# Market Splitting Algorithm for Congestion Management in Electricity Spot Market

Marta Marmiroli*, Masahiko Tanimoto+,Yukitoki Tsukamoto+, Ryuichi Yokoyama*
*Department of Electrical Engineering
Tokyo Metropolitan University
1-1 Minami Osawa, Hachioji-shi, Tokyo
JAPAN
+Mitsubishi Electric Corporation
JAPAN

*Abstract:* - The market splitting method is a well-known approach to congestion management for day ahead spot market. The result of a market split gives a price signal that clearly informs the market participants about the congested areas. The method is based on a simple model and it is easy to understand for market participants. This paper introduces an algorithm to solve the market splitting problem for complex radial networks. The proposed algebraic approach is an alternative to the linear programming or other iteratively approaches. The algorithm was tested on large-scale systems and its high performances were proved.

*Key-Words*: - spot market, auction, congestion management, market splitting, price signal, trading

## 1 Introduction

The deregulation of the electricity industry and the consequent open access to the transmission network forced the power system developers and operators to face with the problem of congestion.

The problem of transmission congestion becomes particularly relevant in case of interconnections among countries and furthermore power system control areas in general. It is a matter of fact that interconnections among control areas were developed with the objective to increase the stability of the power system. The interconnected networks may reciprocally support frequency control and emergency procedures. In short, the capacity of the interconnection was designed for the previous described scope.

The deregulation of the market and the increase of the number of transmission users caused a change in the utilization of the interconnections. Market players dispatch generating resources with the objective to increment their own economic benefit. For this reason, it is common that the transfer capacity requested to wheel cheap electricity from a control area to another one exceeds the capability of the interconnections between the two areas.

In case of insufficient transfer capacity among control areas, it is necessary to introduce rules to manage the congestion on the transmission lines.

A congestion management method has to be able to relief the overflow on the transmission lines and, at the same time, to give a strong economical signal to the transmission users.

Several power exchanges around the world adopted a market split approach to manage congestion in the spot market. In case overflow on an interconnection line between control areas, the spot market is divided into separated sub-markets, and a different price is assigned to each sub-market [1]. The market price is lower in the sub-market that is supposed to export electricity than the price in the sub-market receiving electricity.

Several algorithm and methods to solve the market split problem are published in literature [2][3].

This paper proposes a new efficient and simple algorithm to individuate the sub-markets that have to be separated to relief congestion.

After an introduction of the auction rules for electricity spot market in section 2, the paper describes the market split concept in section 3, and the algorithm to identify the congested sub-market in section 4. In Section 5, a numerical example is illustrated. After some performance evaluation in section 6, conclusions and open problems are discussed in section 7.

## 2 Auction rules in markets

The single price auction mechanism is often utilized to set the market price in an electricity day-ahead market. The objective is to find the single market price that maximizes the quantity of electricity exchanged in the market and the consequent quantity that may be contracted by each player. The price is built using the market participants' order prices.

An order is defined by the price-quantity pair and the position (offer or bid). An offer order is a will to sell a certain quantity of electricity at a price equal or higher than the offer price. On the contrary, a bid order is a will to buy a certain amount of electricity at a price equal or lower than the bid price.

Market participants' orders are collected until a predetermined time called gate close. After the gate close, the market operators process all the order to find the market price and the contract quantity for each order.

In a single price auction, the market price and the contract quantity for each order have to be set respecting the following conditions described in equation (1) and (2).

The conditions for an offer order are:

$$\begin{array}{ll} price_{offer} < price_{market} & quantity_{contract} = quantity_{offer} \\ price_{offer} > price_{market} & quantity_{contract} = 0 \\ price_{offer} = price_{market} & 0 \le quantity_{contract} \le quantity_{offer} \end{array} \quad (1)$$

,where $price_{offer}$ and $quantity_{offer}$ are the price-quantity pair of the offer order, $price_{market}$ is the price for the market and $quantity_{contract}$ is the quantity that the player sell.

In a similar way, the condition for a bid are:

$$\begin{array}{ll} price_{bid} > price_{market} & quantity_{contract} = quantity_{bid} \\ price_{bid} < price_{market} & quantity_{contract} = 0 \\ price_{bid} = price_{market} & 0 \le quantity_{contract} \le quantity_{bid} \end{array} \quad (2)$$

In a word, the formula expresses the concept that any seller will not sell at a price that is lower than his offer price, and any buyer will not buy at a price higher than his bid price.

