Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp104-112)

Dynamic Modeling and Control in Operationel Space of an Hexapod Robot

***C. Mahfoudi , **K. Djouani,* M. Bouaziz and S. Rechak**
**University Paris12, LIIA laboratory
*Nationale Polytechnical School, Institute of Mechanics, Algiers

Abstract: This paper concerns real-times hexapod robot force control. Based on an operational trajectory planner, a computed torque control for each leg of hexapod robot is presented. This approach takes in to account the real-time force distribution on the robot legs and the dynamic model of the hexapod. First, Kinematic and dynamic modeling are presented. Than, a methodology for the optimal force distribution is given. The force distribution problem is formulated in terms of a nonlinear programming problem under equality and inequality constraints. Then, according to X. Chen et al, the friction constraints are transformed from nonlinear inequalities into a combination of linear equalities and linear inequalities. Therefore, the overall hexapod computed torque control is presented. Simulations are given in order to show the effectiveness of the proposed approach. Finally, some remarks and perspectives are given.

Key-Words: hexapod Robot, Optimal Force Distribution, Dynamic modeling and Control.

1 Introduction

Hexapod robots, as part of legged vehicles, can be used in work spaces with rough terrain, e.g. map building on an uneven ground, hazardous tasks like land mine searching and removing, volcano data collection, etc.

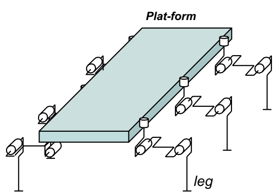

Figure 1: **View of the hexapod**

AS shown in [1], interests to walking machine, from research and application point, of views, are twofold. First, the complexity nature of legged locomotion has been very attractive and challenging to many pioneering researches. Due to the complexity of the legged robots, applications in real world are not significant. Major problems concerns real time dynamic control of the legged robot under several constraints. To overcome such problems, dynamic model should be integrated in every control strategy.
Before we address the hexapod robot's dynamic modeling it is helpful to have an overall view of how the robot is controlled. In the task planing stage, a trajectory planner is used to determine a path that guides the hexapod from its initial position to a given final position. Then, a gait, which gives the position and events for placing and lifting the robot legs is selected [2]. The inverse kinematic model is, than used in order to to compute the desired trajectory (positions and velocities) in joint space. A joint computed torque control strategy is used for the hexapod real time control. The proposed approach is based on the computation of the force distribution on the legs. Due to the existence of three actuated joints in each leg, the hexapod robot has redundant actuation leading to more active joints (18) than the robot platform degree-of-freedom (6 dof), figure(1). Thus, when formulating the force distribution problem, we find fewer force moment balancing equations than unknown variables. So, the solution of these equations is not unique. Moreover, some physical constraints, that concern the contact nature, friction, ...etc, must be taken into account in the calculation of force distribution. In addition, joints torque saturation must also be considered. Thus the Force Distribution Problem (FDP) can be formulated as a nonlinear constrained programming

problem under nonlinear equality and inequality constraints. Several approaches have been proposed for solving such a problem [3], [4],[5],[6], [7], [8] [9],[10] and [11].

The robot crawling is divided into 3 phases. The first phase, only 3 legs are supporting the robot, for instance legs $1-2-3$, leading to a force distribution problem with 9 unknown variables. Furthermore, in the second phase, all the six legs are supporting the robot leading to a force distribution problem with 18 unknown variables. In order to reduce the problem complexity, we consider that the contact forces on the legs 1-2-3 can be deduced from the first phase by introducing a continuous, decreasing function that varies from 1 to 0. Thus, the problem dimension, in the second phase, can be reduced from 18 to 9. The third phase is similar to the first one, with the legs 4-5-6 supporting the robot. In the three phases, the force-distribution problem is the same and solved with the same algorithm. The rest of the paper is organized as follows. Direct and inverse geometrical models of the hexapod are presented in section 2. In section 3 the dynamic model of an hexapod robot is derived. Section 4 concerns the force distribution problem. For simulation a real time control is presented in section 5.

2 Geometrical Modelling

Before presenting the direct and inverse geometrical model, let us consider the hexapod architecture. As the hexapod legs are identical, only one leg modelling is considered, the leg j architecture is given in figure (10). Every leg "j" (j=1,...,6) is fixed at the plate-forme by a revolute joint situated at l_j distance from the center of gravity of the plate-forme (the body). The angle ϕ_j represents the orientation of the coordinate frame $(x_{1,j}, y_{1,j}, z_{1,j})$ fixed at the first articulation of the leg and the coordinate fixed coordinate frame of the body (x_0, y_0, z_0). A walking robot is considerate as an arborescent robot with some closed loops. So to study this kind of robots we use the method defined by Wissama and Klifinger [12].

The transformation matrix from ith joint's attached coordinate frame to the (i-1)th joint's attached coordinate frame is given by figure (11):

$$ {}^{i-1}\boldsymbol{T}_i = R(Z,\gamma)T(Z,b)R(X,\alpha)T(X,d) \\ R(Z,\theta)T(Z,r) \quad (1)$$

