Parameter Estimation of Induction Motor Based on Continuous Time Model

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Abstract: - The paper presents an on-line recursive algorithm for parameter estimation of the squirrel-cage induction motor. The algorithm uses the continuous parametric model of the induction motor, with certain advantages in the estimation precision and parameters evaluation. The method is based on a technique that uses the Poisson moment functional theory. This technique allows an easy connection between the estimated parameters and the parameters of the equivalent phase circuit. The experimental results obtained through numerical simulation prove the effectiveness of the proposed on-line estimation algorithm.

Key-Words: - Identification, estimation, induction motor, vector control, continuous model.

1 Introduction

The induction motor is preferred in many industrial applications because is robust and because its low price. Recently, it is used even in motion control, where the performances are more severe. For the user, it is absolutely necessary to know precisely the electrical, magnetical and mechanical parameters in order to ensure an optimal tuning of the regulators disregarding the control method. Usually the producer does not mention all the parameters in the data sheets. That means that a designer of an electrical drive control system have to approximately determine the unknown parameters with the nominal values of the motor [1] or to organize specific identification experiments [2].

Several identification methods can be used. The most common experiment is no load and locked rotor test. The precision of the estimated parameters is rather low and the experiment is time consuming. Moreover, the parameters determined this way characterise only the operation of the motors feed by the sinusoidal waveform mains. Induction motors feed by static converters have different operating conditions due to the broad spectrum of the command signal. Because a part of the electrical and magnetical parameters depend on the frequency, the spectrum of the signal used in identification must be close as possible to the one of the real one.

It is known that the estimation techniques based on nonparametric models are less precise than the methods based on parametric models [3]. Although they imply sometimes more complex experimental devices, the latter avoid the conversion errors, which appear in a nonparametric-parametric change of representation. In digital control, where a zero-order hols is used, a discrete parametric model is generally preferred. Thus, usually the parameter estimation methods use the discrete models of processes, ensuring several facilities: both data measuring and processing have discrete nature, discrete models are simpler than the continuous ones, and it is simpler to use them for simulation, control, and prediction.

The main disadvantage is that the obtained parameters have a synthetic character and their physical interpretation is not simple for models with the order greater than two.

On the other hand, the identification methods based on continuous models offer the following advantages: an easier use of the model obtained by the supervision level in adaptive systems, the change of the sample rate of control system, which is usually different from the sampling rate used for identification, an easy physical interpretation and evaluation of process parameters (time constants, eigenvalues, damping coefficients), the possibility to embed a priori knowledge about partially known process in terms of poles, zeroes or physical quantities like mass, stiffness, resistances or capacities.

In the controllers design, estimators use the continuous model parameters and not the discrete ones. Thus the improvement of the parameters precision in the case of high performance drives can be achieved with a direct estimation technique.

2 The Model of the Souirrel Cage Induction Motor

In order to estimate the electrical and magnetic parameters of the induction motor it is useful to highlight a linear form in the parameters by using only measurable quantities:

$$y(t) = \varphi^T(t)\theta \tag{1}$$

The analysis starts with the well-known electromagnetic space-vector model of the induction motor in a general reference frame, given in (2).

$$\begin{cases} \underline{u}_{sg} = R_s \underline{i}_{sg} + \dfrac{d}{dt}\underline{\Psi}_{sg} + j\omega_g \underline{\Psi}_{sg} \\ 0 = R_r \underline{i}_{rg} + \dfrac{d}{dt}\underline{\Psi}_{rg} + j(\omega_g - \omega_r)\underline{\Psi}_{rg} \\ \underline{\Psi}_{sg} = L_s \underline{i}_{sg} + L_m \underline{i}_{rg} \\ \underline{\Psi}_{rg} = L_m \underline{i}_{sg} + L_r \underline{i}_{rg} \end{cases} \tag{2}$$

Power invariant transformation is applied. Simbols u, i, ψ denote voltage, current, and flux linkage, respectively, electrical angular speeds are ω_r for rotor and ω_g for general reference frame. Indices s and r stand for stator and rotor, and index m denotes parameters and variables associated with magnetizing flux.