The objective of maximization of the exchange quantity does not ensure the uniqueness of the solution. For this reason it is necessary to introduce additional rules to set the final market solution. In this paper it will be assumed that in case of multi-solution the lowest price that satisfies the previous described constraints is selected as the market price.

In some cases, at the market price, the total offer quantity may not be equal to the bid one. The excess of quantity in the offer or bid side is defined as a surplus. The surplus is distributed among order with the price equal to the market price in a proportional way.

The authors implemented a sequential approach to calculate the market price and the contract quantity for each order. The method can be described in 4 steps as bellows:

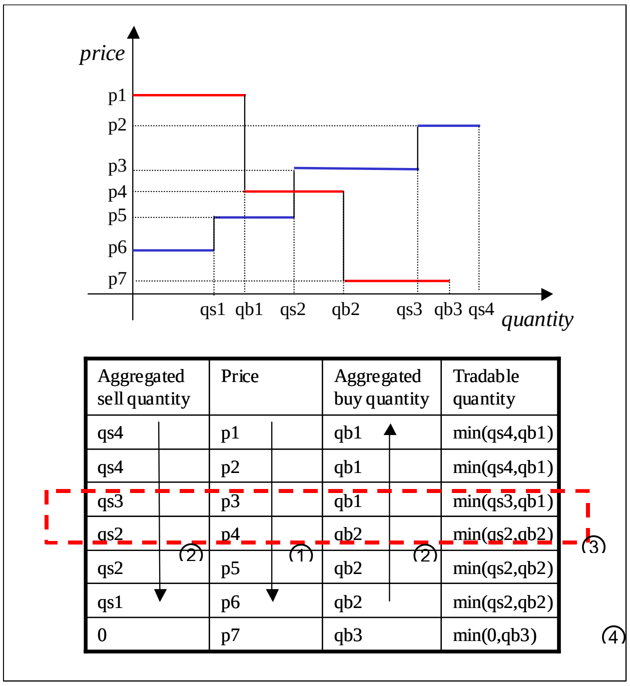
1) sort all the bid and offer prices from the highest to the lowest.
2) calculate the aggregated sell and buy quantity for each price.
3) calculate the tradable quantity for each price as the minimum between the aggregated sell and buy quantity
4) select the price that correspond the maximum tradable quantity. If two or more prices have the same maximum tradable quantity, the price with the smallest surplus (aggregated sell/buy quantity) is selected.

Once the price and the total trade are individuated, the contract quantity for each player is calculated using the condition expressed in (1) and (2).

Fig. 1 is a graphical representation of the sequential process results. The market price, which is a solution of the described algorithm, satisfies the condition to be the cross point between bids and offers curves.

The proposed method is a practical algorithm that can be easily implemented on a computer system.

Similar algorithms are implemented and performed daily in commodities and stocks exchanges [4].



| Aggregated sell quantity | Price | Aggregated buy quantity | Tradable quantity |
|---|---|---|---|
| qs4 | p1 | qb1 | min(qs4,qb1) |
| qs4 | p2 | qb1 | min(qs4,qb1) |
| qs3 | p3 | qb1 | min(qs3,qb1) |
| qs2 | p4 | qb2 | min(qs2,qb2) |
| qs2 | p5 | qb2 | min(qs2,qb2) |
| qs1 | p6 | qb2 | min(qs2,qb2) |
| 0 | p7 | qb3 | min(0,qb3) |

Fig. 1: Market price calculation

## 3 Market splitting under congestion

One of the main differences between electric energy and the other commodities is that electricity has to be transferred trough transmission lines. However, the auction algorithm, described in the previous section, does not consider any constraints related with the transmission network. In reality, an order, in addition to the quantity and the price, includes also another important information: the location. From a view point of physical power system, the location of an order can be considered as:

- the source point in case of a sell order
- the sink point in case of a buy order.

It may happen that, the solution from the previous described algorithm may not respect some of the constraints on the transmission lines. In this case it is necessary to split the market into sub-markets with different prices to ensure that the condition expressed by equations (1) and (2) are respected and the transmission lines are optimally utilized.

Fig. 2 represents a simple example with 2 areas connected by a inter-tie line. In Area 1, one player offers a quantity of electricity equal $Q_{offer}$, in area 2 a player bids a quantity of electricity equal $Q_{bid}$. The trade quantity $Q_{flow}$ is the minimum between $Q_{offer}$ and $Q_{bid}$. If the trade quantity $Q_{flow}$ exceeds the transmission capacity $Q_{limit}$, the solution is not acceptable. In this case, the market has to be split into two sub-markets and the auction calculation executed again for each of them.

