Fuzzy Sliding Mode PI Controller for Nonlinear Systems

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Abstract: - in this paper, we present a fuzzy logic controller to combine sliding mode controller and PI controller for nonlinear systems with external disturbances. A sliding mode controller can give good transient performance and PI offer zero steady state error. However in the sliding mode controller, the steady state performance is poor due to the presence of discontinuous control action which causes chattering problem. Hence, combining these two controllers by a fuzzy logic can combine their advantages and remove their disadvantages. The proposed method can efficiently to eliminate the chattering in sliding phase so that high performance can be achieved. Some simulation results prove the validity of the proposed method.

Key-words: - Fuzzy logic, PI control, sliding mode control, chattering, nonlinear system.

1 Introduction

The control of nonlinear systems has been an important research topic and many approaches have been proposed [1][2].The sliding mode control theory of the variable structure system provides a method to design a system in such a way that the controller system should be insensitive to parameter variations and external disturbances [3]. Essentially, the sliding mode control uses discontinuous control action to drive the state trajectory toward a specific hyper plane in the state space, and then the state trajectory is maintained to slide on the specific hyper plane until the origin of the state space is reached. In the sliding mode control, the hitting time of the system state reaches the switching plane will affect the speed of the system with the desired dynamic behaviour. Sliding mode control (SMC) is well known for handling matched uncertainties [3][4].A sliding-mode control law is formulated using a Lyapunov approach to guarantee that the system state first reaches the prescribed sliding mode in finite time from any initial state, and then remains on it there after by a discontinuous control. However, SMC suffers from a well known problem chattering due to the high gain and high-speed switching control. The undesirable chattering may excite previously unmodeled system dynamics and damage actuators, resulting in unpredictable instability. Smoothing techniques such as the boundary layer approach have been employed to reduce its effects at the cost of giving concessions from performance [5]. Therefore, a compromise must be sought between the desired control accuracy and controller bandwidth. As a model free design method, fuzzy systems have been as a model free design method [7], fuzzy systems have been successfully applied to control complex or ill-defined processes whose mathematical models are difficult to obtain [6][7].The ability of converting linguistic descriptions into automatic control strategy makes it a practical and promising alternative to the classical control scheme for achieving control of complex nonlinear systems. To eliminate steady-state error, a PI controller should be employed. This paper proposes a fuzzy logic controller (FLC) to combine an SMC and a PI controller. As the SMC and PI controllers can give good transient and steady-state performance respectively, the role of the FLC is to schedule them under different operation conditions [7][8]. The remaining of this paper is organised as follow. In section 2, problem statements. The proposed fuzzy SMC and PI control will be developed in Section 3. Section 4, a plant is used to test the proposed FLC method and some compared results demonstrate its feasibility. Finally, Section 5 concludes the paper.

2 Problem statements

In this section, the variable structure system with sliding mode control is briefly reviewed.

Consider a general class of SISO n-th order nonlinear systems as follow:

$$x^{(n)} = f(\underline{x},t) + g(\underline{x},t)u + d(t) \quad (1)$$
$$y = x$$

where f and g are nonlinear functions, $\underline{x} = \left[x, \dot{x}, \ldots, x^{(n-1)}\right]^T = \left[x_1, x_2, \ldots, x_n\right]^T \in R^n$ is the state vector of the systems which is assumed to be available for measurement, $u \in R, y \in R$ are the input and the output of the system, respectively, and $d(t)$ is the unknown external disturbance. We have to make an assumption that $d(t)$ have upper bound D, that is, $|d(t)| \le D$. We require the system (1), to be controllable, the input gain $g(\underline{x},t) \neq 0$ is necessary. Hence, without loss of generality, we are assumed $g(\underline{x},t) \succ 0$.The control problem is to obtain the state $\underline{x}$ for tracking a desired state $\underline{x_d}$ in the presence of model uncertainties and external disturbance with the tracking error:

$$\underline{e} = \underline{x} - \underline{xd} = \left[e, \dot{e}, \ldots, e^{(n-1)}\right]^T \in R^n \quad (2)$$

