

Indirect Vector Control of Induction Motor

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Abstract: - The paper emphasis a solution for induction motor speed control. The paper presents a vector control model of induction machine with four parameters only. The main advantage of this minimum number of parameters model consists in the measurements possibilities of them. This parameters result univocally from the identification procedures. This minimum number of parameters model preserves the dynamic behavior of the motor. This model minimizes the uncertainty of the model parameters. The indirect vector control structure was reformulated according to this model, simulated with MATLAB, and implemented with a floating-point DSP processor. The experimental results show good transient and steady-state performances. The accuracy of the results is given by the precision of the motor model used.

Key-Words: - induction motor, vector control, parameter estimation, symmetrical T model, asymmetrical Γ .

1 Introduction

Squirrel-cage induction motor is a simple low cost and robust electrical machine. For the sake of its constructive simplicity this machine is maintenance free, and it is used in hostile environment without modifications.

The principle of torque generation in the induction machine is similar with the one of the d.c. machine. Unfortunately this similarity was not highlighted before '70. This is one of the reasons of the vector control techniques development after this period. The theory of the space vector allowed to simplify and to compress the expression of the electromagnetic torque. This is delivered as a vector product between the an active current and a reactive current in a specific coordinates system.

The absence of the autonomous inductor device and the lack of rotor currents measurements increase the complexity of the control structure. This control structure must allow creating in the machine an equivalent permanent magnetic through the control of rotor flux, and the decoupling between this flux and the electromagnetic torque. This is why the mathematical models used have to describe with the highest possible accuracy the operating of the machine in both steady-state and transient regime.

In the direct vector control schemes the flux position is measured or estimated on the basis of the flux components in the stator coordinates. In the indirect control schemes the flux position is determined indirectly through the rotor speed and the slip estimation. The absence of the rotor flux position

sensors and the ability to operate at low speeds has increased the popularity of the indirect control strategy.

Robustness of the solution adopted is very important, since some parameters can change as a function of time or may be unknown. Several modern non-linear control methods used in the control are reported in the literature as feedback linearization techniques or robust techniques based on variable structure systems.

An analysis of several control algorithms in the field showed a variety of computational and interfacing problems. Some authors reveal that the sample rates as low as hundred of μs are often required to ensure dynamic performances. Therefore a powerful processing unit is required.

Two interface classes can be considered: process interface and user interface. Process interface is the most requesting in terms of hardware resources, while the user interface is strongly software dependent. Most of the industrial solution for digital drives control are closed systems, and do not allow to the user to modify the control algorithm or to embed new supplementary functions. These systems are not useful for research. Papers reporting university research reveal a poor experimental activity or only simulation results are presented.

This paper focuses on an indirect vector control solution implemented on an experimental set-up build in the Technical University "Gh. Asachi", Jassy. The experimental results were obtained with a software package called DMCDs (Digital Motion

Control Development Shell) developed also in the Technical University "Gh. Asachi".

2 Indirect vector control rotor field oriented of induction machine

The analysis of vector control structures highlights several possibilities to control the instantaneous values of the electromagnetic torque of the induction machine. Because the equations that describes the machine becomes significantly less complicated when the reference orthogonal frame is synchronous and synphasic with the rotor flux, almost all practical schemes use this type of orientation [1]. The position of rotor flux can be estimated directly or indirectly. The first type means the use of transducers or flux estimators. The indirect estimation schemes of the rotor flux position ensure a good behaviour in all speed range, and they are the common solution in practice.

The induction machine equations in the flux coordinate frame can be written as follows:

$$\begin{cases} \underline{u}_s^e = R_s \underline{i}_s^e + \frac{d\underline{\psi}_s^e}{dt} + j\omega_e \underline{\psi}_s^e \\ 0 = R_r \underline{i}_r^e + \frac{d\underline{\psi}_r^e}{dt} + j(\omega_e - \omega_r) \underline{\psi}_r^e \\ \underline{\psi}_s^e = L_s \underline{i}_s^e + L_m \underline{i}_r^e \\ \underline{\psi}_r^e = L_m \underline{i}_s^e + L_r \underline{i}_r^e \\ J \frac{d\Omega}{dt} = pL_m (\underline{i}_r^e \times \underline{i}_s^e) - m_r - D\Omega \end{cases} \quad (1)$$

Symbols u , i , ψ denote voltage, current, and flux linkage, respectively, rotor speed is ω_r , leakage inductances are identified with index σ , index m denotes parameters and variables associated with magnetizing flux, index s denotes parameter and variables associated with the stator, and index r is for parameters and variables associated with the rotor. Indices e upperscript denotes the frame synchronous with the reference flux.