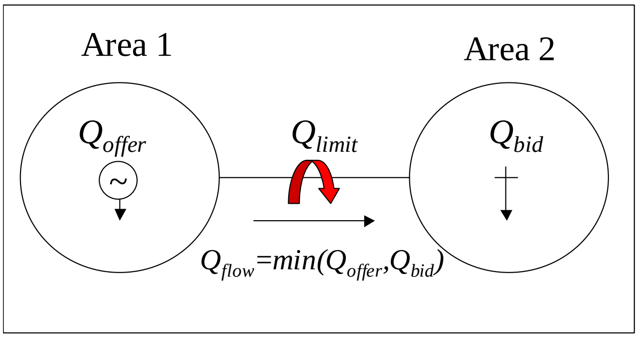


Fig. 2: Congested market example

To guarantee the best utilization of the transmission line, the concept of virtual offer and bid is introduced. The price of a virtual offer is the lowest price in the market and the offer quantity is equal to the $Q_{limit}$. The location of the virtual offer is the importing area. Oppositely, the price of a virtual bid is the highest price in the market and the bid quantity is equal to $Q_{limit}$, the location is the exporting area. Fig. 3 shows how the market may be split introducing a virtual bid and offer.

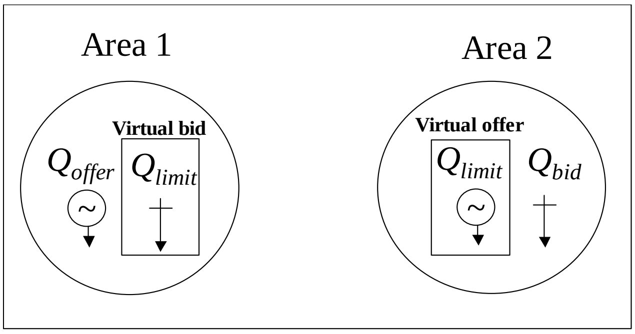


Fig. 3: Market split

After the introduction of the virtual bid and offer, the two sub-markets, area 1 and 2 are independently treated, and the auction algorithm can be performed to calculate the market price and the total trade quantity for each of them.

As represented in Fig. 4, the price $P_{area1}$ in area 1 becomes lower than the market price $P_{market}$ before the market splitting because $Q_{limit}$ is smaller than $Q_{flow}$. Oppositely the price $P_{area2}$ in area 2 becomes higher.

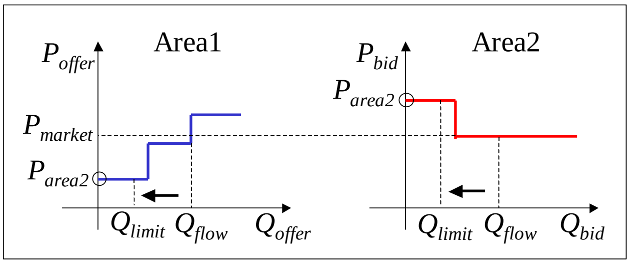


Fig. 4: Prices in the areas

Once the auction algorithm in section 2 is performed and the potential transaction quantity for each order is calculated, the flow on each transmission line can be calculated through a simple power flow calculation. In addition, the net difference between satisfied sell and buy order inside an area is set as in equation (3).

$$p_i = \sum_{sell} Q_{sell}^{satisfied} - \sum_{buy} Q_{buy}^{satisfied} \tag{3}$$

## 4 Congested sub-market identification

In the previous section, the market split concept is described for a market composed of two areas. In this case, the congested path and the location of the market split can be easily recognized. The problem becomes more complex in case of three or more areas. Fig. 5 is an example of market with four areas.

It is not trivial to find the location of the market split that maximizes the trading quantity under the rules described in section 2.

It seems that the inter-tie of area 2-3 is the mostly congested with an overflow of 115MW. In reality, however, solving the congestion on inter-tie of area 1-2 and 4-2 will automatically relieves the congestion on the tie-line area 2-3 without any additional operations to split the market between area 2-3.