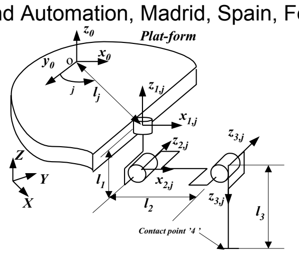

Figure 2: **geometrical model**

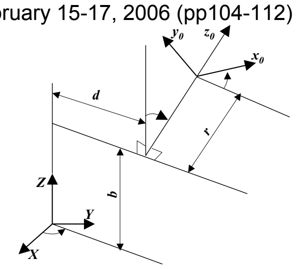

Figure 3: **geometrical parameters**

Thus : ${}^{i-1}\boldsymbol{T}_i =$

$$\begin{pmatrix} C\gamma_i C\theta_i - S\gamma_i C\alpha_i S\theta_i & -C\gamma_i S\theta_i - S\gamma_i C\alpha_i C\theta_i & S\gamma_i C\alpha_j & d_i C\gamma_i + r_i S\gamma_i S\alpha i \\ S\gamma_i C\theta_i + C\gamma_i C\alpha_i S\theta_i & -S\gamma_i S\theta_i + C\gamma_i C\alpha_i C\theta_i & -C\gamma_i S\alpha_i & d_i S\gamma_i - r_i C\gamma_i S\alpha i \\ S\gamma_i S\theta_i & S\gamma_i C\theta_i & C\alpha_i & r_i C\alpha_i + bi \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The table (1) describes the transformation from the world ground coordinate frame (X, Y, Z) to the coordinate frame at the contact point "4" of each leg. The transformation providing the exact position of the contact point"4" of any leg in the absolute coordinate frame fixed at the ground is given by :

frame	α	d	θ	r	b	γ
plat-form	α	d	θ	r	h	β
liaison"1"	0	l_j	$\theta_{1,j}$	0	0	ϕ_j
liaison"2"	$-\pi/2$	0	$\theta_{2,j}$	0	$-l1$	0
liaison"3"	0	l_2	$\theta_{3,j}$	0	0	0
contact Pt"4"	0	l_3	$\theta_{4,j}$	0	0	0

Table 1: **geometrical parameters**

$$ {}^{R}\boldsymbol{T}_4 = {}^{R}\boldsymbol{T}_0^0\boldsymbol{T}_1^1\boldsymbol{T}_2^2\boldsymbol{T}_3^3\boldsymbol{T}_4 \quad (2)$$

When the position and the orientation of the last coordinate frame fixed to the end of each leg "j" are known ,We apply the method proposed by Paul [13]. It provides the values of the joints coordinates $\theta_{i,j}$ $(i = 1, 2, 3)$ $(j = 1, .., 6)$.

3 Hexapod dynamics model

3.1 introduction

The robot dynamics is given by [14] [12] [15] :

$$\Gamma = f(\theta, \dot{\theta}, \ddot{\theta}) \tag{3}$$

Where, θ, $\dot{\theta}$, $and\,\ddot{\theta}$ are respectively the generalizes coordinates, speeds and acceleration.
The explicit forme of Eq(3) can be expressed as follows for any leg "j":

$$\Gamma = \boldsymbol{M}(\theta)\ddot{\theta} + \boldsymbol{C}(\theta,\dot{\theta})\dot{\theta} + \boldsymbol{Q}(\theta) + \boldsymbol{J}^T \boldsymbol{f} \tag{4}$$

where $\boldsymbol{M} \in \Re^{3\times3}$, $\boldsymbol{C} \in \Re^{3\times1}$, $\boldsymbol{Q} \in \Re^{3\times1}$ and $\boldsymbol{J}^T \boldsymbol{f} \in \Re^{3\times1}$.

- $\boldsymbol{M}(\theta)$, matrix $(n \times n)$ representing the inertia of the robot, which is deduced from the kinetic energy.

- $\boldsymbol{C}(\theta,\dot{\theta})$, a vector $(n \times 1)$ representing coriolis torques and centrifuges forces.

- $\boldsymbol{Q}(\theta) = [\boldsymbol{Q}_1, ..., \boldsymbol{Q}_n]$, a vector of gravity torque and forces.

- $\boldsymbol{f}$ the reaction of the ground

- E and U are respectively the kinetic and the potential energy of the system

The Eq(3) can be rewritten as:

$$\Gamma = \boldsymbol{M}(\theta)\ddot{\theta} + \boldsymbol{H}(\theta,\dot{\theta}) + \boldsymbol{J}^T \boldsymbol{f} \tag{5}$$

with :

$$\boldsymbol{H}(\theta,\dot{\theta}) = \boldsymbol{C}(\theta,\dot{\theta})\dot{\theta} + \boldsymbol{Q}(\theta)$$

Then, we can write :

$$\boldsymbol{H}(\theta,\dot{\theta}) = \Gamma \quad ; \quad if\ \ddot{\theta} = \boldsymbol{0}\ and\ \boldsymbol{f} = \boldsymbol{0}$$

So if we use Newton-Euler formalism with $\ddot{\theta} = \boldsymbol{0}\ and\ \boldsymbol{f} = \boldsymbol{0}$ we obtained the value of $\boldsymbol{H}(\theta,\dot{\theta})$.
This transformation is very important and permits to:

- extract the acceleration vector $\ddot{\theta}$:

$$\ddot{\theta} = \boldsymbol{M}^{-1}(\theta)(\Gamma - \boldsymbol{J}^T \boldsymbol{f} - \boldsymbol{H}(\theta,\dot{\theta}))$$

- avoid the computation of the vector $\boldsymbol{C}(\theta,\dot{\theta})$ which has redundant algorithm.