If the position of the rotor flux is known and the coordinate frame is symphasic with this one, then

$$\underline{\psi}_r^e = \psi_{rd}^e = \psi_r^e; \psi_{rq}^e = 0 \quad (2)$$

and (1) can be rewritten as follows.

$$\begin{cases} \frac{d\underline{\psi}_r^e}{dt} + \frac{1}{T_r} \underline{\psi}_r^e = \frac{L_m}{T_r} \underline{i}_{sd}^e; \omega_{sl} = \frac{R_r L_m}{L_r} \frac{\underline{i}_{sq}^e}{\underline{\psi}_r^e}; m_e = p \frac{L_m}{L_r} \underline{\psi}_r^e \underline{i}_{sq}^e \\ \theta_s = \int \omega_e dt = \int \left(\omega_r + \frac{L_m}{T_r} \frac{\underline{i}_{sq}^e}{\underline{\psi}_r^e} \right) dt = \int \left(p\Omega + \frac{L_m}{T_r} \frac{\underline{i}_{sq}^e}{\underline{\psi}_r^e} \right) dt \end{cases} \quad (3)$$

$$\begin{cases} \underline{u}_{sd}^e = \sigma L_s \frac{d\underline{i}_{sd}^e}{dt} + \left(R_s + R_r \frac{L_m^2}{L_r^2} \right) \underline{i}_{sd}^e - \omega_e \sigma L_s \underline{i}_{sq}^e - \frac{L_m}{L_r} R_r \underline{\psi}_r^e \\ \underline{u}_{sq}^e = \sigma L_s \frac{d\underline{i}_{sq}^e}{dt} + \left(R_s + R_r \frac{L_m^2}{L_r^2} \right) \underline{i}_{sq}^e + \omega_e \sigma L_s \underline{i}_{sd}^e + \omega_r \frac{L_m}{L_r} \underline{\psi}_r^e \end{cases} \quad (4)$$

On the basis of this equivalent model of vector controlled induction machine, the equations of the decoupling and performance regulators can be computed. Eq. (4) is used to design current regulators and the decoupling regulator of the command voltages. Taking into account only the

permanent magnetic state $\left(\frac{d\underline{\psi}_r^e}{dt} \cong 0 \right)$, eq. (4)

becomes:

$$\begin{cases} \underline{u}_{sd}^e = \sigma L_s \frac{d\underline{i}_{sd}^e}{dt} + R_s \underline{i}_{sd}^e - \omega_e \sigma L_s \underline{i}_{sq}^e \\ \underline{u}_{sq}^e = \sigma L_s \frac{d\underline{i}_{sq}^e}{dt} + R_s \underline{i}_{sq}^e + \omega_e L_s \underline{i}_{sd}^e \end{cases} \quad (5)$$

The control structure based on the inverse dynamic model of the induction machine assumes knowledge about five parameters of the machine: R_s , R_r , L_m , L_s și L_r

The parameters of the machine are usually established with the short-circuit and no load tests of the induction machine. The value of the stator and rotor leakage inductance is difficult to obtain because from this tests data is possible to determine only the sum of these parameters $L_{\Sigma\sigma}$, but not each one of them. The test standards recommend several empirical distributions of this parameter:

The equivalent T circuit is incompletely specified because the values of the stator and rotor leakage inductance have not unique values.