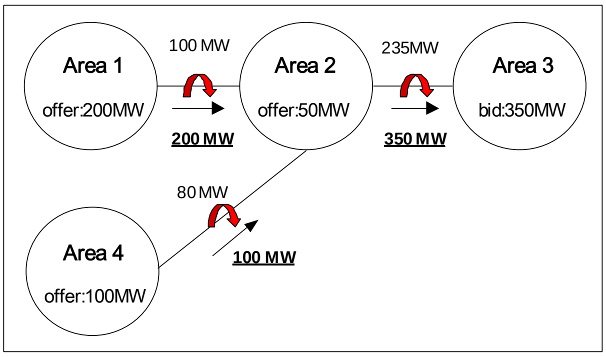


Fig. 5: Market with 4 areas and 3 congested interconnections

To individuate the position of the splits, the authors develop a new algorithm.

### 4.1 Definition of area type

First of all, the concept of ***import***, ***export***, and ***neutral*** constrained area is introduced. A parameter $t_i$, which is called congestion index, is calculated for each area.

$$t_i = p_i + \sum_j \min(Q_j^{limit}, Q_j^{flow}) - \sum_k \min(Q_k^{limit}, Q_k^{flow}) \quad (4)$$

,where $i$ is the index of the area, $p_i$ is the net quantity of the area $i$, which is bigger than 0 if the total amount of contracted quantity for offers is bigger than the total amount for bids, $j$ is the index for inter-tie with electricity flowing into the area, and $k$ with the electricity flow from the area, $Q^{limit}$ is the maximum capacity of the inter-tie in the direction of the flow and $Q^{flow}$ is the quantity that will flow on the line as a result of the auction calculation without any constraint on the network.

An area is defined congested in ***import*** (I) if $t$ is smaller than 0, congested in ***export*** (E) if the congestion index $t$ is bigger than 0, and further ***neutral*** (N) in case of $t$ equal 0.

From equation (4), it is easily understood that an area, which is defined as congested in ***export***, is an area for which the importing quantity of electricity plus the net quantity of the area itself is bigger than the quantity that it can transfer or export to others area. Similarly for an ***import*** area the net quantity of the area itself summed to the quantity that may transfer to other areas is lower than the quantity that it is able to import due to the transmission constraints. In Figure, 6 examples of each kind of areas are described.

It can be demonstrated that in a system if one or more areas are ***export*** areas, then at least one area is an ***import*** one and vice versa.

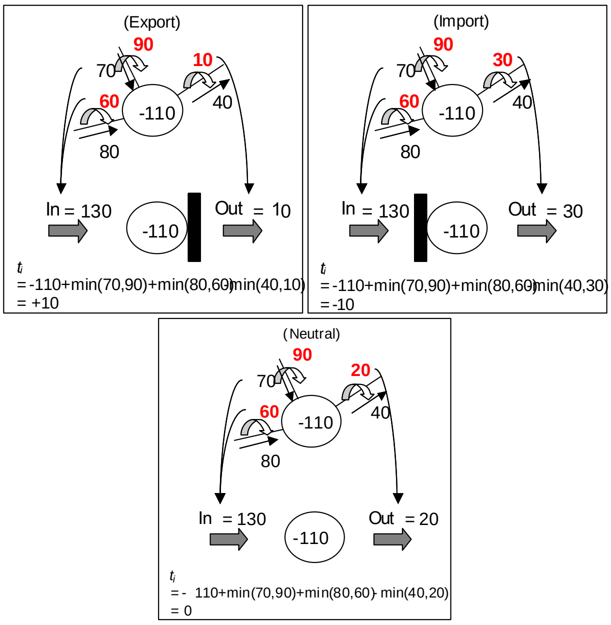


Fig. 6: Example of ***export***, ***import*** and ***neutral*** areas

### 4.2 System reduction and splitting rules

In this section, the rules for splitting are introduced. Two areas interconnected by the same line should be split into two markets if the one area is export constrained and the other is import constrained and the electricity is flowing from the ***export*** area to the ***import*** one.

The rule can be explained intuitively with the help of a graphical support. The possible combinations of areas as node of the same interconnection are listed in Fig. 7. An ***import*** area is represented with a round mark and a briar on the left that means the impossibility to get all the electricity required, an ***export*** area by a round mark and briar on the right that means the impossibility to transfer all electricity that is required. A ***neutral*** area does not have any briar. The arrow between the areas represents the direction of the power flow on the interconnection line. From the figure, it can be guessed that in the case of ***Export-Import*** combination, the split of the two areas in two markets will has as consequence a reduction of the power flowing between the two areas. The ***export*** area will then export less power and the ***import*** area, import less power reducing, in this way the congestion condition of both two areas. On the contrary, in any other case in Fig. 7, the split of the two areas will not be of benefit for both or neither for one of the area (for example in the ***import-export*** combination).