The model becomes linear if the angular speeds are considered constant and become constant parameters. Applying the Laplace transform in (2) and substituting $\underline{i}_{rg}(s)$ and $\underline{\psi}_{rg}(s)$, it yields:

$$\underline{i}_{sg}(s) = \frac{T_r(s + j(\omega_g - \omega_r)) + 1}{L_s[\sigma T_r(s + j(\omega_g - \omega_r)) + 1]}\underline{\psi}_{sg}(s) \tag{3}$$

where $T_r = \dfrac{L_r}{R_r}$.

The operational inductance is obtained as:

$$L(s) = \frac{\underline{\psi}_{sg}(s)}{\underline{i}_{sg}(s)} = L_s \frac{\sigma T_r(s + j(\omega_g - \omega_r)) + 1}{T_r(s + j(\omega_g - \omega_r)) + 1} \tag{4}$$

With this new parameter the circuit impedance will be:

$$Z(s) = \frac{\underline{u}_{sg}(s)}{\underline{i}_{sg}(s)} = R_s + sL_s \frac{\sigma T_r(s + j(\omega_g - \omega_r)) + 1}{T_r(s + j(\omega_g - \omega_r)) + 1} \tag{5}$$

The transfer function (admittance) of the stator circuit becomes:

$$H(s) = \frac{1}{R_s + sL_s \dfrac{\sigma T_r(s + j(\omega_g - \omega_r)) + 1}{T_r(s + j(\omega_g - \omega_r)) + 1}} \tag{6}$$

or

$$H(s) = \frac{\underline{i}_{sg}(s)}{\underline{u}_{sg}(s)} = \frac{\underline{b}_1 s + \underline{b}_0}{s^2 + \underline{a}_1 s + \underline{a}_0} \tag{7}$$

The coefficients of the linear model (7) have the following physical interpretation:

$$\begin{aligned} &\underline{b}_1 = \frac{1}{\sigma L_s} = b_{1r}; \\ &\underline{b}_0 = \frac{1}{\sigma L_s T_r} + j\frac{1}{\sigma L_s}(\omega_g - \omega_r) = b_{0r} + jb_{0i}(\omega_g - \omega_r) \\ &\underline{a}_1 = \frac{1}{\sigma T_r} + \frac{1}{\sigma T_s} + j(2\omega_g - \omega_r) = a_{1r} + j(2\omega_g - \omega_r) \\ &\underline{a}_0 = \frac{1}{\sigma T_s T_r} - \omega_g(\omega_g - \omega_r) + j\left(\omega_g\left(\frac{1}{\sigma T_s} + \frac{1}{\sigma T_r}\right) - \omega_r \frac{1}{\sigma T_s}\right) \\ &\quad = a_{0r} - \omega_g(\omega_g - \omega_r) + j(\omega_g a_{0i1} - \omega_r a_{0i2}) \end{aligned} \tag{8}$$

where:

$$\begin{aligned} &b_{1r} = \frac{1}{\sigma L_s};\; b_{0r} = \frac{1}{\sigma L_s T_r};\; b_{0i} = \frac{1}{\sigma L_s}; \\ &a_{1r} = \frac{1}{\sigma T_r} + \frac{1}{\sigma T_s};\; a_{0r} = \frac{1}{\sigma T_s T_r};\; a_{0i} = \frac{1}{\sigma T_s} \end{aligned} \tag{9}$$

and $T_s = \dfrac{L_s}{R_s}$, $\sigma = 1 - \dfrac{L_m^2}{L_s L_r}$

For the particular case, when $\omega_g = 0$ (stator reference frame), the complex coefficients of the vector-space differential equation are:

$$\begin{aligned} &\underline{b}_1 = \frac{1}{\sigma L_s} = b_{1r};\; \underline{b}_0 = \frac{1}{\sigma L_s T_r} - j\frac{1}{\sigma L_s}\omega_r = b_{0r} - jb_{0i}\omega_r \\ &\underline{a}_1 = \frac{1}{\sigma T_r} + \frac{1}{\sigma T_s} - j\omega_r = a_{1r} - j\omega_r \\ &\underline{a}_0 = \frac{1}{\sigma T_s T_r} - j\omega_r \frac{1}{\sigma T_s} = a_{0r} - ja_{0i}\omega_r \end{aligned} \tag{10}$$

