Optimum Synthesis of Mechanisms For Path Generation Using a New Curvature Based – Deflection Based Objective Function

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Abstract: - This paper proposed a new curvature based path-description method for path-generation of planar mechanism, in which the position vector of a point on coupler, which follows most closely a predefined path, is directly calculated as a function of coupler motion properties. In addition this curvature based method makes it possible to define an objective function independent of rotation and translation transformations.
Therefore, in this method, initial position, orientation, and coupler point vector parameters are not the optimization variable. These five parameters could be calculated as a function of other design parameters and they are found when these optimal values are obtained. Reducing five parameters of the optimal variables leads to a smaller design space and so less computation time to find the optimal result.
Application of the method is also shown and its computation time and accuracy are compared with the result reported in the literature. It is found that this method is superior as it converges faster without loosing any accuracy.

Key-Words: - Mechanism synthesis; Optimization; Path Generation; Curvature based objective function; Deflection based objective function;

1 Introduction

Path generation is one of the important functional requirements in mechanical engineering design. The use of optimum synthesis is inevitable whenever the mechanism needs to follows a number of positions exceeding a certain number, 6 points for path generation of a planar four-bar linkage. An objective function is generally defined as a function of design parameters, which it is then optimized to find the optimal values of these parameters. For any optimization method, the calculation time depends on the number of design parameters. The lower the number of parameter, the faster is the calculation.

To reduce the calculation time, researchers try to reduce the number of the design parameters. To do so, they use different mathematical method to obtain the path characteristics to measure the goodness of matching between the desired and generated path. Their interest was mainly in removing the rotation, translation and scale design parameters. Ullah et al[7]. used Fourier Descriptors and compare the amplitude of characteristic spectrum of generated and required path as a measure of their matching goodness. In this method the points on the curve need to be sampled in the constant time intervals. Yong Liu et al.[11] resolved this point by employing Refined Numerical Representation.

Other researchers have tried different optimization methods in the synthesis of rigid body mechanisms. Nolle et al.[4] used gradient and least square method; Kunjur et al[3]. used genetic algorithm; Liu et al[11] applied artificial immune system; and Smaili et al.[1] employed tabu-gradient-search to find optimum solution to the mechanism synthesis.

This paper presents a curvature based path description method to define a new objective function which is independent of the vector position of coupler point, translation and rotation translations parameters. To the best of the authors' knowledge this is the first attempt at using such error function to be independent of vector position of the coupler point.

Here the basic idea of the method is introduced. Two illustrative examples are provided. In the first example the result is compared with what is reported by Smaili et al.[1]. In the second example the validity of the assumption is discussed.

2 Formulation

The main idea of this article is to employ the curvature to define the path, which enable us to introduce a new error function with fewer number of design parameters.

The curvature of a path is defined as:

$$\kappa = \frac{\left|\vec{\dot{p}} \times \vec{\ddot{p}}\right|}{\dot{p}^3} \quad (1)$$

Wherein $\dot{p}, \ddot{p}$ are the velocity and acceleration of the moving point p tracing the path.

When p is a point on a rigid body its position vector, with respect to the base, is obtained as a function of the position of another point f on the body, its position respect to f and the orientation of the body (fig.1):

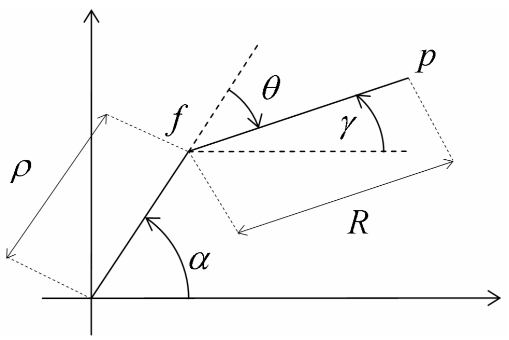
$$p = f(t) + Re^{i\gamma} \quad (2)$$

Replacing f as a function of ρ and α, the velocity and acceleration vector of p are obtained as follows:

$$p = \rho e^{i\alpha} + Re^{i\gamma}$$
$$\dot{p} = \left(\dot{\rho} + i\dot{\alpha}\rho\right)e^{i\alpha} + iR\dot{\gamma}e^{i\gamma}$$
$$\ddot{p} = \left(\ddot{\rho} + 2i\dot{\alpha}\dot{\rho} + i\ddot{\alpha}\rho - \dot{\alpha}^2\rho\right)e^{i\alpha} + iR\ddot{\gamma}e^{i\gamma} - R\dot{\gamma}^2e^{i\gamma} \quad (3)$$