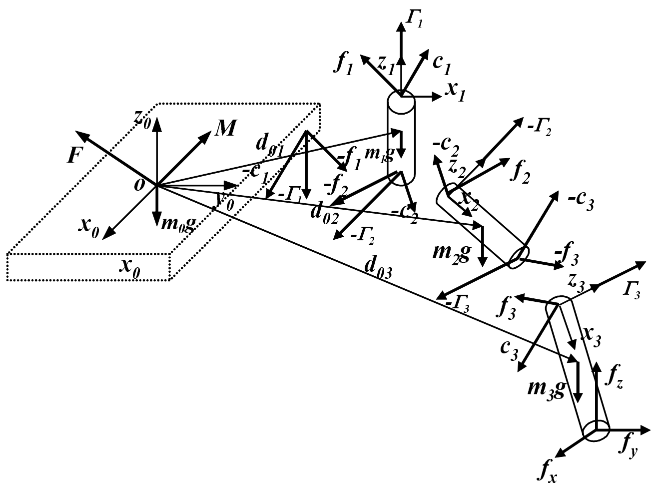

Figure 4: **Forces acting on the hexapod system**

3.2 Newton-Euler Formalism

Remark: The contact forces on the ground, $\boldsymbol{f}_x, \boldsymbol{f}_y, \boldsymbol{f}_z$ are 0 if the leg is lifted and $\neq 0$ otherwise.
The Newton-Euler algorithm can be established as follow [12] :

3.2.1 Velocities and accelerations computation

let C_i be any link of a leg "j", figures (4)(5), $\boldsymbol{v}_i$ the velocity of the gravity center G_i and ω_i the angular velocity of the link C_i. Let $\gamma_i = \dot{\mathbf{v}}_i$ the acceleration of G_i and $\boldsymbol{a}_i = \dot{\omega}_i$ the angular acceleration of the link C_i. Then:

$$\boldsymbol{P}_{oi} = \boldsymbol{P}_{o,i-1} + \boldsymbol{P}_{i-1,i} \tag{6}$$

and

$$\boldsymbol{d}_{oi} = \boldsymbol{P}_{oi} + \boldsymbol{d}_{i,i} \tag{7}$$

after derivation :

$$\dot{\boldsymbol{P}}_{oi} = \dot{\boldsymbol{P}}_{o,i-1} + \omega_{i-1} \wedge \boldsymbol{P}_{i-1,i} + \sigma_i \dot{\theta}_i \boldsymbol{Z}_i \tag{8}$$

In our case, we have only rotational articulations so $\sigma_i = 0$, then:

$$\boldsymbol{v}_i = \dot{\boldsymbol{d}}_{oi} = \dot{\boldsymbol{P}}_{oi} + \omega_{i-1} \wedge \boldsymbol{d}_{i,i}$$

The second derivation gives:

$$\ddot{\boldsymbol{P}}_{oi} = \ddot{\boldsymbol{P}}_{o,i-1} + \boldsymbol{a}_{i-1} \wedge \boldsymbol{P}_{i-1,i} + \omega_{i-1} \wedge (\omega_{i-1} \wedge \boldsymbol{P}_{i-1}) \\ +\sigma_i(2\dot{\theta}_i \omega_{i-1} \wedge \boldsymbol{Z}_i + \ddot{\theta}_i \boldsymbol{Z}$$

Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp104-112)

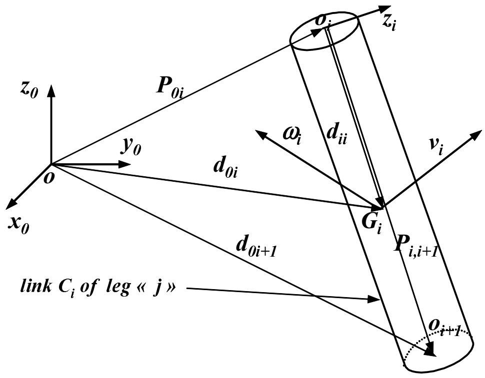

Figure 5: **Kinematics parameters representation of a link "i" in a leg "j"**

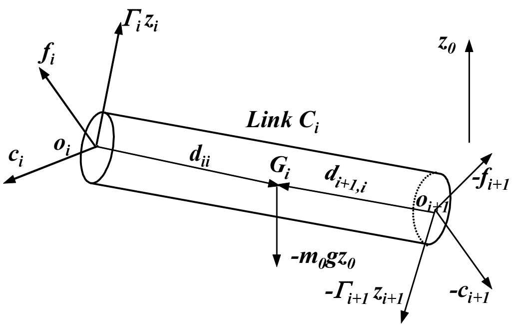

Figure 6: **Equilibrium of a link "i" in a leg "j"**

(9)

And

$$\gamma_i = \dot{\boldsymbol{v}}_i = \ddot{\boldsymbol{d}}_{oi} = \ddot{\boldsymbol{P}}_{oi} + \boldsymbol{a}_{i-1} \wedge \boldsymbol{d}_{i,i} + \omega_i \wedge (\omega_{i-1} \wedge \boldsymbol{d}_{ii}) \tag{10}$$

γ_i represents the gravity center acceleration.
There fore we can define the global acceleration with the following relation:

$$\alpha_i = \ddot{\boldsymbol{P}}_{oi} + g\boldsymbol{Z}_0 \tag{11}$$

where, g is the gravity acceleration with :

$$\boldsymbol{a}_i \wedge \star + \omega_i \wedge (\omega_i \wedge \star) = \underline{\boldsymbol{b}}_i \star \tag{12}$$

In condensed forme:

$$\underline{\boldsymbol{b}}_i = \widehat{\underline{\boldsymbol{a}}} + \widehat{\underline{\omega}}.\widehat{\underline{\omega}} \tag{13}$$

$\widehat{\underline{\boldsymbol{a}}} and \widehat{\underline{\omega}}$: represent the skew symmetric matrixes .
Eq(11) can be represented in condensed forme:

$$\alpha_i = \sigma_i \boldsymbol{t}_i + r_{i-1} \tag{14}$$

with:

$$\begin{cases} \boldsymbol{t}_i = 2\dot{\theta}_i \omega_{i-1} \wedge \boldsymbol{Z}_i + \ddot{\theta}_i \boldsymbol{Z}_i \\ \boldsymbol{r}_{i-1} = \alpha_{i-1} + \underline{\boldsymbol{b}}_{i-1} \boldsymbol{P}_{i-1,i} \end{cases}$$