Define a sliding surface in the space of the error state as:

$$\sigma = c_1 e + c_2 \dot{e} + \cdots + c_{n-1} e^{(n-2)} + e^{(n-1)} = \underline{c}^T \underline{e} \quad (3)$$

Where $\underline{c} = \left[c_1, c_2, \ldots, c_{n-1}, 1\right]^T$ are the coefficients of the Hurwitz polynomial $H(z) = z^{n-1} + c_{n-1} z^{n-2} + \cdots + c_1$, i.e., all the roots are in the open left half-plane and z are a Laplace operator. If the initial condition $\underline{e}(0) = 0$, the tracking problem $\underline{x} = \underline{x_d}$ can be considered as the state error vector remaining on the sliding surface $\sigma = 0$ for all $t \succ 0$.

A sufficient condition to achieve this behaviour is to select the control strategy such that.

$$\frac{1}{2}\frac{d}{dt}(\sigma^2) \le -k_d |\sigma|, \qquad k_d \ge 0 \quad (4)$$

The system is controlled in such a way that the state always moves towards the sliding surface and hits it. The sign of the control value must change at the intersection between the state trajectory and sliding surface.

Consider the control problem of nonlinear systems (1), if $f(\underline{x},t)$ and $g(\underline{x},t)$, are known. The SMC input u .

$$u = \frac{1}{g(\underline{x},t)}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x},t) + x_d^{(n)} - k_d \,\mathrm{sgn}(\sigma)\right]$$

(5)

Where

Let the Lyapunov function candidate defined as : $V_1 = \frac{1}{2}\sigma^2$ (6)

Differentiating (6) with respect to time, $\dot{V}_1$ along the system trajectory as

$$\begin{aligned}\dot{V}_1 &= \sigma\dot{\sigma} \\ &= \sigma.(c_1\dot{e} + c_2\ddot{e} + \cdots + c_{n-1}e^{(n-1)} + x^{(n)} - x_d^{(n)}) \\ &= \sigma.(\sum_{i=1}^{n-1} c_i e^{(i)} + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)}) \\ &\le -k_d|\sigma|\end{aligned}$$

(7)

Hence the SMC input u guarantees the sliding condition of (4). It is obvious that in order to satisfy the sliding condition, a hitting control term u_{sw} must be added i.e. $u = u_{eq} - u_{sw}$.

Where

$$u_{eq} = g(\underline{x},t)^{-1}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x},t) + x_d^{(n)}\right]$$

(8)

$$u_{sw} = g(\underline{x},t)^{-1}.k_d \,\mathrm{sgn}(s)$$

(9)

The result in the control law (5) for a nonlinear plant, the switching-type control term u_{sw} will cause chattering problem. To solve these problems, we propose the fuzzy sliding mode control algorithm using the fuzzy logic system and the PI control law in section 3.

3 Design fuzzy sliding mode PI control

A SMC and a PI controller are combined into a single FLC to control a nonlinear system (1).

We employ PI control term in order to avoid chattering problem. The input and output of the continuous time PI controller is in the form of:

$$u_{PI} = k_p\sigma + k_i\int \sigma\, dt$$

(10)

We define a state δ which will be used on analysing the system with the PI controller as:

$$\begin{cases} \delta = \int \sigma \text{ dt when the PI controller is active} \\ \delta \text{ is constant when the PI controller is inactive} \end{cases}$$

(11)

δ_r is the reference value of δ. It is constant to cancel out the effect of the unknown disturbance d when the sliding plane is hit. Hence we have

$$\delta_r = \frac{d}{k_i} \qquad (12)$$

Where k_i is a gain to be designed later. In practice, due to integral action as given by (11), the state δ will automatically become δ_r under proper designer of the controller when the sliding plane is hit. The maximum bound δ_{rb} can be evaluated as:

$$\delta_{rb} = \frac{\max(d)}{k_i} \qquad (13)$$

With $\delta_{rb} \succ 0$. To carry out the stability analysis, we choose an upper bound for δ:

$$|\delta| \prec 10 \ \delta_{rb} \qquad (14)$$

The state error is defined as follows:

$$e_\delta = \delta_{rb} - \delta \qquad (15)$$

Then from (11).