Now we define $r = Re^{i\theta}$, where $\theta = \gamma - \alpha$ (fig.1), and introduce $\dot{p}$ and $\ddot{p}$ into eq.1 we have:

Fig. 1 Parameters to define a point



$$\kappa = \frac{X}{Y} \quad (4)$$

Where

$$X = r^2\gamma'^3 - \rho\left(\rho + \rho''\right) + r\begin{bmatrix}\left(\gamma''\rho - \gamma'^2\rho' - 2\gamma'\rho'\right)\sin\theta + \\ \left(\gamma''\rho' + \gamma'^2\rho + \gamma'\left(\rho - \rho''\right)\right)\cos\theta\end{bmatrix}$$

$$Y = \left[r^2\gamma'^2 - 2r\gamma'\left(\rho'\sin\theta - \rho\cos\theta\right) + \rho'^2 + \rho^2\right]^{\frac{3}{2}}$$

$\ddot{\alpha}$ was also assumed to be zero without loosing any generality.

This equation could be written as follows:

$$\left[\kappa^2 B_2^3\right]r^6 + \left[3\kappa^2 B_2^2 B_1\right]r^5 +$$
$$\left[3\kappa^2\left(B_2^2 B_0 + B_1^2 B_2\right) - A_2^2\right]r^4 +$$
$$\left[\kappa^2\left(B_1^3 + 9B_2 B_1 B_0\right) - 2A_2A_1\right]r^3 +$$
$$\left[3\kappa^2\left(B_1^2 B_0 + B_0^2 B_2\right) - \left(A_1^2 + 2A_2A_0\right)\right]r^2 +$$
$$\left[3\kappa^2 B_0^2 B_1 - 2A_1A_0\right]r + \left[\kappa^2 B_0^3 - A_2^2\right] = 0 \quad (5)$$

Where

$$A_0 = -\rho\left(\rho + \rho''\right)$$
$$A_1 = \left(\gamma''\rho - \gamma'^2\rho' - 2\gamma'\rho'\right)\sin\theta + \left(\gamma''\rho' + \gamma'^2\rho + \gamma'\left(\rho - \rho''\right)\right)\cos\theta$$
$$A_2 = \gamma'^3$$
$$B_0 = \rho'^2 + \rho^2$$
$$B_1 = -2\gamma'\left(\rho'\sin\theta - \rho\cos\theta\right)$$
$$B_2 = \gamma'^2$$

Supposing that p is the point on coupler that follows the desired path with curvature equal to κ at that point, the above equation could be employed to obtain r, when the kinematics characteristics of the