In our case, $\sigma_i \boldsymbol{t}_i = 0$ because $\sigma_i = 0$, then we will have:

$$\gamma_i + g\boldsymbol{Z}_0 = \alpha_i + \underline{\boldsymbol{b}}_i \boldsymbol{d}_{ii} \tag{15}$$

3.2.2 Forces and torques computation

- Let $\boldsymbol{c}_i$, the interaction torque exerted by the link C_{i-1} on the link C_i figure(6).
- $\boldsymbol{f}_i$, the interaction force exerted on o_i by the link C_{i-1} on the link C_i.
- $\boldsymbol{F}^i = \boldsymbol{f}^i + \sigma_i \Gamma^i \boldsymbol{Z}_i$, the force exerted, on o_i by the i^{th} actionnary on the link C_i, where Γ^i is a scalars, representing the moment of the motor.
- $\boldsymbol{C}^i = \boldsymbol{c}^i + \overline{\sigma}_i \Gamma^i \boldsymbol{Z}_i$: the moment exerted, by the link C_{i-1} and the i^{th} actionnary on the link C_i.

The equilibrium of the link C_i can be represented by:

$$\boldsymbol{F}^i_{res} = \boldsymbol{F}^i - \boldsymbol{F}^{i+1} - m_i g \boldsymbol{Z}_0 = m_i \gamma_i \tag{16}$$

$$\boldsymbol{C}^i_{res} = \boldsymbol{C}^i - \boldsymbol{C}^{i+1} - \boldsymbol{d}_{i,i} \wedge \boldsymbol{F}^i + \boldsymbol{d}_{i+1,i} \wedge \boldsymbol{F}^{i+1} \tag{17}$$

and

$$\boldsymbol{C}^i_{res} = \varphi^i \boldsymbol{a}_i - \omega_i \wedge (\varphi^i \omega_i) \tag{18}$$

Where ,φ^i is the inertia matrixes of the link C_i. from Eq(16) :

$$\boldsymbol{F}^i = \boldsymbol{F}^{i+1} + m_i(\alpha_i + \underline{\boldsymbol{b}}_i \boldsymbol{d}_{ii}) \tag{19}$$

Then, from Eq(17) and (18) we obtained:

$$\boldsymbol{C}^i = \boldsymbol{C}^{i+1} + \varphi^i \boldsymbol{a}_i + \omega_i \wedge (\varphi^i \omega_i)$$

$$+\boldsymbol{d}_{ii} \wedge (\boldsymbol{F}^i - \boldsymbol{F}^{i+1}) + \boldsymbol{P}_{i,i+1} \wedge \boldsymbol{F}^{i+1}$$

3.2.3 The kinetic energy computation

The total kinetic energy of the system is given as follows :

$$E = \sum_{j=1}^{n} E_j \tag{20}$$

where E_j, kinetic energy of the link C_j,expressed by the following equation :

$$\boldsymbol{E}_j = \frac{1}{2}(\omega_j^T \varphi^j \omega_j + m_j \boldsymbol{V}_{Gj}^T \boldsymbol{V}_{Gj}) \tag{21}$$

ω_j : the instantiate rotation velocity of the link "C_j" expressed in the coordinate frame "j".
$\boldsymbol{V}_{Gj}$: linear velocity of the gravity center of the link "C_j" expressed in the coordinate frame "j". as the figure (7) show:

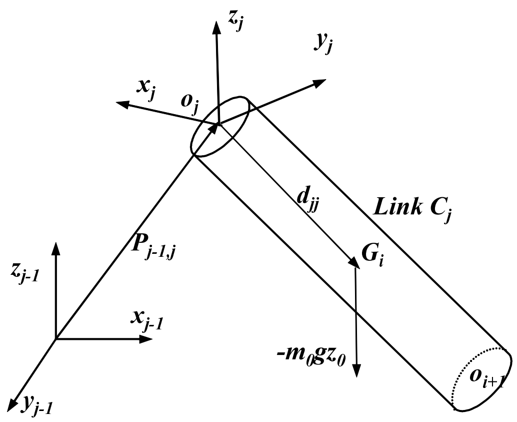

Figure 7: **Modelling a link of a leg**

$$\boldsymbol{V}_{Gj} = ^j \boldsymbol{V}_j + ^j \omega_j \boldsymbol{d}_{jj} \tag{22}$$

as we know:

$$\boldsymbol{k}_j = \varphi^j - m_j \widehat{\boldsymbol{d}}_{jj} \widehat{\boldsymbol{d}}_{jj} \tag{23}$$

Eq(21) can be transformed :

$$\boldsymbol{E}_j = \frac{1}{2}(\omega_j^{T\,j} \boldsymbol{k}_j\, ^j\omega_j + m_j\, \boldsymbol{V}_j^{T\,j} \boldsymbol{V}_j + 2m_j\, ^j \boldsymbol{d}_{jj}^T (^j \boldsymbol{V}_j \wedge ^j\omega_j)) \tag{24}$$

with:

$$^j\omega_j = ^j \boldsymbol{A}_{j-1}\, ^{j-1}\omega_{j-1} + \overline{\sigma}_j \dot{\theta}_j^j \boldsymbol{Z}_j = ^j \omega_{j-1} + \overline{\sigma}_j \dot{\theta}_j\, ^j \boldsymbol{Z}_j \tag{25}$$

and,

$$^j \boldsymbol{V}_j = ^j \boldsymbol{A}_{j-1}\,(^{j-1} \boldsymbol{V}_{j-1} + ^{j-1}\omega_{j-1} \wedge \boldsymbol{P}_{j-1,j}) + \sigma_j \dot{\theta}_j\, ^j \boldsymbol{Z}_j \tag{26}$$

$^j\boldsymbol{A}_{j-1} \in \Re^{3\times 3}$: represented the orientation matrix .