$$\dot{e}_\delta = \begin{cases} -\sigma \text{ when the PI controller is active} \\ 0 \text{ when the PI controller is active} \end{cases} \qquad (16)$$

Hence $|e_\delta| \prec 11 \ \delta_{rb}$ (17)

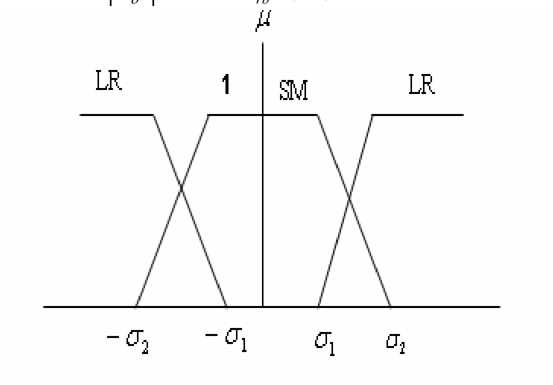

Fig.1 Membership functions

A SMC and a PI controller are combined into FLC, and then the fuzzy rules are givens as:

Rule1: ***if*** σ *is* *SM* ***then***

$$u = u_1 = g(\underline{x},t)^{-1}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x},t) + x_d^{(n)} - k_i\delta - k_p\sigma\right]$$

(18)

Rule2: ***if*** σ *is* *LR* ***then***

$$u = u_2 = g(\underline{x},t)^{-1}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x},t) + x_d^{(n)} - k_d \operatorname{sgn}(\sigma)\right]$$

(19)

Where SM and LR are membership functions as shown in figure.1. k_i, k_p and k_d are gains to be designed.

3.1 PI sub-system analysis

From Rule 1, (1) and (18) we have

$$x^{(n)} = f(\underline{x},t) + g(\underline{x},t)[g(\underline{x},t)^{-1}(-\sum_{i=1}^{n-1} c_i e^{(i-1)} - f(\underline{x},t) + x_d^{(n)} - k_i\delta - k_p\sigma)] + d(t)$$

$$\dot{\sigma} = k_i\delta - k_p\sigma + d(t)$$
$$= k_i(e_\delta - \delta_{rb}) - k_p\sigma + d(t)$$

(20)

Hence, from (15),

$$\dot{\sigma} = k_i e_\delta - k_p\sigma \ (21)$$

$$\begin{bmatrix} \dot{e}_\delta \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ k_i & k_p \end{bmatrix}\begin{bmatrix} e_\delta \\ \sigma \end{bmatrix} = A\begin{bmatrix} e_\delta \\ \sigma \end{bmatrix} \ (22)$$

We can define a symmetric positive definite matrix Q, if the real part of eigenvalues of A. Hence we can be found a symmetric positive (16) definite matrix P, satisfying the following equation:

$$A^T P + PA = -Q \ (23)$$

Also from (23), we have define a common Lyapunov function V such that

$$V = [e_\delta \quad \sigma]\begin{bmatrix} P_1 & P_2 \\ P_2 & P_4 \end{bmatrix}\begin{bmatrix} e_\delta \\ \sigma \end{bmatrix}$$

Where

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_4 \end{bmatrix} \ (24)$$

Obviously from (23) and (24), $\dot{V} \le 0$ such that this PI sub-system is stable.

3.2 SMC sub-system analysis

With reference to Fig.1, the are two sub-region defined by $|\sigma| \ge \sigma_2$ and $\sigma_1 \le |\sigma| \le \sigma_2$. From (24), we have

$$\dot{V} = P_1 e_\delta \dot{e}_\delta + P_2\sigma\,\dot{e}_\delta + P_2 e_\delta\dot{\sigma} + P_4\sigma\dot{\sigma} \ (25)$$

To ensure the system stability of SMC sub-system under the lyapunov function of (24) that $\dot{V} \le 0$ in this sub-system, we divide study in two sub-regions.