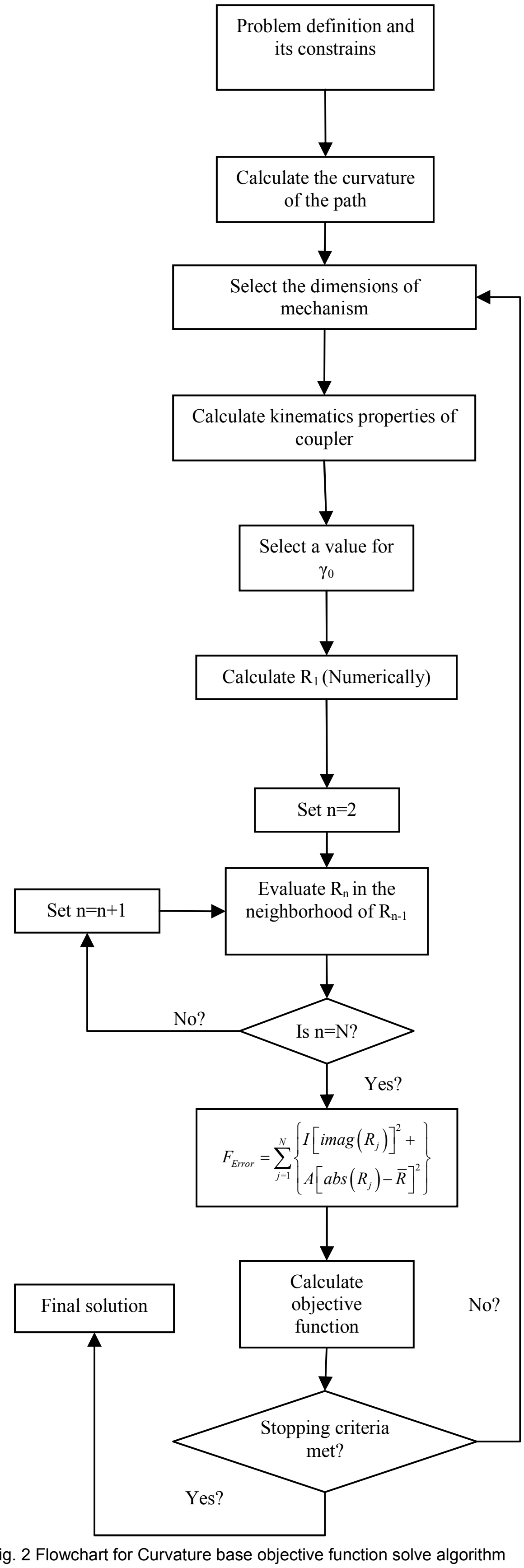


Fig. 2 Flowchart for Curvature base objective function solve algorithm

coupler are known. Letting $\gamma = \int \gamma' d\alpha + \gamma_0$, the resulting r is found as a function of γ_0 and α, and therefore R is calculated from $R = re^{-i\theta}$. Thus for any value of γ_0, a set of R values are obtained. A semi-analytical procedure is used to find R from the above relations. At the first step R_1 is found numerically from eq.5 and analytical method is used to find the subsequent values of R. For all mechanisms that generate path around to the desired one, R is weakly varied with α. If R_n is the answer for α_n, R_{n+1} is obtained analytically as a function of α_{n+1} in the neighborhood of R_n.

For N points on the desired path, N values for R are calculated by the above procedure, but R must be real and remains constant as the coupler is a rigid body without any deformation. Therefore we define the error function as:

$$F_{Error} = \sum_{j=1}^{N} Error_j$$

Where

$$Error_j = I\left[imag\left(R_j\right)\right]^2 + A\left[abs\left(R_j\right) - \overline{R}\right]^2$$

and

$$\overline{R} = \frac{\sum_{j=1}^{N} R_j}{N}$$

I and A are weighting factors here.

This error function is an indication of the angular and longitudinal deformations of the coupler and its minimum correspond to the best mechanism that generates a path most close to the desired one. Any optimization technique could be use to find the optimum answer. The calculation procedure to obtain the design variables including γ_0 is shown in fig.2.

Meanwhile, numerical experiment of this procedure shows that the minimum value of error corresponds approximately to the point with maximum value

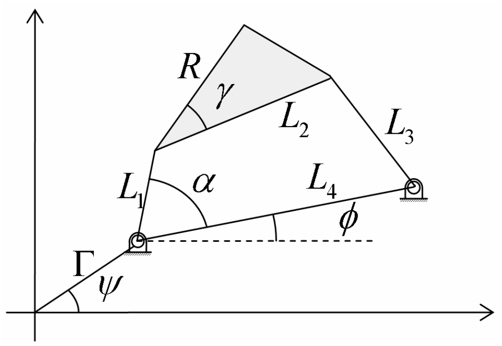


Fig. 3 Design parameter needed to define a four bar mechanism

i	1	2	3	4	5	6	7	8	9	10
xid	1	1.5	2	2.5	3	3.5	4	3.5	2.5	1.5
yid	1.5	2	2	2	2	2	1.5	1	1	1
αi	0°	35°	87.5°	105°	140°	175°	192.5°	262.5°	280°	315°

of γ'. Thus the value of γ_0 is calculated such that the minimum value of error occurs when the value of γ' is maximum. To simplify the calculation, γ_0 is obtained by minimizing the imaginary part of R at the point with maximum value of γ', as this part is dominant at this condition. This value is designated as γ_0^*.