With the new parameters, the differential equation system becomes:

$$\begin{cases} \dfrac{d^2 i_{sd}}{dt^2} + a_{1r}\dfrac{di_{sd}}{dt} + \omega_r \dfrac{di_{sq}}{dt} + a_{0r} i_{sd} + a_{0i}\omega_r i_{sq} = \\ \qquad = b_{1r}\dfrac{du_{sd}}{dt} + b_{0r}u_{sd} + \omega_r b_{0i} u_{sq} \\ \dfrac{d^2 i_{sq}}{dt^2} + a_{1r}\dfrac{di_{sq}}{dt} - \omega_r \dfrac{di_{sd}}{dt} + a_{0r} i_{sq} - a_{0i}\omega_r i_{sd} = \\ \qquad = b_{1r}\dfrac{du_{sq}}{dt} + b_{0r}u_{sq} - \omega_r b_{0i} u_{sd} \end{cases} \tag{11}$$

and taking into account that $b_{0i} = b_{1r}$, this system can be arranged in the matrix form:

$$\begin{bmatrix} \dfrac{d^2 i_{sd}}{dt^2} + \omega_r \dfrac{di_{sq}}{dt} \\ \dfrac{d^2 i_{sq}}{dt^2} - \omega_r \dfrac{di_{sd}}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{di_{sd}}{dt} & -i_{sd} & -\omega_r i_{sq} & \dfrac{du_{sd}}{dt} + \omega_r u_{sq} & u_{sd} \\ -\dfrac{di_{sq}}{dt} & -i_{sq} & \omega_r i_{sd} & \dfrac{du_{sq}}{dt} - \omega_r u_{sd} & u_{sq} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} \tag{12}$$

with

$$\begin{aligned} &\theta_s^T = [\theta_{s1} \;\; \theta_{s2} \;\; \theta_{s3} \;\; \theta_{s4} \;\; \theta_{s5}]^T = \\ &= \left[\left\{\frac{1}{\sigma T_r} + \frac{1}{\sigma T_s}\right\} \;\; \frac{1}{\sigma T_s T_r} \;\; \frac{1}{\sigma T_s} \;\; \frac{1}{\sigma L_s} \;\; \frac{1}{\sigma L_s T_r}\right]^T \end{aligned} \tag{13}$$

and $\theta_{s3} = \dfrac{\theta_{s2}\theta_{s4}}{\theta_{s5}}$

The link between the parameters of the linear model and the physical parameters of the induction motor is immediate.

$$\begin{cases} R_s = \dfrac{\theta_2}{\theta_5};\;\; L_s = \dfrac{\theta_1}{\theta_5} - \dfrac{\theta_2\theta_4}{\theta_5^2}; \\ T_r = \dfrac{\theta_4}{\theta_5};\; \sigma = \dfrac{1}{\theta_4 L_s}; T_s = \dfrac{L_s}{R_s} \end{cases} \tag{14}$$

3 The Pprinciple of Continuous Model Parameter Estimation

Let's take a linear dynamic system described by:

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{j=0}^{m} b_j \frac{d^j u(t)}{dt^j} \tag{15}$$

where a_i, b_j are constant or slowly variable coefficients and $a_0 = 1$.

Because this system is linear in parameters, all the estimation methods for discrete models can be used. But unlike ARMA models used for discrete systems, the model (15) is not only a combination of input and output samples. It contains also pure time derivatives of these signals. A direct measure of the pure derivatives signal can be done only if the noise level is very low, usually under 5%. The solution is to perform linear dynamical operations (LDO) in both terms of eq. (15) and changing the initial model into an estimation model which verifies a differential equation identical with the original one but do not contain pure derivatives of the input-output signals [4],[5]. The Poisson moment functional method can be interpreted as a technical application of the modulating function. Let consider a chain of k continuous identical filters, each of them in the form:

$$G_1(s) = \frac{1}{s+\lambda} \tag{16}$$

with the equivalent transfer function

$$G_k(s) = \frac{1}{(s+\lambda)^k} \tag{17}$$

and the weight function

$$g_k(t) = L^{-1}\{G_k(s)\} = \frac{t^{k-1}}{(k-1)!}e^{-\lambda t} \tag{18}$$

The convolution theorem allows to define the LDO of k degree the operation performed by a chain of k+1 continuous filters on a signal denoted "$\circ$" and applied at the input of the filters chain. This is known as Poisson moment functional (PMF) of k degree.