Case1: $|\sigma| \ge \sigma_2$

From (16), $\dot{e}_\delta = 0$. Then (25) can be reduced to

$\dot{V} = P_2 e_\delta \dot{\sigma} + P_4 \sigma\dot{\sigma}$ (26)

It can be proved that $\dot{V} \le 0$ if kd, k_i and k_p satisfied the following three conditions:

$$k_d \succ k_i \delta_{rb} \quad (27)$$

$$\sigma_2 \succ 11\delta_{rb} \quad (28)$$

$$|P_2| \prec P_4 \quad (29)$$

Proof 1:

Form Rule 2, (1) and (3), we have

$\sigma\dot{\sigma} = \sigma(d - kd\ sign(\sigma))$ (30)

$\le |\sigma||d| - kd|\sigma|$

From (27) and (14),

$kd \succ k_i \delta_{rb} = \max(d)$

Hence $\sigma\dot{\sigma} \prec 0$ (31)

Also, since $|\sigma| \ge \sigma_2$. From (28) and (17), then $|\sigma| \succ |e_\delta|$.

Consider (26),

$$\begin{aligned}
\dot{V} &= P_2 e_\delta \dot{\sigma} + P_4 \sigma\dot{\sigma} \\
&\le |P_2 e_\delta \dot{\sigma}| + P_4 \sigma\dot{\sigma} \\
&= |P_2||e_\delta||\dot{\sigma}| + P_4 \sigma\dot{\sigma} \\
&\prec |P_2||\sigma\dot{\sigma}| - P_4 \sigma\dot{\sigma} \\
&= |P_2||\sigma\dot{\sigma}| - P_4 |\sigma\dot{\sigma}| \\
&\prec 0
\end{aligned}$$

Case2: $\sigma_1 \le |\sigma| \le \sigma_2$

From (16), $\dot{e}_\delta = -\sigma$ .then (25) becomes

$\dot{V} = -P_1 e_\delta \sigma - P_2 \sigma^2 + P_2 e_\delta \dot{\sigma} + P_4 \sigma\dot{\sigma}$ (32)

To ensure (32), is can be negative if the following conditions

$kd = \frac{2m}{P_4}$ (33)

Where, $m = 11 P_1 \delta_{rb} + |P_2|\sigma_2 + P_4 k_i \delta_{rb}$.

$\sigma_1 = 11\delta_{rb}|P_2|\left(\frac{k_i}{m} + \frac{2}{P_4}\right)$ (34)

Proof 2:

From $\sigma_1 \le |\sigma| \le \sigma_2$, $\dot{e}_\delta = -\sigma$ .then (25) becomes:

$$\begin{aligned}
\dot{V} &= -P_1 e_\delta \sigma - P_2\sigma^2 + P_2 e_\delta \dot{\sigma} + P_4 \sigma\dot{\sigma} \\
&= -P_1 e_\delta \sigma - P_2\sigma^2 + P_2 e_\delta \dot{\sigma} + P_4 \sigma d - P_4 kd|\sigma| \\
&\le |\sigma|\max(-P_1 e_\delta \sigma - P_2\sigma - P_4 kd|\sigma|) - P_4 kd|\sigma| + P_2 e_\delta \dot{\sigma}
\end{aligned}$$

Let $m = 11 P_1 \delta_{rb} + |P_2|\sigma_2 + P_4 k_i \delta_{rb}$ and $kd = \frac{2m}{P_4}$

From (13) and (17),

$$\begin{aligned}
&11 P_1 \delta_{rb} \succ P_1|e_\delta| \\
&P_4 k_i \delta_{rb} = \max(P_4 d) \\
&\sigma_2 \succ |\sigma| \\
&m = 11 P_1 \delta_{rb} + |P_2|\sigma_2 + P_4 k_i \delta_{rb} \\
&\quad \succ P_1|e_\delta| + |P_2 \sigma| + \max(P_4 d) \\
&\quad \ge \max(-P_1 e_\delta - P_1\sigma + P_4 d)
\end{aligned}$$