3 Examples

Method is coded in MATLAB® and applied to the optimum synthesis of path generating planar four bar mechanisms. Ten design parameters generally involve describing these mechanisms as shown in fig.3. In the abovementioned procedure only five parameters, namely L1, L2, L3, L4 and γ0 are needed to find the solution. Two examples are introduced. In the first example the result of this method is compared with the result of Smaili et al[1]. In the second one an approximation procedure to estimate γ_0 is studied. In these two examples the weighting factors I and A put equal to 1 and the Genetic Algorithm applied to find the optimum solution.

Example1.

The planar four bar mechanism that follows 10 prescribe path in coordinate with crank rotation is designed. The desired points are given in table 1. The result of this method is shown in table 2 and fig. 4and compared with the result of Smaili et al[1]. 499 iterations are needed to find the optimum result that took 11 s run time on a Pentium IV PC. Smaili et al. introduce this example to compare tabu-search and the tabu-gradient algorithms. It was reported that 318 iterations with 17 s run time on Pentium IV are needed. This indicates that the method proposed here is faster, ever thought, as reported in [1], the tabu-gradient-search claimed to be the fastest optimization method in mechanism synthesis, and converges much faster than genetic algorithm.

Example2.

In this example we compare the result of the

Table 2 Classic objective fuinction TS solution and curvatre based objective function GA solution

	Classic O.F. TS	C.B. GA
L1	2.7132	1.4987
L2	4.4178	6.0102
L3	0.5976	4.5001
L4	2.7016	7.4895
R	8.5473	3.5876
γ	-64.61	-5.0124

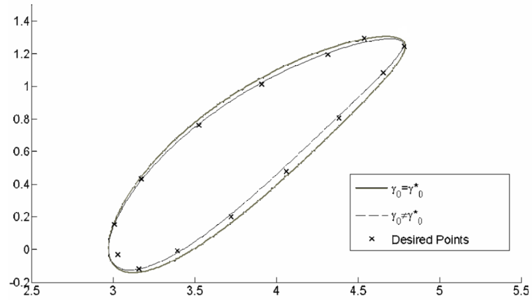


Fig. 4 Desired coupler points and coupler curve generated with and without assuming $\gamma_0 = \gamma_0^*$

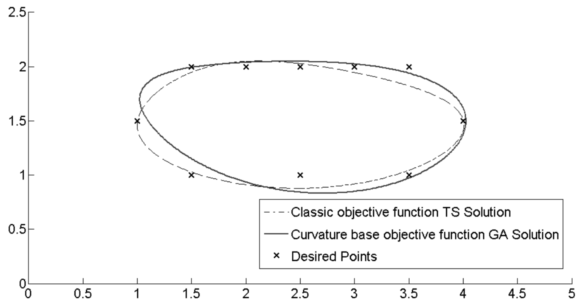


Fig. 5 Coupler curve generated by Classic objective function, TS and Curvature based objective function, GA solutions optimal mechanism

optimization, with and without letting $\gamma_0 = \gamma_0^*$ discussed in the previous section. Herein the 14-point path is provided.

Considering γ_0 as one of the optimization parameters the calculation time was 35s with 1550 iterations. Meanwhile, the optimization needs about 890 iterations and takes 23s of run time, by letting $\gamma_0 = \gamma_0^*$. Fig.5 compares the results of these optimizations. This example indicates that assuming $\gamma_0 = \gamma_0^*$ the method could be made much faster but accuracy is slightly lower.

4 Conclusion

This paper presents an error function which could measure the goodness of matching between the predefined curve and generated curve, as a function of coupler motion properties but independent of vector position of coupler point. This error function defined on the basis of curvature of the desired and generated curve. As a result, the mechanism, and so the design parameters, will be independent of rotation and translation.

The method is applied to four bar planar mechanism and showed that it needs lesser calculation time. This is achieved due to lower number of design

parameters needed to define the objective function.
Although the illustrative examples were four bar planar mechanism, the objective function is constructed so that the algorithm can be easily applied to other path generation mechanisms.

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