$$M_k\{\circ\} \overset{\text{def}}{=} \int_0^{t_0} \frac{(t_0-t)^k}{k!} e^{-\lambda(t_0-t)} \circ \, dt \tag{19}$$

On the basis of this definition, a signal filtered by a single filter is:

$$y_0^0(t) = M_0\{y(t)\} = \int_0^{t_0} e^{-\lambda(t_0-t)} y(t)dt \tag{20}$$

If the signal is filtered by k+1 filters

$$y_k^0(t) = M_k\{y(t)\} = \int_0^{t_0} \frac{(t_0-t)^k}{k!} e^{-\lambda(t_0-t)} y(t)dt \tag{21}$$

In other words, one can say that if the input signal of the k+1 filters is the n-th order derivative of the signal y(t) on obtains:

$$y_k^n(t) = M_k\left\{\frac{d^n y(t)}{dt^n}\right\} = \int_0^{t_0} \frac{(t_0-t)^k}{k!} e^{-\lambda(t_0-t)} \frac{d^n y(t)}{dt^n} dt \tag{22}$$

An equivalent model of (15) can be obtained applying the k-order PMF, where k must be choose according to the system's order.

The equivalent system

$$\sum_{i=0}^{n} a_i M_k\left\{\frac{d^i y(t)}{dt^i}\right\} = \sum_{j=0}^{m} b_j M_k\left\{\frac{d^j u(t)}{dt^j}\right\} \tag{23}$$

or

$$\sum_{i=0}^{n} a_i y_k^i(t) = \sum_{j=0}^{m} b_j u_k^j(t) \tag{24}$$

can be arranged in a matrix form as in (25).

$$\begin{aligned} & y_k^0(t) = \varphi^{*T}(t)\theta \\ & \varphi^{*T}(t) = \left[-y_k^1(t) - y_k^2(t) \,..\, - y_k^n(t)\; u_k^0(t)\; u_k^1(t) \,..\, u_k^m(t)\right] \\ & \theta^T = [a_1\; a_2\; ....\; a_n\; b_0\; b_1\;\; b_m] \end{aligned} \tag{25}$$

Using a standard procedure for the off-line least squares algorithm, the vector of the estimated parameters is

$$\hat{\theta} = \left[\sum_{t=1}^{N} \varphi^*(t)\varphi^{*T}(t)\right]^{-1} \left[\sum_{t=1}^{N} \varphi^*(t) y_n^0(t)\right] \tag{26}$$

As in the case of discrete model parameter estimation, if the vector $\varphi^{*T}(t)$ is known or can be built, the vector of parameters θ is obtained following this procedure. The main problem is to determine $\varphi^{*T}(t)$ at any moment from available signal in a recursive manner.

$$\begin{cases} y_k^p = f\left(y_0^0(t), y_1^0(t), \;.....y_{k-1}^0(t), y_k^0(t)\right); p = \overline{1,n} \\ u_k^q = f\left(u_0^0(t), u_1^0(t), \;.....u_{k-1}^0(t), u_k^0(t)\right); q = \overline{1,m} \end{cases} \tag{27}$$

Integrating eq. (22) for n=1 one obtains:

$$\begin{aligned} y_k^1(t) = M_k\left\{\frac{dy(t)}{dt}\right\} = \int_0^{t_0} \frac{(t_0-t)^{k-1}}{(k-1)!} e^{-\lambda(t_0-t)} y(t)dt - \\ -\lambda \int_0^{t_0} \frac{(t_0-t)^k}{k!} e^{-\lambda(t_0-t)} y(t)dt - \frac{t_0^k}{k!} e^{-\lambda t_0} y(0) \end{aligned} \tag{28}$$

