Analysis of PWM Converters Using MATLAB

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Abstract: - This paper aims to show how the MATLAB environment can be used to analyse three topologies of DC-to-DC forth-order PWM converters such as Cuk, SEPIC and Zeta converters. The steady state and dynamic analysis is made for these converters with coupled or separate inductors and parasitic included operating either in continuous conduction mode (CCM) or in discontinuous conduction mode (DCM). The space-state averaging method was used as basically modelling approach that was easily modified in order to provides not only the classical dynamic model of converter but also the canonical averaged dynamic model expressed by its characteristic coefficients. Also, it computes the boundary between DCM and CCM, the decay interval inductor currents, the converter efficiency, the dc and ac small-signal properties of converter. The external characteristics, frequency responses and any functional parameter of converter are computed and plotted too. Some analysis results of SEPIC PWM converter with parasitic included and coupled inductors obtained by running this computer program are given in order to illustrate the usefulness of the program. New models of converters may be added in order to compare their efficiency and accuracy, the results obtained by means of this program based on the space-state averaged models being used as reference in the comparing process.

Key-Words: - computer program, MATLAB environment, DC-to-DC PWM converters

1 Introduction

Investigating the dynamic behaviour of a DC-to-DC pulse width modulated (PWM) converter is a necessary step in topology selecting and then to devise a system with desired performance. This usually comprises deriving a small-signal model for the power stage of converter and designing a suitable controller for it. Depending on the simulation environment and the modelling method, a small-signal of converter given as a functional block diagram could be more suitable than an analytical or equivalent circuit model for computer simulation (with MATLAB or SIMULINK for instance), while designing the control circuit.

Despite its complexity, the state-space averaging (SSA) method is commonly used for establishing the small-signal model of the switching converters as the interested frequency range is about one decade below its switching one [1]. In the SSA method, the power flow to a converter is averaged on a switching period and its dynamics are expressed in terms of a set of state and output equations. The injected-absorbed-current (IAC) method is another approach for obtaining dynamic models of the switching converters. The main technical advantage of this method is that the small-signal model can be represented either by an electrical equivalent circuit with a fixed topology or by a functional block diagram, which cannot be done with another modelling methods. But, if the IAC method can be easily applied to elementary converters (second-order converters) no parasitic included, its directly application to higher-order converter with parasitic included is a tedious task [2].

The fourth-order PWM converters are networks with four storage elements: two inductors and two capacitors. Cuk, Sepic and Zeta converters are examples of basic fourth-order PWM converters with many applications [1] – [8]. These converters are well known for their low-ripple input or output current when they operate in either the continuous conduction mode (CCM) or discontinuous conduction mode (DCM). Applying the SSA method in which we consider the absorbed and injected currents of the power stage as output variables, one can obtain the same canonical dynamic model as that provided by the IAC method. In this way, a systematically derivation of the canonical dynamic model that includes all the parasitic elements of the converter and that can be directly transposed into an equivalent circuit or into a functional block diagram results. This paper presents an algorithm implementation using MATLAB environment to derive the steady-state (dc) model and the canonical dynamic model described by characteristic coefficients. All important dc and ac properties of three converters can be computed and plotted. The behaviour of Cuk, Sepic and Zeta PWM converters with CCM or DCM, with separate or coupled inductors, and all parasitic included can be analysed and various studies regarding some parameter effects of converter can be performed. We chosen the MATLAB environment to built the program because it provides all the operations with matrices and also it permits an easy graphical representation of the computed quantities.

The paper is organized as follows. The averaged models used in program are succinctly presented in Section 2. Then, in Section 3, the program facilities are illustrated by means of the main stages of computation and analysis. An application example on a SEPIC PWM converter is given in Section 4. Conclusions are presented in Section 5.

Generally, the notations in the paper are those commonly used. The capital letters denote the dc components of variables, while the small caps with a tilde above them refer to their small-signal variations. The asterisk is used to represent the Laplace transformations of the small-signal variations of variables or functions on complex frequency s.

2 Converter description and models

A matrix description is adopted, so the technique can be extended to other converter topologies. The modelling method of the fourth-order PWM converters combines two well-known methods: the (SSA) method [1] and the injected-absorbed-current (IAC) method [2].

The state variables of converter are the currents i_1 and i_2 through the inductors L_1 and L_2, and the voltage drops v_1 and v_2 across the capacitors C_1 and C_2. The line voltage v_I is the input quantity of converter. The absorbed and injected currents i_I and i_J, and the output voltage v_O are the output quantities. So, the state vector has the form $x = [i_1 \quad i_2 \quad v_1 \quad v_2]^T$ and the output vector is $y = [i_I \quad i_J \quad v_O]^T$.