Then (32) becomes

$$\begin{aligned}
\dot{V} &\le m|\sigma| - 2m|\sigma| + P_2 e_\delta \dot{\sigma} \\
&= -m|\sigma| + P_2 e_\delta \dot{\sigma}
\end{aligned}$$

A sufficient condition for $\dot{V} \le 0$ is

$|\sigma| \succ \frac{\max(P_2 e_\delta \dot{\sigma})}{m}$ (35)

Since $|\sigma| \succ \sigma_2$, (35) can be satisfied by letting

$$\begin{aligned}
\sigma_1 &= \frac{\max(P_2 e_\delta \dot{\sigma})}{m} \\
&= 11\delta_{rb}|P_2|\frac{\max(\dot{\sigma})}{kd} \qquad \text{(from (17))} \\
&= 11\delta_{rb}|P_2|\frac{\max(d - \frac{2}{P_4}kd\ \mathrm{sgn}(\sigma))}{kd} \\
&= 11\delta_{rb}|P_2|(\max(\frac{d}{kd}) + \frac{2}{P_2}) \\
&= 11\delta_{rb}|P_2|(\frac{k_i\delta_{rb}}{kd} + \frac{2}{P_4}) \qquad \text{(from (13))}
\end{aligned}$$

which gives condition (34). In conclusion, conditions (33) and (34) ensure that $\dot{V} \le 0$.

Theorem 1:

For the dynamic model of (1), if the fuzzy control law is chosen as (18) and (19), and we firstly need to select ki, kp, Q to satisfy (23) and the parameters kd, σ_1 and σ_2 are chosen as in (27), (28) and (34). Then, both SMC and PI sub-system control system is stable in the sense lyapunov approach. The convergence of the tracking error which can still guarantee and the chattering problem of control is eliminated.

4 Simulations example

Consider a nonlinear system of the form [9] as follows:

$$x^{(n)} = f(\underline{x}, t) + g(\underline{x}, t)u + d(t)$$
$$y = x$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$,

$f(\underline{x}, t) = \begin{bmatrix} x_2 \\ -0.1x_2 - x^3{}_1 + 12\cos(t) \end{bmatrix}$,

$g(\underline{x}, t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, d(t) = \begin{bmatrix} 0 \\ 0.1 rand(1,20) \end{bmatrix}$.

The control objective is to regulate y to y_r with $y_r(t) = 2\sin(10^{-2}t)$ .consider the fuzzy control law as described in (18) and (19). Let $ki = 2, kp = 5.$,

and $Q = \begin{bmatrix} -2 & 0 \\ 0 & -400 \end{bmatrix}$ from (23) we have $P1 = 85.4$, $P2 = -1$ and $P4 = 40.2$. $\max(d) = 0.1$.

35)

Also from (18) and (35), $\delta_{rb} = 0.05$. Besides, let $\sigma_2 = 0.6 \succ 11\delta_{rb}$ which satisfies (28). Then from (33) and (34), $kd = 2.567$ and $\sigma_1 = 0.0284$. $x_1(1) = 1; x_2(1) = 0$.

From Fig.5, it can be seen that the one using fuzzy sliding mode PI controller overcomes the chattering problem of control and is eliminated. However, as shown in Fig.3, chattering exists of traditional SMC.

In Fig.4, the stability convergence and robustness. Hence, the high performance can be is achieved.