or

$$y_k^1(t) = y_{k-1}^0(t) - \lambda y_k^0(t) - g_{k+1}(t_0) y(0) \tag{29}$$

For n=2, yields:

$$y_k^2(t) = M_k\left\{\frac{d^2y(t)}{dt^2}\right\} = \int_0^{t_0}\frac{(t_0-t)^{k-2}}{(k-2)!}e^{-\lambda(t_0-t)}y(t)dt - 2\lambda\int_0^{t_0}\frac{(t_0-t)^{k-1}}{(k-1)!}e^{-\lambda(t_0-t)}y(t)dt + \lambda^2\int_0^{t_0}\frac{(t_0-t)^{k}}{k!}e^{-\lambda(t_0-t)}y(t)dt - -\left(\frac{t_0^{k-1}}{(k-1)!}e^{-\lambda t_0} - \lambda\frac{t_0^{k}}{k!}e^{-\lambda t_0}\right)y(0) - \frac{t_0^{k}}{k!}e^{-\lambda t_0}\frac{dy(0)}{dt} \tag{30}$$

or

$$y_k^2(t) = y_{k-2}^0(t) - 2\lambda y_{k-1}^0(t) + \lambda^2 y_k^0(t) - -\left(g_k(t_0) - \lambda g_{k+1}(t_0)\right)y(0) - g_{k+1}(t_0)\frac{dy(0)}{dt} \tag{31}$$

The last terms in (29) and (31) take into account the combined effects of initial conditions and they were ignored in the experiment because the filters are stable and causal.

In matrix form, for $p=\overline{1,n}$ on obtains:

$$\begin{bmatrix} y_n^0(t)\\ y_n^1(t)\\ y_n^2(t)\\ ..\\ y_n^n(t)\end{bmatrix} = \begin{bmatrix} 0 & 0 & .. & 0 & 1\\ 0 & 0 & .. & 1 & *\\ 0 & 0 & .. & * & *\\ .. & .. & .. & .. & ..\\ 1 & * & .. & * & *\end{bmatrix}\begin{bmatrix} y_0^0(t)\\ y_1^0(t)\\ y_2^0(t)\\ ...\\ y_n^0(t)\end{bmatrix} \tag{32}$$

where term $*_{ij}$ is expressed as:

$$*_{ij} = (-1)^{n+j-i}C_{i-1}^{(i+j-n-2)}\lambda^{(i+j-n-2)} \tag{33}$$

The conclusion is that an n-order equivalent estimation model can be obtained if is used PMF with the same order of the initial differential equation of the system.

Assuming that the signals $y_r^0(t)_{r=\overline{1,n}}$ are known, processing them as in (32), one can obtain the terms from (25). Similarly, for signals $u_r^0(t)_{r=\overline{1,n}}$, the terms of (25) can be obtained from (34):

$$\begin{bmatrix} u_n^0(t)\\ u_n^1(t)\\ ...\\ u_n^m(t)\\ ...\\ ...\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & ... & 0 & 1\\ 0 & 0 & 0 & ... & 1 & *\\ 0 & 0 & 0 & .. & * & *\\ 0 & 0 & & & * & *\\ ... & ... & ... & ... & ... & ...\\ ... & ... & ... & ... & ... & ...\end{bmatrix}\begin{bmatrix} u_0^0(t)\\ u_1^0(t)\\ ...\\ u_m^0(t)\\ ...\\ u_n^0(t)\end{bmatrix} \tag{34}$$

In the case of analogue filtering, the state equation of the filters chain for signal y(t) is:

$$\begin{bmatrix} \frac{d}{dt}y_0^0(t)\\ \frac{d}{dt}y_1^0(t)\\ \frac{d}{dt}y_2^0(t)\\ ...\\ \frac{d}{dt}y_n^0(t)\end{bmatrix} = \begin{bmatrix} -\lambda & 0 & .. & 0 & 0\\ 1 & -\lambda & .. & 0 & 0\\ 0 & 1 & .. & 0 & 0\\ .. & .. & .. & .. & ..\\ 0 & 0 & .. & 1 & -\lambda\end{bmatrix}\begin{bmatrix} y_0^0(t)\\ y_1^0(t)\\ y_2^0(t)\\ ...\\ y_n^0(t)\end{bmatrix} + \begin{bmatrix} 1\\ 0\\ 0\\ ...\\ 0\end{bmatrix}y(t) \tag{35}$$