The three distinct time intervals that characterize the operating of the power switching stage in DCM are written d_1T_s, d_2T_s and d_3T_s, with $d_1+d_2+d_3=1$, and T_s the constant switching period [5]. For the converter operating in CCM, $d_3 = 0$ and $d_2 = 1- d_1$. The loss resistance of inductors (r_1 and r_2), the equivalent-series resistance of capacitors (r_3 and r_4) and the conducting state resistance of the transistor (r_5) and diode (r_6) have been considered as parasitic elements of the converter with a switch realized as a transistor and diode combination.

The procedure of primary description, averaging, perturbation and linearization of operating equations of the power stage belongs to SSA method, as well as the steady-state model of the converter, while the canonical dynamic (ac small-signal) model belongs to IAC method. The elements of the canonical dynamic model that are named characteristic coefficients are systematically obtained through the SSA method. In this way, they can be easily derived and computed, for any converter type. For the sake of brevity, only the forms of the dynamic model will be presented.

The canonical dynamic (ac small signal) model of a PWM converter obtained through the IAC method is given in complex domain by two linear equations of the small-signal perturbations of the absorbed ($\tilde{i}_I^*$) and injected ($\tilde{i}_J^*$) currents, namely:

$$\tilde{i}_I^* = A_I^* \tilde{d}^* - B_I^* \tilde{v}_I^* + C_I^* \tilde{v}_O^* \quad (1)$$

$$\tilde{i}_J^* = A_O^* \tilde{d}^* - B_O^* \tilde{v}_I^* + C_O^* \tilde{v}_O^* . \quad (2)$$

The characteristic coefficients are written as $A_I^*, B_I^*, C_I^*, A_O^*, B_O^*, C_O^*$ [2]. Generally, these coefficients are functions of the complex frequency s and depend on the converter topology.

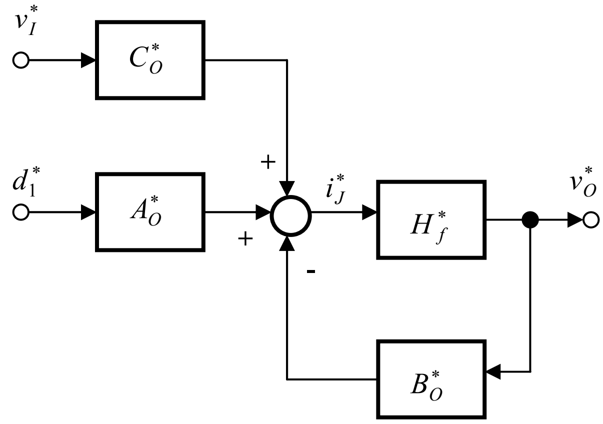

Fig. 1. Functional block diagram of a PWM converter

The equation (1) and (2) represent the canonical dynamic (ac small signal) model of a switching PWM converter, because they keep the form regardless the converter configuration, control mode (voltage or current) and operating mode (DCM or

CCM). These two equations can be transposed either into an equivalent circuit with fixed topology or into a functional block diagram as shown Fig. 1 [2]. The equivalent circuit with fixed topology, which consists only controlled voltage or current sources and impedances and admittances, is obtained by the directly transposition of the equations (1) and (2).

For a fourth-order PWM converter, $\tilde{v}_O^*$ relies on $\tilde{i}_J^*$ by the following transfer function

$$\tilde{v}_O^* = \frac{\tilde{i}_J^* R(1 + sr_4 C_2)}{1 + s(R + r_4)C_2} = \tilde{i}_J^* H_f^* \tag{3}$$

where R is the load resistance of converter, C_2 is the capacity of the output filter capacitor and r_4 is its equivalent-series resistance. Using (2) and (3), a unified functional block diagram of converter can be drawn as that shown in Fig. 1. From this block diagram, $\tilde{v}_O^*$ is found as

$$\tilde{v}_O^* = \frac{H_f^* C_O^*}{1 + H_f^* B_O^*} \tilde{v}_I^* + \frac{H_f^* A_O^*}{1 + H_f^* B_O^*} \tilde{d}^* . \tag{4}$$

The equation (4) contents two transfer functions of the open loop converter: the audio-susceptibility or line-to-output voltage transfer function (H_{OI}^* or G_{vg}^*) and the control-to-output voltage transfer function (H_{OC}^* or G_{vd}^*). These transfer functions have the following expressions:

$$H_{OI}^* = \frac{H_f^* C_O^*}{1 + H_f^* B_O^*} \tag{5}$$

and

$$H_{OC}^* = \frac{H_f^* A_O^*}{1 + H_f^* B_O^*} . \tag{6}$$

More details concerning the computational algorithms concerning the characteristic coefficients of the fourth-order PWM converters can be found in the previous works of authors.