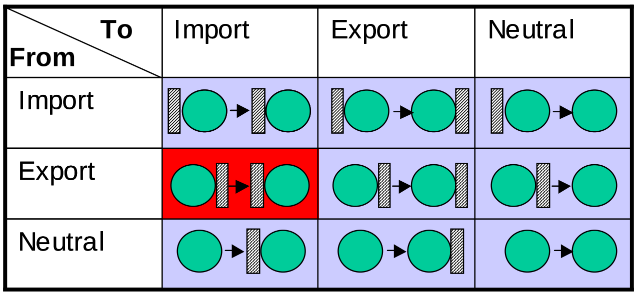


Fig. 7: The matrix of the splitting rules

The authors introduced a system reduction algorithm with the objective to recognize easily the location of the ***export-import*** interconnection that has to be split. For each interconnection in the system, if the areas that delimitated the interconnection are of the same kind, they will be merged in one area. A ***neutral*** area has to be merged with an ***export*** area if it is connected with one, otherwise it can be merged with an ***import*** area. ***Import-export*** combination has also to be reduced in a single area.
In Fig. 8, the example of Fig. 5 is reduced to a simple system.

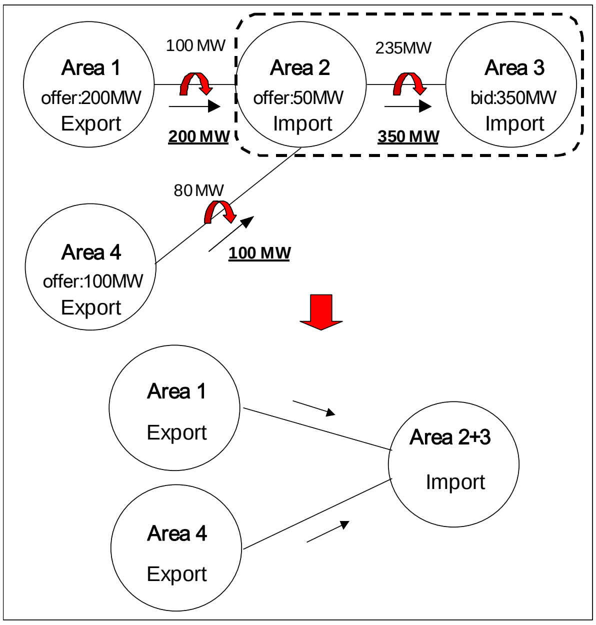


Fig. 8: Example of system reduction

### 4.3 Algorithm flow

After the calculation of the sub-market price and possible contract quantity for each order, it is necessary to check the power flow on the inter-tie among the sub-markets themselves. It may happen that one or more inter-ties are still congested. For this reason, it is necessary to execute iteratively the algorithm until the power flow on each inter-tie is equal or lower to the available line.
Since the maximum number of sub-market is equal to the number of areas that define the market, the maximum number of iteration is equal to the total number of areas.
The entire algorithm implemented to solve the spot market including the transmission constraints is described in the flow chart in Fig. 9.

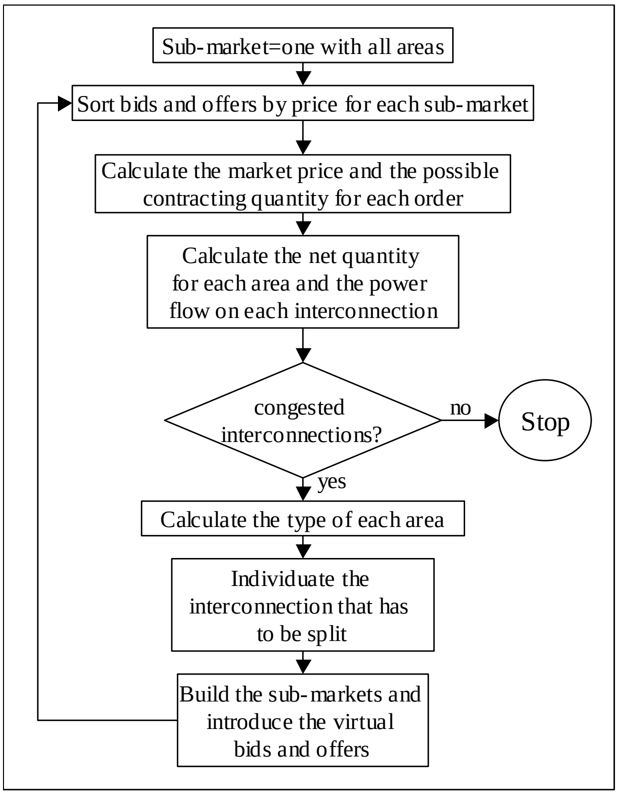
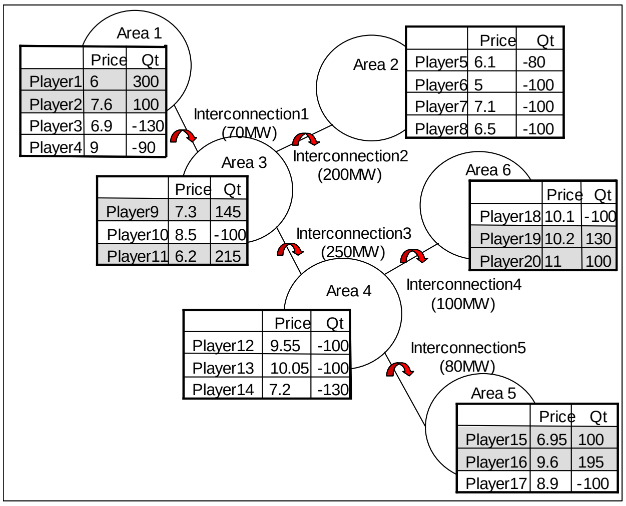