The modern parameter estimation methods allow to compute with high accuracy four parameters associated to the induction motor model: the parameters of the stator circuit (R_s , L_s), the time constant of the rotor T_r , and the global leakage coefficient σ [2],[3]. If the designer knows the value of the stator leakage inductance $L_{\sigma s}$, the other parameters can be computed as follows.

$$L_m = L_s - L_{\sigma s}; L_r = \frac{L_m^2}{(1-\sigma)L_s}; L_{\sigma r} = L_r - L_m; R_r = \frac{L_r}{T_r} \quad (6)$$

The sum of the leakage inductances of the machine is constant [4]. A good approximation of the stator leakage inductance $L_{\sigma s}$ is [5]:

$$L_{\sigma s} \approx \frac{1}{2} \sigma L_s \quad (7)$$

This approximation allows computing the rotor leakage inductance as:

$$L_{\sigma r} \approx \sigma L_s \frac{2 - \sigma}{4(1 - \sigma)} \quad (8)$$

The value of the ratio of stator leakage inductance and the rotor leakage inductance is defined as:

$$k_{\sigma} = \frac{L_{\sigma s}}{L_{\sigma r}} = \frac{2(1 - \sigma)}{2 - \sigma} < 1 \quad (9)$$

There are cases when this ratio can not characterize well enough the leakage in the stator and the rotor of the induction machine. A better approach seems to be the use of a minimum number of parameters which can define the transient electromagnetic state and can be detected without approximation.

3 Squirrel-cage induction motor compact model

The model of the squirrel-cage induction motor has a degree of freedom because rotor voltage is null ($\underline{u}_r=0$). This makes possible the obtain several models which are energetically equivalent with the classical symmetrical T model of the machine.

The induction machine can be assimilated with a transformer. Then a turns ratio can be explained in the same manner as the turns ratio used for a transformer. The classical equivalent circuit of the machine is obtained with the new state variables:

$$\underline{u}'_r = \frac{N_s}{N_r} \underline{u}_r; \quad \dot{i}'_r = \frac{N_r}{N_s} \dot{i}_r; \quad \underline{\psi}'_r = \frac{N_r}{N_s} \underline{\psi}_r \quad (10)$$

and parameters

$$R'_r = \left(\frac{N_s}{N_r}\right)^2 R_r; \quad L'_{\sigma r} = \left(\frac{N_s}{N_r}\right)^2 L_{\sigma r}; \quad L'_m = \frac{N_s}{N_r} L_m \quad (11)$$

This model is completely characterized by five parameters.

Because the induction motor is squirrel-cage, the turns ratio $\alpha = \frac{N_s}{N_r}$ can be chosen arbitrarily and any

other value that ensures the power invariance is allowed except 0 or ∞ . A more general model of the induction motor results from eq. (10)-(11), with the new variables defined in (12) and the parameters defined in (13).

$$\dot{i}'_{rg} = \frac{\dot{i}_{rg}}{\alpha}; \quad \underline{\psi}'_r = \frac{\underline{\psi}_r}{\alpha} \quad (12)$$

$$R'_r = \alpha^2 R_r; \quad L'_r = \alpha^2 L_r; \quad L'_m = \alpha L_m \quad (13)$$

The phasor equations (1) can be rewritten in a matricial form:

$$\begin{bmatrix} \underline{u}'_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + L_s(p+j\omega_e) & L_m(p+j\omega_e) \\ L_m(p+j\omega_e) & R_r \frac{(p+j\omega_e)}{(p+j(\omega_e - \alpha))} + L_r(p+j\omega_e) \end{bmatrix} \begin{bmatrix} \dot{i}'_s \\ \dot{i}'_r \end{bmatrix} \quad (14)$$

It is defined a matrix T

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\alpha} \end{bmatrix} \quad (15)$$

This allows writing the equivalent model of the motor (16).

$$\begin{bmatrix} \underline{u}'_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + L_s(p+j\omega_e) & \alpha L_m(p+j\omega_e) \\ \alpha L_m(p+j\omega_e) & \alpha^2 \left[R_r \frac{(p+j\omega_e)}{(p+j(\omega_e - \alpha))} + L_r(p+j\omega_e) \right] \end{bmatrix} \begin{bmatrix} \dot{i}'_s \\ \dot{i}'_r \end{bmatrix} \quad (16)$$

The structure of the equivalent model is presented in Fig. 1.