The gain of the filters used is 1/λ<1. The attenuation can be high on the entire chain of filters. To avoid this high attenuation one can be used filters in the form:

$$G_k(s) = \frac{\lambda^k}{(s+\lambda)^k} \tag{36}$$

Eq. (32) becomes in this case:

$$\begin{bmatrix} y_n^0(t)\\ y_n^1(t)\\ y_n^2(t)\\ ..\\ y_n^n(t)\end{bmatrix} = \begin{bmatrix} 0 & 0 & .. & 0 & *\\ 0 & 0 & .. & * & *\\ 0 & 0 & .. & * & *\\ .. & .. & .. & .. & ..\\ * & * & .. & * & *\end{bmatrix}\begin{bmatrix} y_0^0(t)\\ y_1^0(t)\\ y_2^0(t)\\ ...\\ y_n^0(t)\end{bmatrix} \tag{37}$$

where the term $*_{ij}$ has the value

$$*_{ij} = (-1)^{n+j-i}C_{i-1}^{(i+j-n-2)}\lambda^{(i+j-n-1)} \tag{38}$$

Now, the state equation of the filters chain is:

$$\begin{bmatrix} \frac{d}{dt}y_0^0(t)\\ \frac{d}{dt}y_1^0(t)\\ \frac{d}{dt}y_2^0(t)\\ ...\\ \frac{d}{dt}y_n^0(t)\end{bmatrix} = \lambda\begin{bmatrix} -1 & 0 & .. & 0 & 0\\ 1 & -1 & .. & 0 & 0\\ 0 & 1 & .. & 0 & 0\\ .. & .. & .. & .. & ..\\ 0 & 0 & .. & 1 & -1\end{bmatrix}\begin{bmatrix} y_0^0(t)\\ y_1^0(t)\\ y_2^0(t)\\ ...\\ y_n^0(t)\end{bmatrix} + \lambda\begin{bmatrix} 1\\ 0\\ 0\\ ...\\ 0\end{bmatrix}y(t) \tag{39}$$

4 On-Line Estimation of Squirrel-Cage Induction Motor

The temporal model (12) is over parameterised and between parameters exists a nonlinear relation as in (13). Thus, the model (12) is restructured in the form:

$$\begin{bmatrix} \frac{d^2i_{sd}}{dt^2} + \omega_r\frac{di_{sq}}{dt} + \omega_r i_{sq}\frac{\theta_2\theta_3}{\theta_4}\\ \frac{d^2i_{sq}}{dt^2} - \omega_r\frac{di_{sd}}{dt} - \omega_r i_{sd}\frac{\theta_2\theta_3}{\theta_4}\end{bmatrix} = = \begin{bmatrix} -\frac{di_{sd}}{dt} & -i_{sd} & \frac{du_{sd}}{dt} + \omega_r u_{sq} & u_{sd}\\ -\frac{di_{sq}}{dt} & -i_{sq} & \frac{du_{sq}}{dt} - \omega_r u_{sd} & u_{sq}\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix} \tag{40}$$

where

$$\theta^T = \left[\left\{\frac{1}{\sigma T_r} + \frac{1}{\sigma T_s}\right\} \quad \frac{1}{\sigma T_s T_r} \quad \frac{1}{\sigma L_s} \quad \frac{1}{\sigma L_s T_r}\right] \tag{41}$$

The least squares algorithm from eq. (26) is used in a recursive form to estimate on-line the parameters in (40). On that purpose are used 4 chains of analog filters with 3 elementary filter cells each. The generic signals $\{y_0^0(t), y_1^0(t), y_2^0(t)\}$, with $y \in \{u_{sd}(t), u_{sq}(t), i_{sd}(t), i_{sq}(t)\}$, are obtained as the solutions of the system

$$\begin{bmatrix} \frac{d}{dt} y_0^0(t) \\ \frac{d}{dt} y_1^0(t) \\ \frac{d}{dt} y_2^0(t) \end{bmatrix} = \lambda \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_0^0(t) \\ y_1^0(t) \\ y_2^0(t) \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} y(t) \tag{42}$$

