Doppler Detection in HF Radars Using Wavelets

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Abstract - In this article a method for parameter estimation of Doppler radar signals is proposed. It is based on the discrete wavelet transform, spline orthogonal wavelets are used from the projection of the mixed signal; this operation acts as a lowpass filter and wavelet analysis play the role of passband filter banks with narrowing frequencies ranges. Finally an example is presented.

Key-Words: - Wavelets, multiresolution, surface wave radar, Doppler.

1 Introduction

The radar operation in the high frequency (HF) band (3 to 30 MHz, with wavelengths between 100 to 10 meters respectively) is used to the target detection beyond the horizon for the RF waves ([7]).

The HF surface wave radar (HFSWR), with ground-wave propagation over the sea is useful to protect maritime sovereignty (200 nautical mile of the exclusive economic zone); the longer range performance is achieved by using sky-wave propagation with "reflection" in some ionospheric layer.

HFSWR's using low frequencies of the HF band and vertically polarized waves, can remove the line-of-sight limitation of the conventional microwave radars because of the low attenuation when these surface-waves propagates over the highly conductive sea water.

In this article we propose a method to calculate the radial velocity v of the target, detected by the frequency shift (Doppler effect) between the delayed received signal s_r and the emitted signal s_e; this shift $\Delta\nu$ is the beat note obtained when the receiver mixes s_r with s_e and passes the output through a low-pass filter during the continuous wave (CW) emission mode. We considered, specially, small shifts produced by low velocities (between $0 - 10$ m/s) of a ship.

The proposed method -explained in the next section- use wavelet transform in a multiresolution framework ([1],[2]). Figure 1 show the orthogonal *Lemarié-Battle* cubic spline (OCS) wavelet ψ ([8]); the scaling function ϕ is depicted in Fig. 2.

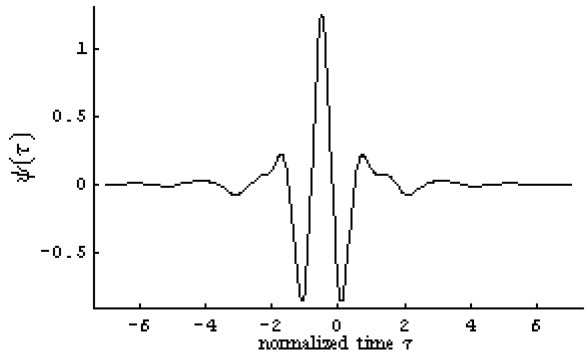

Fig. 1: Orthogonal cubic spline (OCS) wavelet.

We use the following notation:

t	:	is the time.
Δt	:	the sampling rate interval.
$\tau = t/\Delta t$	:	the normalized time, with t and Δt in the same unit (e.g.: sec.).
ν	:	is the frequency.
$\nu_s = 1/\Delta t$	:	the sampling rate frequency.
$f = \nu/\nu_s$	:	the normalized frequency, with ν and ν_s in the same unit (e.g.: Hz).

2 Method Description

Assuming that the radar is rest at the origin in the space, and the transmitting antenna converts the signal emitted $s_e(t)$ to a full electromagnetic wave, this wave $u(X,t)$ satisfy the equation

$$\frac{1}{c^2}u_{tt}(X,t) - \nabla^2 u(X,t) = s_e(t)\delta(X) \qquad (1)$$

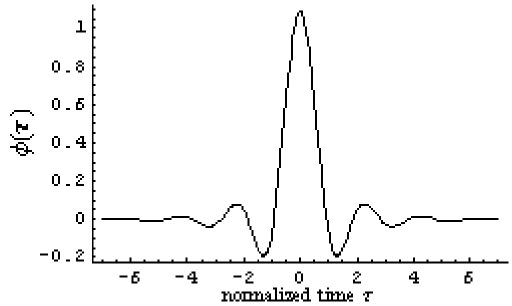

Fig. 2: OCS–scaling function ϕ.

where $X = (x, y, z)$ is the position vector, t is the time, δ is the Dirac-delta, c is the constant propagation speed of the light, and $\nabla^2 u = u_{xx} + u_{yy} + u_{zz}$.