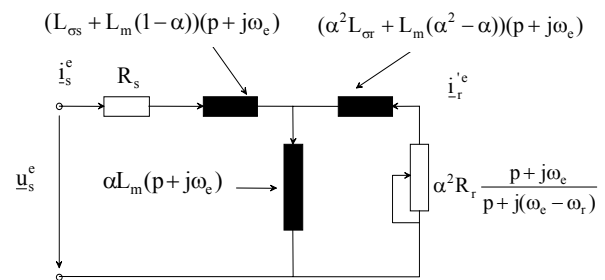


Fig. 1. The generalized phasor circuit of the squirrel-cage induction motor

The turns ratio matrix (15) does not modify the input impedance at the motor terminals. The equivalent circuit generates the same torque and speed for the same input signals. Some remarkable properties can be obtained with “asymmetric” equivalent circuits by choosing appropriate values for α .

A “resistive” circuit is obtained when α is chosen to vanish rotor leakage inductance in the phasor generalized circuit.

$$\alpha^2 L_r - \alpha L_m = 0 \quad (17)$$

For this particular case, the factor α yields:

$$\alpha = \frac{L_m}{L_r} \quad (18)$$

The new asymmetrical Γ model is completely specified by four parameters (19) and is presented in Fig.2.

$$R_{s4} = R_{s5}; \quad L_{s4} = L_{s5}; \quad L_{\sigma 4} = L_{\sigma 5} + \frac{L_{m5}}{L_{r5}} L_{\sigma 5}; \quad R_{r4} = \left(\frac{L_{m5}}{L_{r5}}\right)^2 R_{r5} \quad (19)$$

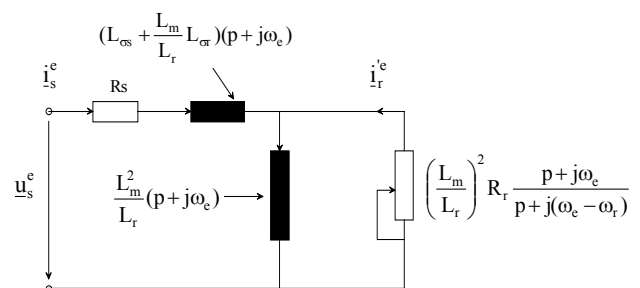


Fig.2. The equivalent asymmetric equivalent circuit.

The leakage inductance has the value

$$L_{m4} = L_{s4} - L_{\sigma s4} = \frac{L_{m5}}{L_{r5}} L_{m5} \quad (20)$$

and the global leakage coefficient is

$$\sigma \stackrel{\text{def}}{=} 1 - \frac{L_{m5}^2}{L_{s5} L_{r5}} = 1 - \frac{L_{m4}}{L_{s4}} \quad (21)$$

The rotor time constant of this new model is

$$T_{r4} = \frac{L_{m4}}{R_{r4}} = \frac{L_{r5}}{R_{r5}} = T_{r5} \quad (22)$$

This denotes a dynamic behavior equivalent with the one of the model with five parameters.

Finally the parameters of this model are univocal determined on the basis of the estimated parameters of the motor ($\hat{R}_s, \hat{L}_s, \hat{T}_r, \hat{\sigma}$).

$$R_{s4} = \hat{R}_s; L_{s4} = \hat{L}_s; L_{m4} = (1 - \hat{\sigma}) \hat{L}_s; L_{\sigma s4} = \hat{\sigma} \hat{L}_s; R_{r4} = \frac{(1 - \hat{\sigma}) \hat{L}_s}{\hat{T}_r} \quad (23)$$

4 Indirect vector control of induction motor

The minimum number of parameters model of induction motor is used for a vector control structure. On this purpose the decoupling regulators must be designed taking into account the new state variables and the new defined parameters.

With (12), (13) and (18), the phasor model from (1) becomes:

$$\begin{cases} \underline{u}_s^e = R_s \underline{i}_s^e + \frac{d\underline{\psi}_s^e}{dt} + j\omega_e \underline{\psi}_s^e \\ 0 = R_{r4} \underline{i}_{r4}^e + \frac{d\underline{\psi}_{r4}^e}{dt} + j(\omega_e - \omega_r) \underline{\psi}_{r4}^e \\ \underline{\psi}_s^e = L_s \underline{i}_s^e + L_{m4} \underline{i}_{r4}^e \\ \underline{\psi}_{r4}^e = L_{m4} (\underline{i}_s^e + \underline{i}_{r4}^e) \\ m_e = p L_{m4} (\underline{i}_{r4}^e \times \underline{i}_s^e) \end{cases} \quad (24)$$