The particular form for (37) is

$$\begin{bmatrix} y_2^0(t) \\ y_2^1(t) \\ y_2^2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \lambda & -\lambda \\ \lambda^2 & -2\lambda^2 & \lambda^2 \end{bmatrix} \begin{bmatrix} y_0^0(t) \\ y_1^0(t) \\ y_2^0(t) \end{bmatrix} \tag{43}$$

With (42) and (43) are determined the values necessary in the estimation equation:

$$y^*(t) = \varphi^{*T}(t)\theta \tag{44}$$

where

$$y^*(t) = \begin{bmatrix} i_{sd2}^{\,2} + \omega_r i_{sd2}^{\,1} + \omega_r i_{sq2}^{\,0} \dfrac{\theta_2\theta_3}{\theta_4} \\ i_{sq2}^{\,2} + \omega_r i_{sq2}^{\,1} - \omega_r i_{sd2}^{\,0} \dfrac{\theta_2\theta_3}{\theta_4} \end{bmatrix} \tag{45}$$

and

$$\varphi^{*T}(t) = \begin{bmatrix} -i_{sd2}^{\,1} & -i_{sd2}^{\,0} & u_{sd2}^{\,1} + \omega_r u_{sq2}^{\,0} & u_{sd2}^{\,0} \\ -i_{sq2}^{\,1} & -i_{sq2}^{\,0} & u_{sq2}^{\,1} - \omega_r u_{sd2}^{\,0} & u_{sq2}^{\,0} \end{bmatrix} \tag{46}$$

The recursive algorithm has the following steps:

1. Acquisition of measured $i_{sa}, i_{sb}, u_{sa}, u_{sb}, \omega_m$ from the induction motor, where ω_m is the mechanical angular speed;
2. Computing of the electric angular speed and of the d-q signals

$$y_d(t) = \sqrt{\frac{3}{2}} y_a(t);\; y_q = \frac{1}{\sqrt{2}}\left(y_a(t) + 2y_b(t)\right);\; \omega_r = p\omega_m$$

where p is the number of pole pairs.

3. Computing of the signal from the estimation equation with (42)-(43);
4. Computing of the vector $y^*(t)$ from (45) and of the regression vector (46);
5. Computing of the prediction error

$$\zeta(k) = y^*(k) - \varphi^{*T}(k)\theta(k-1)$$

6. Computing of the covariance matrix

$$P(k) = \left[I - P(k-1)\varphi^*(k)\left[\beta I + \varphi^{*T}(k)P(k-1)\varphi^*(k)\right]^{-1}\varphi^{*T}(k)\right]\frac{P(k-1)}{\beta}$$

7. Adjusting of the parameters vector

$$\theta(k) = \theta(k-1) + P(k)\varphi^*(k)\zeta(k)$$

8. Wait onesampling period and jump at the step 1.

5 On-Line Algorithm Simulation and Results

The recursive least squares algorithm is attractive because is simple and ensures good performances in standard conditions. In this case this algorithm must be used with some precautions. The algorithm assumes that the noise is uncorrelated with the input and state variables. This hypothesis could be wrong in the case of a nonlinear model used in estimation. The convergence in a parameters space point is ensured if the input signal satisfies the request of persistent excitation of a specified order. Moreover, even if the motor is excited with an inverter with a simple modulation strategy (like hysteresis regulators), the condition of excitation persistence is not satisfied [6]. This is way for testing was implemented a control structure based on indirect rotor field vector control. A voltage-source inverter that uses an asynchronous natural modulation technique feeds the induction motor. The commutation frequency is 1 KHz. The simulated control structure is presented in Fig. 1

Initialisation of the algorithm supposes initial values for the parameters vector θ(0), covariance matrix P(0), weight factor β, and the pole of PMF λ. The parameters vector can be approximated through other off-line experiment for a pre-estimation [7], [8]. The covariance matrix must be chosen carefully because the system is nonlinear and inside the system a parametric reaction is present.