The electromagnetic wave is reflected from a target, and the receiving antenna converts the echo to the receiving signal s_r.

If both antennas are at the origin space and the target (essentially a point) moves directly toward or away from the radar at a constant velocity v, the distance (range) varies with time according to

$$r(t) = r_0 + vt \tag{2}$$

where $r = (x^2 + y^2 + z^2)^{1/2}$, then it can be demonstrated ([3],[4]) that the received signal is

$$s_r(t) = A(r)\, s_e(\sigma t - t_0) \tag{3}$$

where $A(r) \cong A_0/r_0^2$ is the attenuation factor with A_0 constant. The time scaling factor

$$\sigma = \frac{c - v}{c + v} \tag{4}$$

represent the Doppler effect where $\sigma > 0$, and

$$t_0 = \frac{2\, r_0}{c + v}. \tag{5}$$

Under these conditions, the objective consist on estimating r_0 and v; for details, the cited bibliography has been proposed.

In this work we will expose the estimate of the velocity, using the wavelet transform. The analogical procedure to obtained the mentioned beat note in the receiver can be represented by the lowpass filter of the signal

$$s_B(t) = s_e\, s_r(t). \tag{6}$$

This procedure is particularly suitable in case that the emitted signal is sinusoidal; if

$$s_e(t) = \sin(2\pi\nu t) \tag{7}$$

where ν is the emission frequency, then

$$\begin{aligned} s_B(t) = \tfrac{A_0}{2r_0^2}[&\cos(2\pi\nu(1-\sigma)t + 2\pi\nu t_0) - \\ &- \cos(2\pi\nu(1+\sigma)t - 2\pi\nu t_0)] \end{aligned} \tag{8}$$

where $s_B(t)$ is the sum of two monochrome waves associated to the frequencies

$$\nu_0 = \nu\ |1 - \sigma| \ \text{ and } \ \nu_1 = \nu\ (1 + \sigma). \tag{9}$$

Since $\sigma \cong 1$, see Eq. (4) with $|v| << c$, these frequencies are very different. The filtering operations allows to separate the contribution of the high frequency, to specify the low frequency and to estimate ν_0. Using (4), result

$$v = c\ \frac{1 - \sigma}{1 + \sigma} \cong c\ (1 - \sigma)/2. \tag{10}$$

Therewith and ν_0 as per (9), we obtained

$$|v| \cong \frac{c\ \nu_0}{2\ \nu}. \tag{11}$$

Knowing ν_0 and assuming $c = 3\ 10^8$ in m/sec, with (11) we can estimate $|v|$ with relative error less than $3.33\ 10^{-7}$ for velocities less than 100 m/sec.

So, the precision of the obtained $|v|$ value will depend on the frequency approach; in this sense, the convenience of detecting ν_0 using an appropriate passband filter, with adjustable range, it suggests the employment of wavelets. The cutoff frequencies can be adjusted in relation to the sampling frequency of the signal.

The orthogonal spline wavelets, in a multiresolution analysis context, allows us to approach ν_0 in a range or band, associated to certain scale or resolution level ([5],[9]); finally, the approach is obtained using appropriate Fourier matrices ([6]).

3 Implementation

We will suppose that $c = 3\ 10^8$ is the speed of the light in m/sec, and $\nu = 10^p$ is the emission frequency in Hertz with $p > 0$ integer. It can be assumed that

$$0 < v_{min} \leq |v| \leq v_{max}. \tag{12}$$

Then, using (9) and (10),

$$\underbrace{10^p\ (\frac{2\ v_{min}}{c + v_{min}})}_{\alpha} \leq \nu_0 \leq \underbrace{10^p\ (\frac{2\ v_{max}}{c + v_{max}})}_{\beta}. \tag{13}$$