In the reference frame synchronous with the rotor flux phasor, the phasor equations in cartesian coordinates are:

$$\begin{cases} \frac{d\underline{\psi}_{r4}}{dt} + \frac{1}{T_r} \underline{\psi}_{r4} = \frac{L_{m4}}{T_r} \underline{i}_{sd} \\ \omega_{sl} = \frac{L_{m4}}{T_r} \underline{\psi}_{r4} \\ m_e = p \underline{\psi}_{r4} \underline{i}_{sq} \end{cases} \quad (25)$$

$$\begin{cases} \underline{u}_{sd}^e = R_s \underline{i}_{sd}^e + L_{\sigma s4} \frac{d\underline{i}_{sd}^e}{dt} - \omega_e L_{\sigma s4} \underline{i}_{sq}^e \\ \underline{u}_{sq}^e = R_s \underline{i}_{sq}^e + L_{\sigma s4} \frac{d\underline{i}_{sq}^e}{dt} + \omega_e L_s \underline{i}_{sd}^e \end{cases} \quad (26)$$

This equivalent model is a nonlinear process with two inputs ($\underline{i}_{sd}^e, \underline{i}_{sq}^e$) and two coupled outputs ($\underline{\Psi}_r, m_e$). The decoupling regulator is obtained with the inverse model of the machine. This leads to a feedforward structure presented in Fig.4.

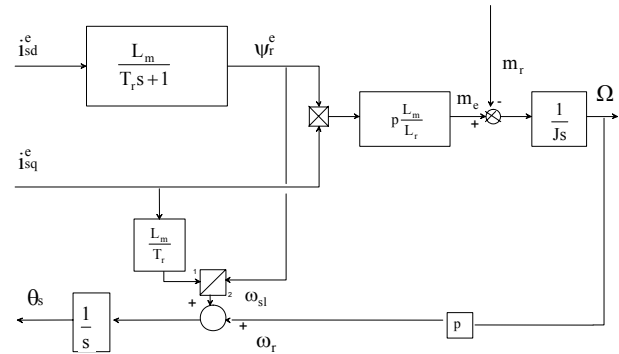


Fig.3. The induction motor model in the rotor flux frame

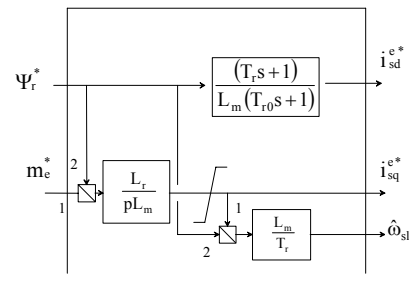


Fig. 4. Structure of the decoupling regulator

The transfer function of the reactive component is not a proper one, and, for practical implementations is replaced with

$$\frac{\underline{i}_{sd}^{e*}}{\underline{\psi}_r^*} = \frac{T_r s + 1}{L_m (T_{r0} s + 1)} \quad (27)$$

The active protection of the group inverter-motor is realised through the limits of the current component, $\pm i_{isq \max}$, which generates the torque. The decoupling of the stator currents allows a cascade control structure like the one used for DC motor speed control. The optimal speed regulator parameters can be obtained with the pole-placement method.

The inverse model used for current reference values leads theoretically at an over-all system without dynamic. In steady-state (27) becomes:

$$\underline{i}_{sd}^{e*} = \frac{1}{L_m} \underline{\psi}_r^* \quad (28)$$

The feedforward control structure induces a limitation of the torque response. The transfer function is given by

$$\frac{m_e}{m_e^*} = \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2}, \quad \omega_m = \frac{1}{T_r} \sqrt{\left(\frac{i_{sq}^*}{i_{sd}^*}\right)^2 + 1},$$

$$\zeta_m = \left(\sqrt{\left(\frac{i_{sq}^*}{i_{sd}^*}\right)^2 + 1} \right)^{-1} \quad (29)$$

The speed loop is analyzed with (29). A PI regulator is sufficient in order to ensure a null steady-state error. The structure of the speed loop is given in Fig.5a. The electromagnetic time constants are small enough to be neglected in this model. The simplified model of the speed loop is given in Fig.5b.