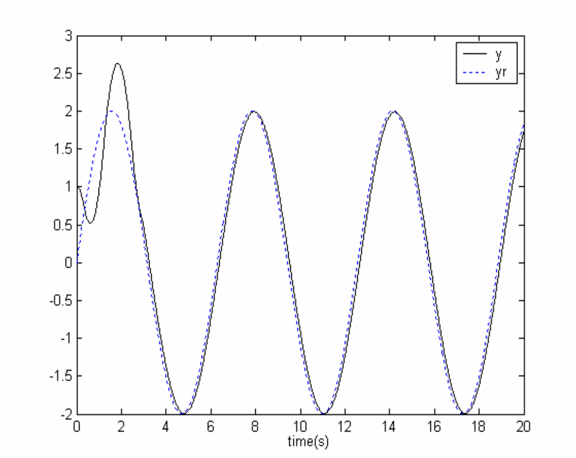

Fig.2 The behaviour of $y(t)$ and $y_r(t)$ with traditional SMC

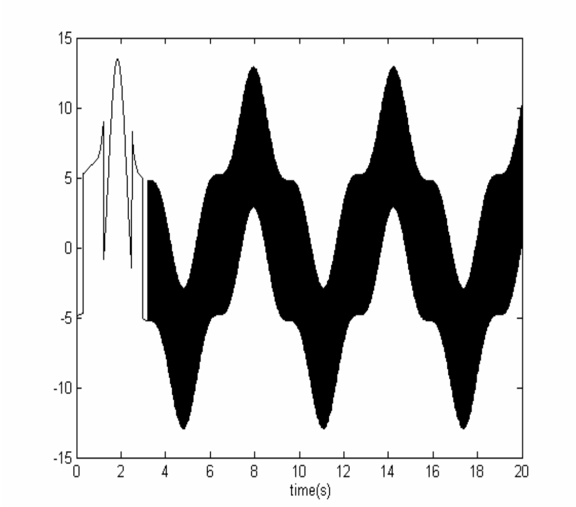

Fig.3 Sliding mode control signal

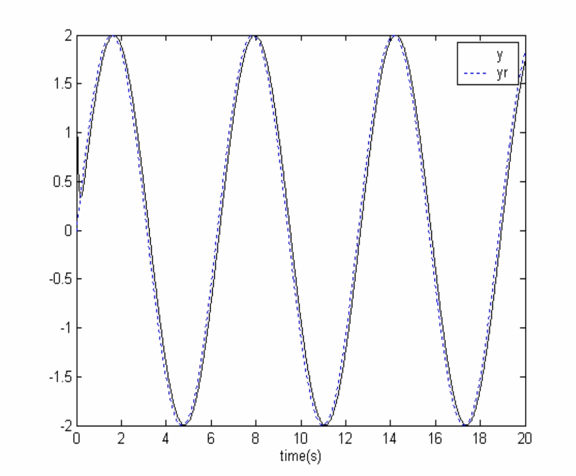

Fig.4 the behaviour of $y(t)$ and $y_r(t)$ with fuzzy SM and PI control

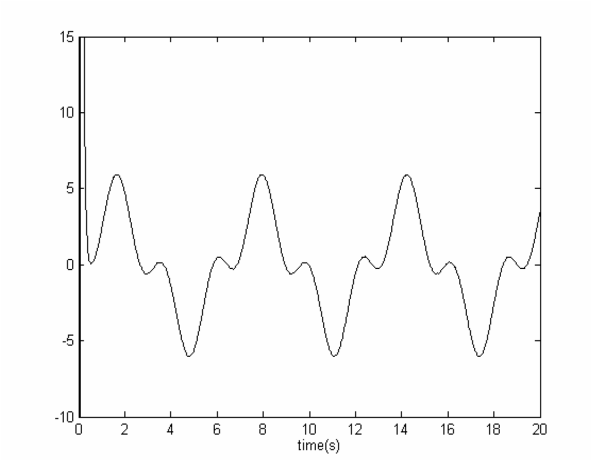

Fig.5 Fuzzy SM and PI control signal

5 Conclusion

This paper proposes an approach to combine a sliding mode and PI controller using a fuzzy logic for nonlinear systems with external disturbance. When, only, the sliding mode technique is used the chattering exists due to the presence of discontinuous control action. The proposed fuzzy logic control law overcomes the chattering problem and high performance can be achieved. The controller does not need the accurate system mathematical model, so it is relatively easy to design. The simulation results verify the validity of the proposed fuzzy logic controller.

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