3.2.4 Elements M_{ij} of the matrix M

:
In the end we computed the matrix $\boldsymbol{M}$ from Eqs(20)(24) :
The elements M_{ii} of the matrix $\boldsymbol{M}$ is equal to the coefficient of $\dot{\theta}_i^2/2$ in the expression of kinetic energy, and the elements M_{ij} , if $i \neq j$ is equal to the coefficient of $\dot{\theta}_i\dot{\theta}_j$. $M_{11} = Iz_1 + S2^2 Ix_2 + C2^2 Iy_2 + S23^2 Ix_3 + C23^2 Iy_3 + m_3 C2^2 l_2^2 + m_3 C23\, C2 l_2 l_3 + Ia_1$
$M_{12} = M_{13} = 0$
$M_{22} = Iz_2 + Iz_3 + m_3 S3^2 l_2^2 + m_3 C3^2 l_2^2 + m_3 C3\, l_2 l_3 + Ia_2$
$M_{23} = Iz_3 + 1/2\, m_3 C3 l_2 l_3$
$M_{33} = Iz_3 + Ia_3$
Whith, $(Ia_1, Ia_2\, and\, Ia_3)$ represent the inertias of the motors. These results are obtained with diagonal inertia matrix of the legs links.
At the end of computation for all legs, we can describe the equilibrium of the plat-form as following, figure (8).

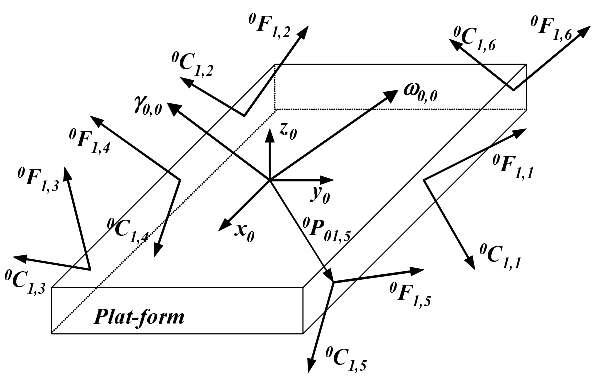

Figure 8: **Equilibrium of the plat-form**

- let $\gamma_{0,0}$ and $\omega_{0,0}$ respectively the plat-form desired linear acceleration and angular acceleration in the coordinates frame (x_0, y_0, z_0).
- $\boldsymbol{F}_{1,j}^0$, the force applied by the leg "j" at the articulation "1" on the plat-form "0".
- $\boldsymbol{C}_{1,j}^0$, the moment applied by the j^{th} leg in the articulation "1" on the plat-form .
- $\boldsymbol{P}_{01,j}^0$, the distance between the articulation "1" of the j^{th} leg and the origin of the coordi-

Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp104-112)

nate frame (x_0, y_0, z_0) expressed in the same coordinate frame.

The application of the dynamic fundamental principe, at the mass center of the plate-form provides the following matrix equation :

$$\begin{pmatrix} m_0 \boldsymbol{I}_3 & \boldsymbol{0} \\ \boldsymbol{0} & \underline{\varphi}^0 \end{pmatrix} \begin{pmatrix} \gamma_{0,0} \\ \omega_{0,0} \end{pmatrix} + \begin{pmatrix} -m_0 \mathbf{g}_0 \\ \omega_{0,0} \wedge (\underline{\varphi}^0 \omega_{0,0}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{M} \end{pmatrix} \tag{27}$$

Where,

- m_0 and $\underline{\varphi}^0$ are respectively the masse and the inertia matrix of the plat-form.
- $\boldsymbol{I}_3$, the identity matrix (3×3).
- $\mathbf{g}_0$: gravity vector

$\boldsymbol{F}$ and $\boldsymbol{M}$ are definite as follow :

$$\begin{cases} \boldsymbol{F} = \sum_{j=1}^{6} \boldsymbol{F}_{1,j}^0 \\ \boldsymbol{M} = \sum_{j=1}^{6} (\boldsymbol{C}_{1,j}^0 + \boldsymbol{P}_{01,j}^0 \wedge \boldsymbol{F}_{1,j}^0) \end{cases} \tag{28}$$

4 Force Distribution Problem

4.1 problem Formulation

The force system acting on a hexapod robot is shown in figure (9). For simplicity, only the force components on the foot are presented here. In the general case, rotational torques at the feet are neglected. Let (x_0, y_0, z_0) be the robot fixed body coordinate frame in which the body is located in the (x_0, y_0) plane and $(x_{1,j}, y_{1,j}, z_{1,j})$ denote the coordinate frame fixed at the foot "j", in which the leg j lies in the $(x_{1,j}, z_{1,j})$ plane and its z axis is normal to the support surface of the foot which is assumed to be parallel to the (x_0, y_0, z_0) plane. $\boldsymbol{F} = [F_X F_Y F_Z]^T$ and $\boldsymbol{M} = [M_X M_Y M_Z]^T$ denote respectively the robot body force vector and moment vector, which results from the gravity and the external force acting on the robot body. Define $f_{x,j}, f_{y,j}$, and $f_{z,j}$ as the components of the force acting on the supporting foot "j" in the directions of x_0, $y_0\, and\, z_0$, respectively. The number of supporting feet, n, can vary between 3 and 6 for an hexapod robot. The robot's quasi-static force/moment equation can be written as :

$$\begin{cases} \sum_{j=1}^{n} \boldsymbol{f}_j = \boldsymbol{F} \\ \sum_{j=1}^{n} \boldsymbol{OP}_j \wedge \boldsymbol{f}_j = \boldsymbol{M} \end{cases} \tag{29}$$