Neglecting the viscosity coefficient, D , the transfer function of the control system is:

$$\frac{\Omega}{\Omega^*} = \frac{\frac{\omega_v^2}{z}(s+z)}{s^2 + 2\zeta_v \omega_v s + \omega_v^2}$$

$$\omega_v = \sqrt{\frac{k_{iv}}{J}}, \quad \zeta_v = \frac{k_{pv}}{2\sqrt{Jk_{iv}}}, \quad z = \frac{k_{iv}}{k_{pv}} \quad (30)$$

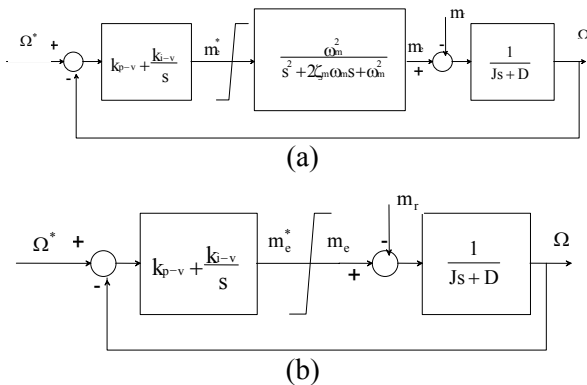


Fig.5. Speed control loop structure

Neglecting the viscosity coefficient, D , the transfer function of the control system is:

$$\frac{\Omega}{\Omega^*} = \frac{\frac{\omega_v^2}{z}(s+z)}{s^2 + 2\zeta_v \omega_v s + \omega_v^2}$$

$$\omega_v = \sqrt{\frac{k_{iv}}{J}}, \quad \zeta_v = \frac{k_{pv}}{2\sqrt{Jk_{iv}}}, \quad z = \frac{k_{iv}}{k_{pv}} \quad (30)$$

The design objective for the speed regulator is to obtain a minimum transient time for step input. Due to the limited capacity of the torque response of the machine, the cut-off frequency of the loop can not be bigger than the one of the transfer function (29). If the cut-off frequencies are equal, the parameters of the speed regulator are:

$$k_{pv} = J \frac{1}{T_r} \left(1 + \frac{1}{\sigma^2}\right); \quad k_{iv} = J \left(\frac{1}{2T_r} \left(1 + \frac{1}{\sigma^2}\right)\right)^2 \quad (31)$$

The torque generation mechanism is underdamped. In this case it is desirable to impose a cut-off frequency for the speed loop much smaller than the one of the torque generation mechanism:

$$\omega_{Bv} = K\omega_{Bm}; \quad 0 < K < 1 \quad (32)$$

Eq. (31) can be rewritten with this coefficient:

$$k_{pv} = J \frac{K}{T_r} \left(1 + \frac{1}{\sigma^2}\right); \quad k_{iv} = J \left(\frac{K}{2T_r} \left(1 + \frac{1}{\sigma^2}\right)\right)^2 \quad (32)$$

The value of this coefficient can be experimentally determined.

This control structure can be used for current-fed inverters. If it is used a voltage-fed inverter the orthogonal components of the stator current will be indirectly controlled through the stator voltage components. Two supplementary loops must be added.

In the steady-state magnetic regime, the system becomes the system described by (26).

The tuning of the PI current regulators can be done on the basis of the open loop transfer function

$$H(s) = K_{Ri} \frac{T_{ii}s+1}{T_{ii}s} \frac{K}{Ts+1}; \quad K = \frac{1}{R_s}; \quad T = \frac{\sigma L_s}{R_s} \quad (33)$$

The time constant of the electric circuit, T , is compensated with the integral time constant of the PI regulator.

$$H_0(s) = \frac{1}{\tau s + 1}; \quad \tau = \frac{T}{K_{Ri}K} \quad (34)$$

For a specific time constant of the current loop, the tuning parameters of the current loop can be computed:

$$K_{Ri} = \frac{\sigma L_s}{\tau_{adm}}; \quad T_{ii} = T = \sigma T_s \quad (35)$$

Finally the complete control structure is presented in Fig.6.