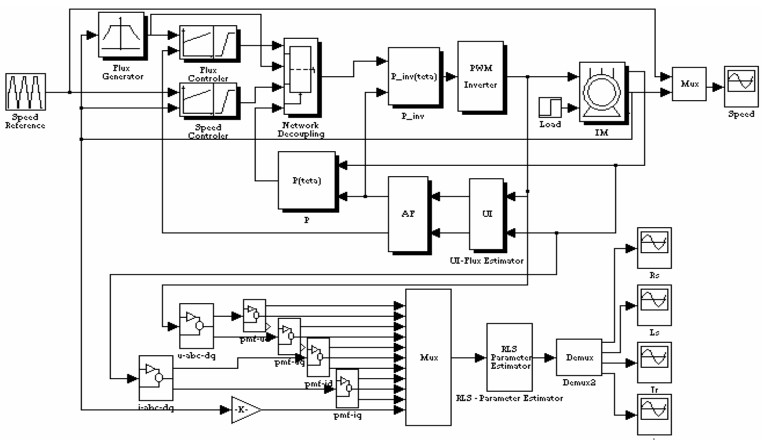

Fig. 1 The control structure simulated in MATLAB-SIMULINK

The sampling period is 500μs. The experiment presented in this paper was done with $\beta = 1$, $\lambda = 120$ and $\theta(0) = [180\ 720\ 24\ 160]^T$.

The motor parameters are:
$U_{nf} = 220V$, $I_n = 4.6A$, $f_n = 50Hz$,
$R_s = 4.498\Omega$, $L_s = 0.485H$, $L_m = 0.468H$,
$R_r = 3.338\Omega$, $L_r = 0.494H$, $J = 0.0665Kgm^2, p = 2$

The results of the numerical simulation are performed in MATLAB-SIMULINK. The simulation program calculates also the physical parameters according to eq. (14). In Fig. 2-6 are presented the speed profile used and the values of the estimated parameters.

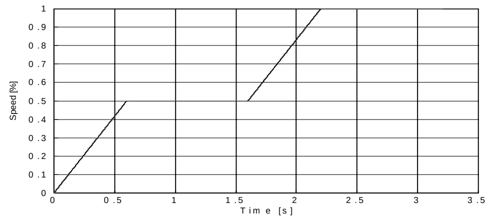

Fig. 2. The imposed/realised speed profile

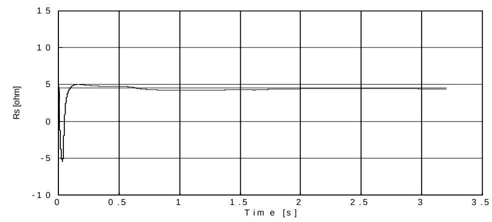

Fig. 3. The estimated/modelled values of R_s

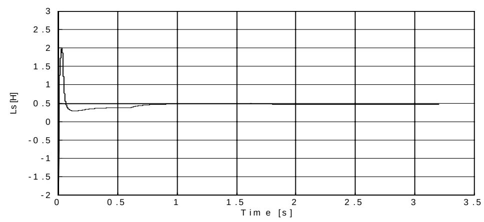

Fig. 4. The estimated/modelled values of L_s

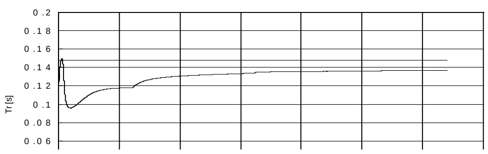

Fig. 5. The estimated/modelled values of T_r

Fig. 6. The estimated/modelled values of the leakage factor σ

6 Conclusion

The paper presents an on-line estimation algorithm based on continuous time model and applied to an induction motor. An originality element is the use of PMF technique in the estimation of induction motor parameters. Because the process model is nonlinear, but can be treated as a linear model in parameters in steady-state mechanical regime, some precautions must be taken into account when the recursive least squares algorithm is used. A supervision level might be useful to discern between the steady-state and transient mechanical regime. The model was built in the hypothesis of constant rotor speed. This assumption is not verified in the transient state and the estimated parameters might be affected.

The experimental results show a good match between estimated parameters and real motor parameters.

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