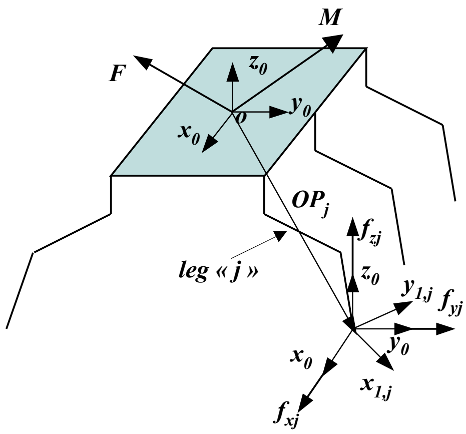

Figure 9: **Orientation of coordinate frame**

where $\boldsymbol{OP}_j$ is the position vector joining contact point of the leg "j" and the gravity center of the body. The general matrix form of this equation can be written as :

$$\boldsymbol{A}\,\boldsymbol{G} = \boldsymbol{W} \tag{30}$$

with :

$$\begin{cases} \boldsymbol{G} = [\boldsymbol{f}_1^T \boldsymbol{f}_2^T \cdots \boldsymbol{f}_n^T]^T & \in \Re^{3n} \\ \boldsymbol{f}_j^T = [f_{x,j}\, f_{y,j}\, f_{z,j}]^T & \in \Re^3 \\ \boldsymbol{W} = [\boldsymbol{F}^T\, \boldsymbol{M}^T]^T & \in \Re^6 \end{cases}$$

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{I}_3 & \cdots & \cdots & \boldsymbol{I}_3 \\ \boldsymbol{B}_1 & \cdots & \cdots & \boldsymbol{B}_n \end{pmatrix} \quad \in \Re^{6 \times 3n}$$

$$\boldsymbol{B}_j \equiv \widehat{\boldsymbol{OP}}_j \equiv \begin{pmatrix} 0 & -P_{z,j} & P_{y,j} \\ P_{z,j} & 0 & -P_{x,j} \\ -P_{y,j} & P_{x,j} & 0 \end{pmatrix} \quad \in \Re^{3 \times 3}$$

where $\boldsymbol{I}_3$ is the identity matrix and $\mathbf{G}$ is the foot force vector, corresponding to three ($\boldsymbol{G} \in \Re^9$) or six ($\boldsymbol{G} \in \Re^{18}$) supporting legs. $\boldsymbol{A}$ is a coefficient matrix which is a function of the positions of the supporting feet, and $\boldsymbol{B}_j$ is a skew symmetric matrix consisting of $(P_{x,j}, P_{y,j}, P_{z,j})$, which is the position coordinate of the supporting foot "j" in (x_0, y_0, z_0). $\boldsymbol{W}$ is a total body force/moment vector. It is clear that Eq(30) is an underdetermined system and its solution is not unique. In other words, the feet forces have many solutions according to the equilibrium equation. However, the feet forces must meet the needs for the following physical constraints, otherwise they become invalid :

Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp104-112)

1. Supporting feet should not slip when the robot walks on the ground. It results in the following constraint:

$$\sqrt{f_{x,j}^2 + f_{y,j}^2} \le \mu \, f_{z,j} \tag{31}$$

   where μ is the static coefficient of friction of the ground

2. Since the feet forces are generated from the corresponding actuators of joints, the physical limits of the joint torques must be taken into account. It follows that :

$$-\tau_{jmax} \le \boldsymbol{J}_j^{T\,j} \boldsymbol{A}_{0j} \begin{pmatrix} f_{x,j} \\ f_{y,j} \\ f_{z,j} \end{pmatrix} \le \tau_{jmax} \tag{32}$$

   for $(j = 1, ..., n)$, where $\boldsymbol{J}_j \in \Re^{3\times 3}$ is the Jacobian of the leg "j", $\tau_{jmax} \in \Re^{3\times 1}$ is the maximum joint torque vector of the leg "j", and $\boldsymbol{A}_{0j} \in \Re^{3\times 3}$ is the orientation matrix of $(x_{1,j}, y_{1,j}, z_{1,j})$ with respect to (x_0, y_0, z_0).

3. In order to have definite contact with the ground, there must exist a $f_{z,j}$ such that :

$$f_{z,j} \ge 0 \tag{33}$$

   In the following, we propose an approach for problem size reduction, linearisation and solving for the hexapod case. Clearly, it is difficult to solve such a nonlinear programming problem for real-time feet force distribution with complex constraints.

The solution of this system is developed in [2].

5 Computed-torque control

Suppose that desired trajectory $X_d(t)$ has been selected for the arm motion. To ensure trajectory tracking by the joint variable,errors [14] [12][16] [15] .

$$\boldsymbol{e}(t) = X_d - X(t) \tag{34}$$

Then the overall robot arm input becomes:

$$\Gamma = \boldsymbol{A}\boldsymbol{J}^{-1}(\ddot{\boldsymbol{X}} - \dot{\boldsymbol{J}}\dot{\theta}) + \boldsymbol{H} \tag{35}$$

$$\boldsymbol{X}(t) = \ddot{\boldsymbol{X}} + \boldsymbol{k}_v(\dot{\boldsymbol{X}}_d - \dot{\boldsymbol{X}}) + \boldsymbol{k}_p(\boldsymbol{X}_d - \boldsymbol{X}) \tag{36}$$

This controller is shown in figures (12),(13)