3 Analysis stages

The flowchart of our program comprises the main stages of modelling and analysis of the three PWM converter topologies (Cuk, SEPIC and Zeta):

1. Establish the conduction mode of operating depending on the parameter values. For this, the boundary between CCM and DCM is computed as the critical load resistance, R_{crit}, or equivalently the critical conduction parameter, K_{crit}. If the operating parameters correspond to DCM, that is $R > R_{crit}$ or $K < K_{crit}$, the parameter D_2 that gives the width of the decay interval of inductor currents must be calculated. Plots of the boundary as functions of different converter parameters can be traced too.
2. Compute the elements of matrices that describes the switched networks and the averaged behaviour of converter (for CCM or DCM, after case).
3. Compute the dc model. In this stage, the dc components of state and output variables, and the external characteristics are calculated. Also, various plots showing the effects of the parameter variation can be obtained in this stage.
4. Compute the polynomial coefficients of the characteristic coefficients.
5. Compute the magnitude and phase of the line-to-output voltage and control-to-output voltage transfer functions. Plot the frequency characteristics of the transfer functions.
6. Compute the magnitude and phase of the input and output impedance of converter. Plot the frequency characteristics of the input and output impedance.

4 Application example

A Sepic PWM converter with DCM and parasitic included, with coupled inductors is used in order to exemplify the features of the computational algorithms and the analysis program. Some results obtained by running this program are presented here. The SEPIC PWM converter has the following specifications: L_1=1 mH, L_2=1 mH, C_1=18 μF, C_2=1000 μF, D_1=0.37, R=82 Ω, V_I=15 V, r_1= r_2=0.3 Ω, r_3= r_4= r_5=r_6=0, f_s=20 kHz. The coupling coefficients of the inductors is k_c=0.37.