In this manner we have an estimate of the frequency ν_0; now, we will consider the integers $j_0,\ j_1$

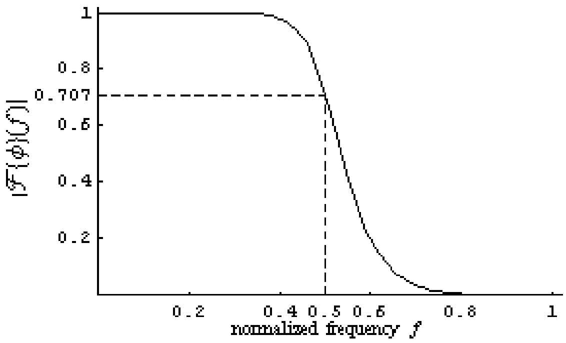

Fig. 3: OCS–lowpass filter with scaling function.

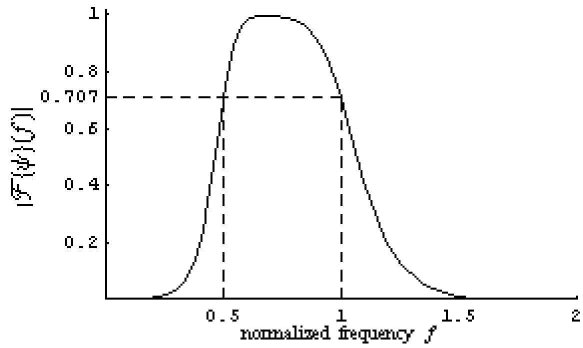

Fig. 4: OCS-passband filter with wavelet function.

such that $2^{j_1-2} \leq \alpha < \beta \leq 2^{j_0-2}$, where $\nu_s = 2^{j_0}$ is the sampling frequency in Hz. That is to say,

$$2^{j_1-2} \leq \nu_0 \leq 2^{j_0-2} \tag{14}$$

it is the localization band of ν_0.

Let's assume that, the function ϕ is the spline orthogonal scaling function of m integer odd order. As well it is known, the family

$$\phi_{j_0,k}(t) = 2^{j_0/2}\,\phi(2^{j_0}t-k) \quad k \in \mathcal{Z} \tag{15}$$

is an orthonormal basis of the spanned subspace V_{j_0}.

On the other hand, $|\hat{\phi}(\nu)|$ is a lowpass filter , almost ideal filter, in the range $|\nu| \leq 1/2$, with $|\hat{\phi}(\nu)| \cong 1$ if $|\nu| \leq 1/4$; Fig. 3 show that behavior for the scaling function depicted in Fig. 2. Then, we get $|\hat{\phi}(2^{-j_0}\nu)| \cong 1$ in the localization region of ν_0.

We project the signal $s_B(t)$ in this space V_{j_0}, that is to say

$$s_0(t) = \sum_k < s_B, \phi_{j_0,k} > \ \phi_{j_0,k}(t) \tag{16}$$

But, if $\phi(t)$ is centered in $t = 0$ as show Fig. 2, we can approach

$$< s_B(t), \phi_{j_0,k} > \cong s_B(2^{-j_0}k) \tag{17}$$

then the procedure consists on sampling the signal in the points $t_k = 2^{-j_0}k$.

From these $s_B(2^{-j_0k})$ values, we proceed with the wavelets analysis, using the Mallat algorithm ([5]), for $j = j_0-1, j_0-2, ...j_0-N$.

Clearly, we should take $0 \leq k \leq 2^N$, that is to say, an associated emission time $T_e = 2^{N-j_0}$ in seconds.

In each step, for $j < j_0$, the mentioned algorithm calculates 2^{N-j_0+j} wavelets coefficients:

$$c_{j,k} =< s_B, \psi_{j,k} > \tag{18}$$

where, ψ is the orthogonal spline wavelet of m order and

$$\psi_{j,k}(t) = 2^{j/2}\,\psi(2^jt-k). \tag{19}$$

These wavelets acts as passband filters localized in the $2^{j-1} \leq |\nu| \leq 2^j$ range. Figure 4 show the module of the frequency response corresponding to the orthogonal cubic spline wavelet.