5 Experimental results

This control structure was digitally implemented and tested. The voltage-feed induction motor with a PWM inverter has the nominal parameters: $P_N=1.5\text{kW}$, $U_N=380\text{V}$, $I_N=3.8\text{ A}$, $\cos \varphi=0.79$, $\Omega_N=1410\text{ rev/min}$, $f_N=50\text{ Hz}$ [8]. The results were obtained by numerical simulation with Matlab, and with aDSP board C31MCDB [6].

The experimental results are presented in Fig. 7-14. Fig. 7-10 show the motor speed, the motor estimated torque and the values of the orthogonal components of the motor current when the electric drive is not loaded.

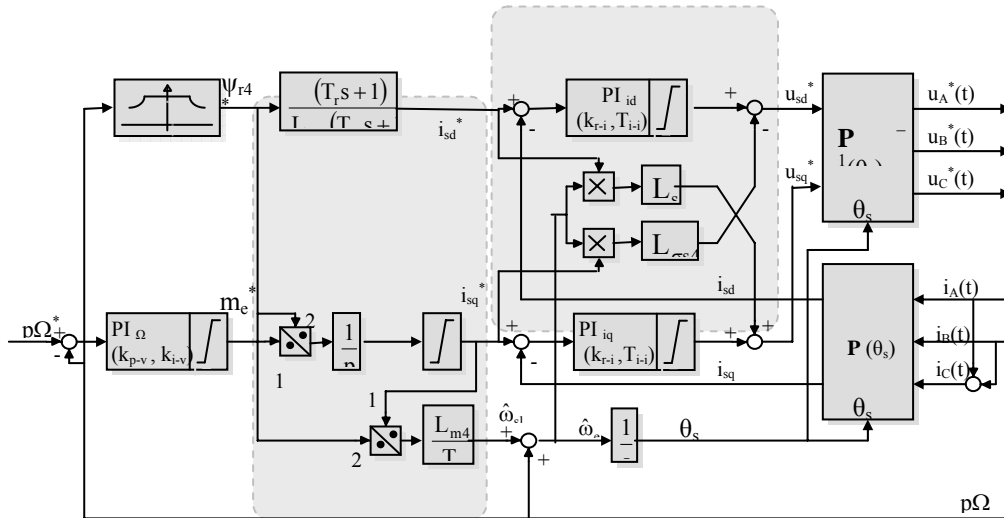


Fig.6. The analog control structure

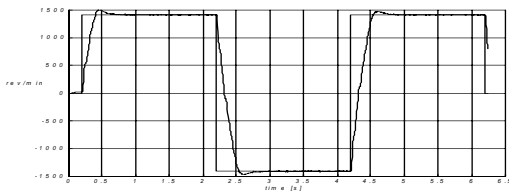


Fig.7. Speed of the vector control induction motor drive

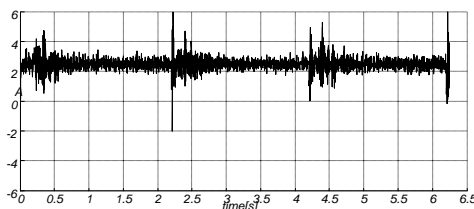


Fig.8. The reference/measured value of the reactive component of the stator current

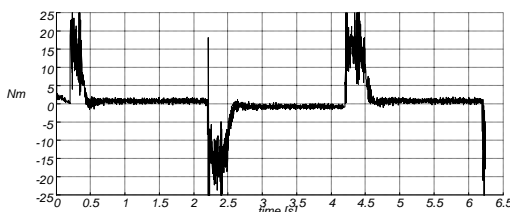


Fig.9. The estimated value of motor electromagnetic torque

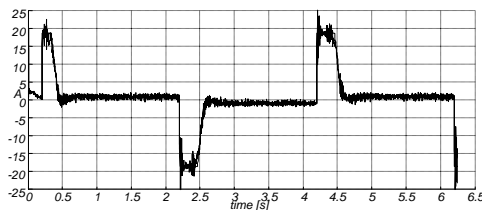


Fig. 10 The reference/measured value of the active component of the stator current

6 Conclusion

The induction motor is not a system completely observable. As a consequence the symmetrical phasor circuit can not be completely specified on the basis of the measurements at the motor terminals configured in a single topology. The phasor model must be reformulated in order to be completely specified with a minimum set of parameters. The new vector control scheme based on an asymmetrical equivalent model uses only the information obtained from the parameter identification experiments without approximation.

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