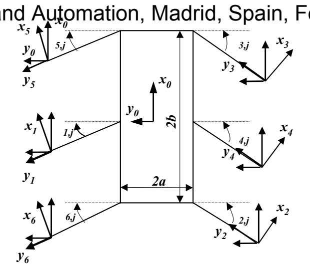

Figure 10: **bottom View of the hexapod**

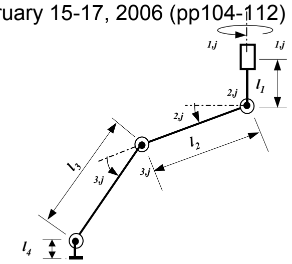

Figure 11: **basic mechanism of the leg**

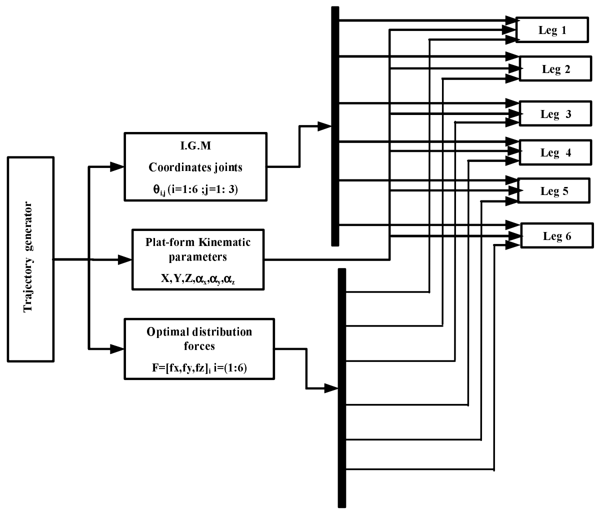

Figure 12: **Control model of the hexapod**

5.1 Choice of PD Gains

It is usual to take the $n \times n$ matrices diagonal so that:

$$\boldsymbol{k}_v = diag[k_{vi}], \quad \boldsymbol{k}_p = diag[k_{pi}]$$

and $k_{pi} = \omega_n^2$, $k_{pi} = 2\xi\omega_n$ with ξ the damping ratio and ω_n the naturel frequency. The PD gains are usually selected for critical damping ξ=1. Then, to avoid exciting the resonant mode, we should select natural frequency to half the resonant frequency $\omega_n < \omega_r/2$.

5.2 Simulation results

In order to show the effectiveness of proposed approach, some simulations were conducted under Matlab. We consider that the hexapod robot is crawling in a linear trajectory (Y=3X), on an uneven ground, in the X-Y plane. Furthermore, the

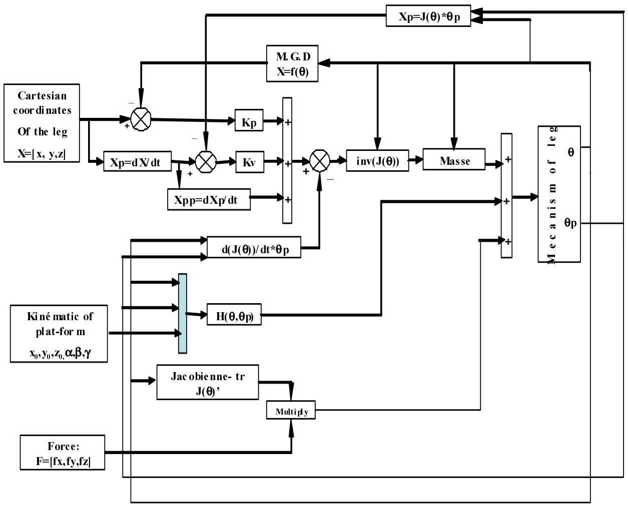

Figure 13: **Control model of a leg "j"**

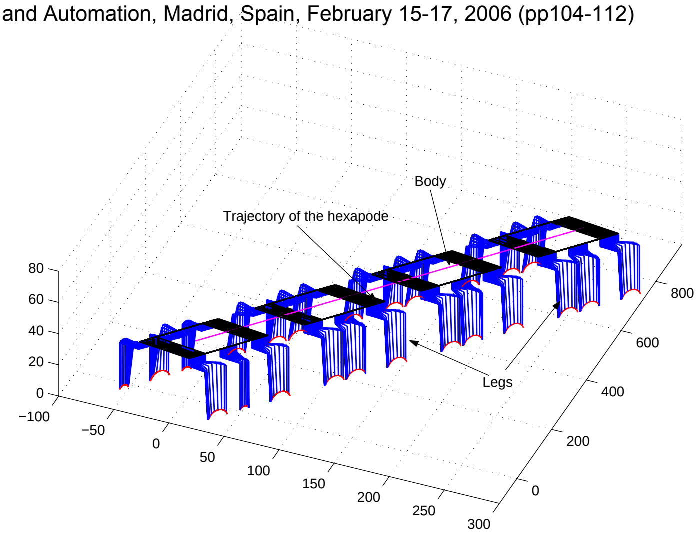

Figure 14: **View of crawling hexapod**

force tensor acting at the body center are (Fx = -3, Fy = 5, Fz = -50 [N], Mx = 0 , My = 2 , Mz = 1 [Nm]). The basic mechanism, size and parameters of Hexapod robot are shown in Figure (10) and (11), where a = 0.25 [m], b = 0.6 [m], l_1 = 0.05 [m], l_2 = 0.20 [m], l_3 = 0.30 [m] and $l_4 \simeq 0$ [m]. There are tree actuated joints $\theta_{1,j}$, $\theta_{2,j}$, $and\,\theta_{3,j}$ in the leg "j", for (j=1,...,6). The masses and the inertia of links are respectively (m1 = 0.1, m2 = 0.07, m3 = 0.03 [kg]) and (Ix1 = 1.36, Iy1 = 0.297, Iz1 = 1.6, Ix2 = 2.1, Iy2 = 2.29, Iz2 = 0.33, Ix3 = 0.001, Iy3 = 0.05, Iz3 = 0.05 [kg cm^2]); The simulation is presented for two cycles of walking corresponding to 15 seconds. The lifted legs do a cycloid trajectory

6 Conclusion

In this paper, we have presented a dynamic decoupled control for a hexapod walking machine. The proposed approch is based on a trajectory planner in operational space, a real time computing of the force distribution on the hexapod legs and a joint computed torque control strategy at the level. The force distribution problem has been formulated in terms of non linear programming problem has been solved as a quadratique optimization problem. Simulations where given in order to show the efectiveness of the proposed approach.