On the other hand, we can obtain the coefficients until the level $j_{min} = j_0 - N$.

With this procedure, detecting the top energy $E_j = \sum_k |c_{j,k}|^2$, some $2^{j_\nu-1} \leq \nu_0 \leq 2^{j_\nu}$ level is selected, and the information is summarized in $2^{N-j_0+j_\nu}$ coefficients. The rest of the signal is filtered.

Then, we have $2^{N-j_0+j_\nu}$ coefficients in a range of wide $2^{j_\nu-1}$.

Since the signal is monochrome, it is possible to detect ν_0, using the information of the coefficients $c_{j,k}$.

Indeed, it can be demonstrated that using appropriate orthogonal Fourier matrices, of dimension $2^{N-j_0+j_\nu}$, is possible to specify $2^{N-j_0+j_\nu-1}+1$ frequencies in the range $[2^{j_\nu-1}, 2^{j_\nu}]$, with phase distinction ([6]). In other words, we can obtain ν_0 with

$$\Delta\nu_0 = 2^{-N+j_0-1} \tag{20}$$

precision.

Summarizing, the accuracy of the estimate will depend on the ν emission frequency, that defines j_0, and of emission time, that determines the number of data N.

Let's suppose that $0 < v_{min} \leq |v| \leq v_{max}$ and we want to estimate the velocity in m/sec with

$$\Delta v = 10^{-q}, \ q \geq 0. \tag{21}$$

Thus, with $|v|$ as from (11),

$$\Delta\nu_0 \leq 10^p\,\frac{2\Delta v}{c} = \frac{2}{3}10^{p-q-8}. \tag{22}$$

Next, it can be deduced j_0, and according with (20) and (22), we have that

$$\Delta\nu_0 = 2^{j_0-N-1} \leq \frac{2}{3}10^{p-q-8} \qquad (23)$$

Then,

$$N \geq j_0 - 1 - \log_2(\frac{2}{3}10^{p-q-8}). \qquad (24)$$

Therefore, the sample size should be $2^N + 1$, sampled in

$$t_k = 2^{-j_0}k,\ 0 \leq k \leq 2^{N-j_0}. \qquad (25)$$

Let's illustrate.

Example Let's suppose that $v = 3.3$, $v_{min} = 1$ and $v_{max} = 5$ in m/sec. The frequency emission in Hz is $\nu = 10^7$ and we want to estimate v with $\Delta v = 0.1$ in m/sec.

Using (13) and (14) we have that

$$10^7\,(\frac{2}{3\times 10^8+1}) \leq \nu_0 \leq 10^7\,(\frac{10}{3\times 10^8+5});$$

this is, practically

$$2^{j_1-2} \leq \frac{2}{3}10^{-1} \leq \nu_0 \leq \frac{1}{3} \leq 2^{j_0-2},$$

and we deduce $j_0 = 1$, $j_1 = -2$.

The frequency localization is $[0.0625,\ 0.5]$. Let's observe that the correct value is $\nu_0 \cong 0.22$, corresponding at $j_\nu = -2$ level.

Then, the maximum will be detected. Since we want precision $\Delta v = 0.1$, we have $N \geq 9$; that is to say, $2^9 + 1$ data, sampled in $t_k = 2^{-2}k,\ 0 \leq k \leq 2^9$. That is to say, the emission time is 2^7 sec.

4 Conclusion

The algorithm used is based on multiresolution analysis, i.e. a digital filter bank with decimation. The frequency bands is narrowed, and, as consequence, noises are eliminated when their frequencies components are out of the band used for the estimation of Doppler shift.

Thus, as an important product of the procedure introduces, noise suppression is obtained.

The filter used are almost ideal, given the wavelet chosen. The complexity of the algorithm is $n\log(n)$, and the online implementation is possible.

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