributed by impulsive noise.

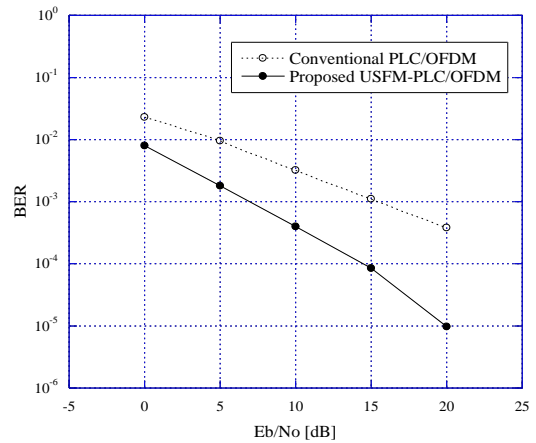


Figure 3: BER performance of UFM-PLC/OFDM and the conventional PLC/OFDM system for power line channels weakly distributed by impulsive noise.

PLC/OFDM system performs better than the conventional PLC/OFDM by 8 dB.

## 4 Conclusion

We have investigated the performance improvement of UFM-PLC/OFDM over the multipath and impulsive noised PLC channel, and evaluated the BER performance. The proposed UFM-PLC/OFDM system can achieve 6 and 8dB gains compared with the conventional PLC/OFDM for heavily and weakly distributed impulsive noise channel, respectively.

## References

- [1] M. Crussier, J. Baudais, and J. Helard, "Robust and high bit rate communications over PLC channel: a bit loading multi-carrier spread spectrum solution," *Proc. of ISPLC2005*, pp.37-41, 2005.
- [2] Y. Kim, S. Kim, and H. Oh, "OFDM receiver performance analysis with measured power line channel model for coded OFDM system," *Proc. of ISPLC2005*, pp.201-205, 2005.
- [3] M. Katayama, "Introduction to robust, reliable, and high-speed power line communication systems," *IEEE Trans. on Fundamentals.*, vol.E84-A, no.12, pp.2958-2965, Dec., 2001.
- [4] H. Dai, and H. V. Poor, "Advanced signal processing for power line communications," *IEEE Communication Magazine*, pp.100-107, May, 2003.
- [5] L. Cimini, "Analysis and Simulation of a Digital Mobile channel using OFDM," *IEEE Trans. on Comm.*, vol.33, pp.665-675, July 1985.
- [6] J.A. C. Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time has come," *IEEE Comm. Mag.*, vol.28, pp.5-14, May, 1990
- [7] C. L. Giovaneli, B. Honary, and P. Farrell, "Optimum space-diversity receiver for class A noise channel," *Proc. of ISPLC2004*, pp.189-194, 2004.
- [8] C. L. Giovaneli, B. Honary, and P. Farrell, "Space-frequency coded OFDM system for multi-wire power line communications," *Proc. of ISPLC2005*, pp.191-195, 2005.
- [9] T.L.Marzetta, and B.M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. on Information Theory*, vol.45, no.1, pp.139-157, Jan., 1999.
- [10] B.M. Hochwald, and T.L.Marzetta, "Unitary space-time modulation for multi-antenna communications in Rayleigh flat fading," *IEEE Trans. on Information Theory*, vol.46, no.2, pp.543-563, Mar., 2000.
- [11] B.M. Hochwald, T.L.Marzetta, T.J. richardson, W. Swendens, and R. Urbanke, "Systematic design of unitary space-time constellation," *IEEE Trans. on Information Theory*, vol.46, no.6, pp.1962-1973, Sept., 2000.
- [12] X. Liang, and X. Xia, "Some unitary signal constellation for differential space-time modulation," *Proc. of Asilomar conference on signals, systems and computers*, pp.716-720, 2000.
- [13] D. Middleton, "Stastical-physical model of electromagnetic interference," *IEEE Trans. on Electromagn. Compat.*, vol.19, no.3, pp.106-126, August, 1977.