For these input data, the program provides the following results:

```
Rcrit = 3.224e+001
D2 = 6.0000e-001

Xd =
  6.9124e-002
  1.1770e-001
  1.5017e+001
  9.1916e+000

Yd =
  6.9124e-002
  1.1209e-001
  9.1916e+000

Md = 6.1277e-001
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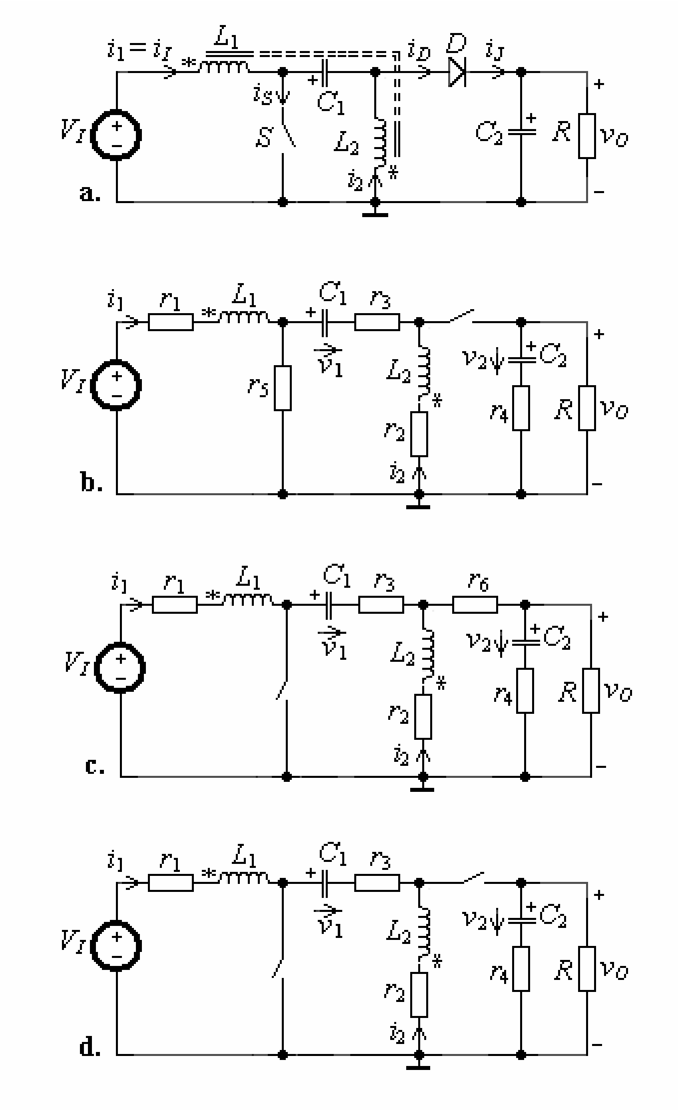

Fig. 2. SEPIC PWM converter: **a.** schematic diagram of power stage; **b.** switched network for the d_1T_s interval; **c.** switched network for the d_2T_s interval; **d.** switched network for the d_3T_s interval

The above results means that SEPIC converter operates in DCM ($R > R_{crit}$) and the decay inductor currents is given as product of D_2 = 0.6 and the switching period T_s.

The steady-state model is given as the state vector $[I_1, I_2, V_1, V_2]^T$ and output vector $[I_I, I_J, V_O]^T$. The dc voltage conversion ratio is $M_d = 0.612$.

The expressions of the characteristic coefficients of interest for the transfer functions of the Sepic converter with DCM are as follows:

$$A_O^* = \frac{f_{d43}s^3 + f_{d44}s^2 + f_{d45}s + f_{d46}}{s^4 + f_{d15}s^3 + f_{d16}s^2 + f_{d17}s + f_{d18}}$$

$$B_O^* = -\frac{f_{d47}s^3 + f_{d48}s^2 + f_{d49}s}{s^4 + f_{d15}s^3 + f_{d16}s^2 + f_{d17}s + f_{d18}}$$

$$C_O^* = \frac{f_{d50}s^3 + f_{d51}s^2 + f_{d52}s}{s^4 + f_{d15}s^3 + f_{d16}s^2 + f_{d17}s + f_{d18}}.$$

The following values of the polynomial coefficients characterise the converter with separate inductors:

denominator:
f_{d15}=7.0000e+002;f_{d16}=2.9634e+007;
f_{d17}=1.0329e+010; f_{d18}=0
nominator of A_O^*:
f_{d43}=1.5103e+004;f_{d44}=1.1744e+007;
f_{d45}=8.1587e+011; f_{d46}=-1.8079e-001;
nominator of B_O^*:
f_{d47}=7.4420e+002;f_{d48}=2.6047e+005;
f_{d49}=2.0672e+010
nominator of C_O^*:
f_{d50}=5.9780e+002;f_{d51}=2.0923e+005;
f_{d52}=1.2539e+010.

For the converter with coupled inductors (k_c=0.37), the polynomial coefficients of A_O^*, B_O^* and C_O^* have the values:

denominator:
f_{d15}=7.9882e+002;f_{d16}=4.8400e+007;
f_{d17}=1.4423e+010; f_{d18}=0
nominator A_O^*:
f_{d43}=1.0291e+004;f_{d44}=1.1429e+007;
f_{d45}=8.2447e+011; f_{d46}=-6.2910e-002;
nominator B_O^*:
f_{d47}=6.9504e+002;f_{d48}=3.7042e+005;
f_{d49}=2.9398e+010
nominator C_O^*:
f_{d50}=4.4825e+002;f_{d51}=2.3889e+005;
f_{d52}=8.5212e+009.