References

[1] B. S. Lin and S. Song. Dynamic modeling, stability, and energy efficiency of a quadrupedal walking machine. " *Journal of Robotics Systeme*, 18:657–670, 2001.

[2] C. Mahfoudi K Djouani S. Rechak and M. Bouaziz. Optimal force distribution for the legs of an hexapod robot. *2003 IEEE Conference on Controle application CCA 2003*, June 23-25, Istambul, Turkey.

[3] D. E. Orin and S. Y. Oh. Control of force distribution in robotic mechanisms containing closed kinematic chains. " *Trans. of the ASME, J. of Dynamic Systems, Measurement, and Control*, 102:134–141, 1981.

[4] C. A. Klein and S. Kittivatcharapong. Optimal force distribution for the legs of a walking machine with friction cone constraints. *IEEE Trans. on Robotics and Automation*, 6:73–85, 1990.

[5] F. T.Cheng and D. E. Orin. Efficient formulationof the force distribution equations for simple closed-chain. *IEEE Trans. on Systems, Man, and Cybernetics*, 21:25–32, 1991.

[6] M. A. Nahon and J. Angeles. Optimization of dynamic forces in mechanical hands. *Trans. of the ASME, J. of Mechanical Design*, 113:167–173, 1999.

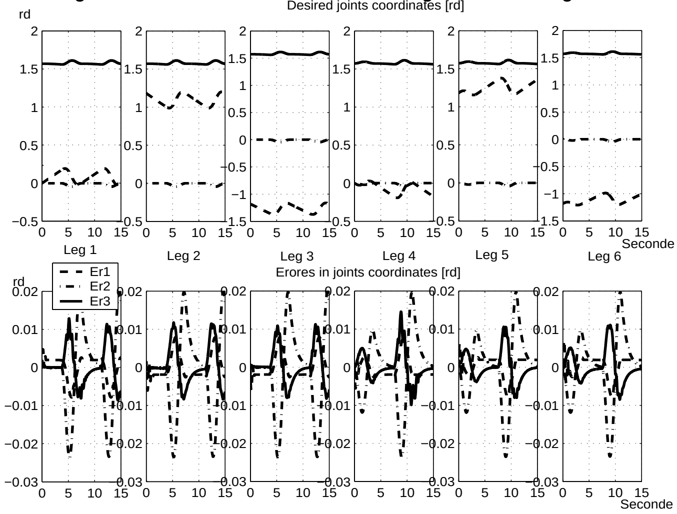

Figure 15: **Errors in coordinates joints**

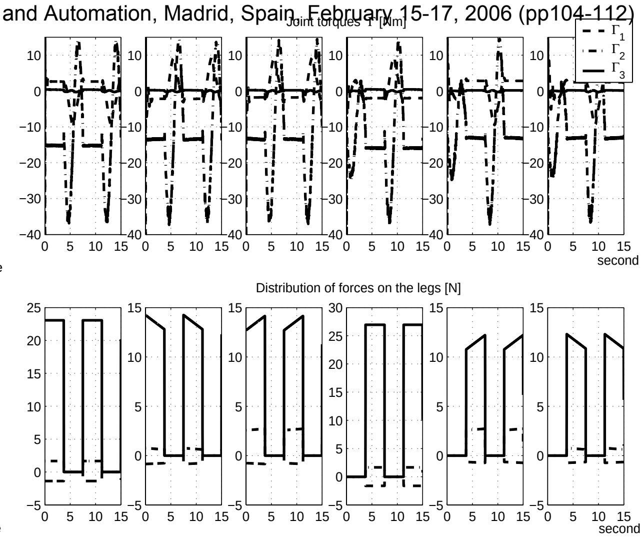

Figure 16: **Articulations torques and reactions of the ground**

[7] M. A. Nahon and J. Angeles. Real-time force optimization in parallel kinematic chains under inequality constraints.

[8] H. Liu and B. Wen. Force distribution for the legs of quadruped robot. *of Robotique Systemes*, 14:1–8, 1997.

[9] V. Kumar and K. J.Waldron. forces distribution in walking vehicules. *Trans. of the ASME, J. of Mechanical Design*, 112:90–99, 1990.

[10] X. Chen and K.Watanabe. Optimal force distribution for the legs of quadruped robot. *Machine inteligence and robotique control*, 1:87–94, 1999.

[11] W. Kwon and B.C. Wen. A new optimal force distribution scheme of multiple cooperating robots using dual method. *J. of Intelligent and Robotic Systems*, 21:301–326, 1998.

[12] W. Khalil and E.Dombre. *modelisation identification et commande des robots.* Hermes sciences, paris, 1999.

[13] R.C Paul. Robots manipulators, mathematics, programing and control. *MIT press*, 1981.

[14] Q. Xiding and G. Yimin. Analysis of the dynamics of a six-legged vehicle. *" The International Journal of Robotics Research*, 14:1–8, 1995.

[15] F. Pfeiffer J. Eltze and H. Weidman. Six-legged technical walking considering biological principles. *" Robotics and Autonomous Systems*, 14:223–232, 1995.

[16] P. Alexandre and A. Preumont. On the gait control of a six-legged walking machine. *" International Journal of Systemes Sience*, 27(8):713–721, 1996.