Using these characteristic coefficients, a study of the effect of the inductor coupling over the line-to-output voltage and control-to-output voltage transfer functions of converter (magnitude and phase) can be made. The frequency characteristics of the open-loop transfer functions of SEPIC PWM converter with coupled inductors and parasitics are shown in Fig. 2 and 3. Running the program with k_c set at zero, the above properties for SEPIC PWM converter with separate inductors are obtained.

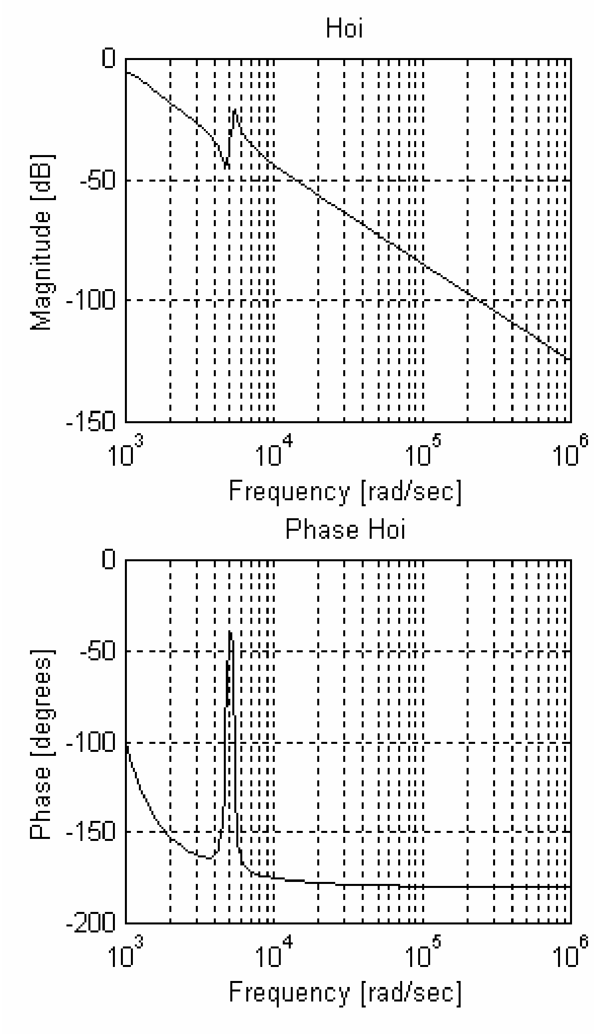

Fig. 2 Frequency characteristics of the line-to-output voltage transfer function of Sepic PWM converter with DCM and coupled inductors

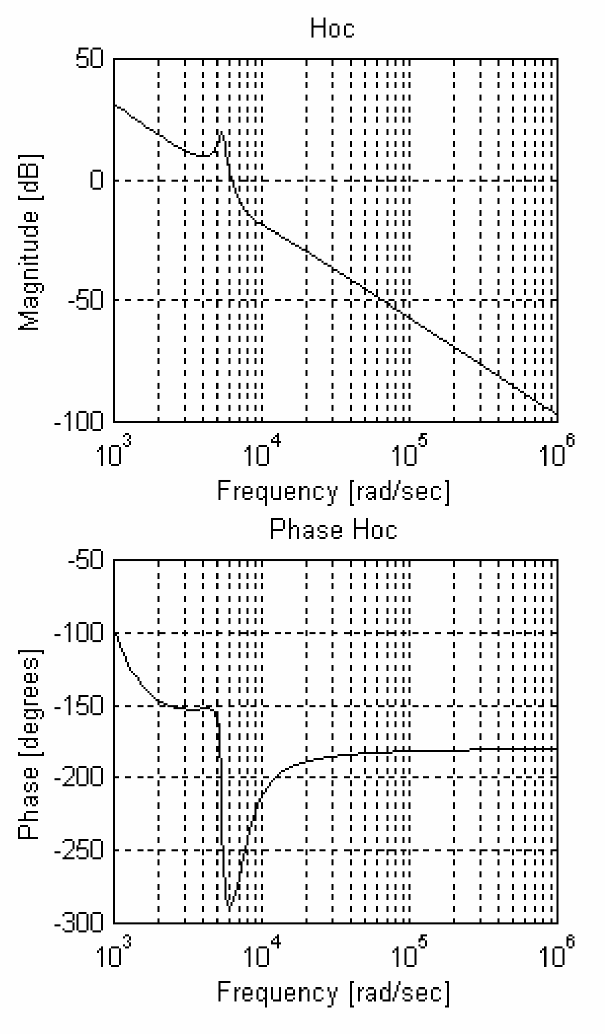

Fig. 3 Frequency characteristics of the control-to-output voltage transfer function of Sepic PWM converter with DCM and coupled inductors

5 Conclusion

Using the MATLAB environment, a program for analysis of some fourth-order PWM converters was built. The computational algorithms implemented are based on an averaged model of PWM converter derived by means of the state-space averaging method combined with the injected-absorbed current method. The used method yields a full-order of converter in CCM and a reduced-order of converter in DCM. This program computes the steady-state model, the boundary between the continuous and discontinuous conduction mode, the parameter that defines the decay interval inductor currents, the external characteristics, the converter efficiency, etc. Also, by this program, all the six characteristic coefficients of the canonical dynamic model can be computed. Three of these characteristic coefficients are used to obtain the functional block diagram of the converters and to perform their small-signal dynamic analysis. The analysis results are similar to those obtained with usual circuit simulators.

Such program allows studying the effect of the parasitic elements, the coupling coefficient of the inductors and the operating point over the dynamic properties of the converter: open-loop or closed-loop transfer functions (audio-susceptibility, control-to-output voltage) and input and output impedance.

New models of converters may be added in order to compare their efficiency and accuracy, the results obtained by means of this program based on the space-state averaged models being used as reference in the comparing process.

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