Fig. 9: The algorithm flow chart

## 5 Numerical Application

In this section, an example for a system with six areas, five inter-ties and twenty players is presented. Since the number of areas and players is limited, the solution can be performed easily with a spreadsheet.
In Fig. 10, the player orders, and the capacity for each interconnection are displayed. A negative quantity in an order means that the order is an offer (sell), on the contrary a positive quantity is associated to a bid order (buy).

The available capacity for each inter-tie is between 70 MW and 250MW shown as Fig.10.



| | Price | Qt |
|---|---|---|
| Player1 | 6 | 300 |
| Player2 | 7.6 | 100 |
| Player3 | 6.9 | -130 |
| Player4 | 9 | -90 |

| | Price | Qt |
|---|---|---|
| Player5 | 6.1 | -80 |
| Player6 | 5 | -100 |
| Player7 | 7.1 | -100 |
| Player8 | 6.5 | -100 |

| | Price | Qt |
|---|---|---|
| Player9 | 7.3 | 145 |
| Player10 | 8.5 | -100 |
| Player11 | 6.2 | 215 |

| | Price | Qt |
|---|---|---|
| Player12 | 9.55 | -100 |
| Player13 | 10.05 | -100 |
| Player14 | 7.2 | -130 |

| | Price | Qt |
|---|---|---|
| Player15 | 6.95 | 100 |
| Player16 | 9.6 | 195 |
| Player17 | 8.9 | -100 |

| | Price | Qt |
|---|---|---|
| Player18 | 10.1 | -100 |
| Player19 | 10.2 | 130 |
| Player20 | 11 | 100 |

Fig. 10: Example system

At first the auction is performed for the 20 orders without any consideration of the line constraints. The price for this first auction is 7.3 ¥/kWh for all the system. Offer orders with price lower than 7.3 ¥/kWh and bid order with price higher than 7.3 ¥/kWh are cleared. Fig. 11 represents the power flow on each interconnection and the type of each area for the solution of this first auction.

It is easily recognizable that several interconnections are congested. Area 2 is exporting cheap electricity to area 6 and 5 through area 3 and 4. After the reduction process (Area 3 and 4 are firstly reduced in one ***export*** area, then area 1 and 2 are integrated with area 3 and 4), three sub markets compose the system; the first sub market contains area 1, 2, 3 and 4, the second sub market covers area 5, and the third contains area 6. The interconnections of areas 4-5 and areas 4-6 are delimitated by ***export-import*** combination. The market has to be split in the three sub markets to relieve congestion.

In Fig. 12, the solution for the three sub markets is presented. The prices in area 5 (8.9 ¥/kWh) and area 6 (10.2¥/kWh) increase as consequence of the congestion relief. On the contrary, the price in the sub market composed with area 1, 2, 3 and 4 decreases to 7.1¥/kWh.

In the sub-market composed with area 1, 2, 3, and 4 there is still congestion. The interconnection between area 2 and 3 is in overflow. It is then necessary to iterate the process: area 2 has to be split from area 3, 4 and 5, and the auction has to be preformed again.

The final solution (Fig. 13) shows the presence of 4 sub markets. The sub-market of area 6 has the highest market price, and on the contrary, area 2 has the lowest one.

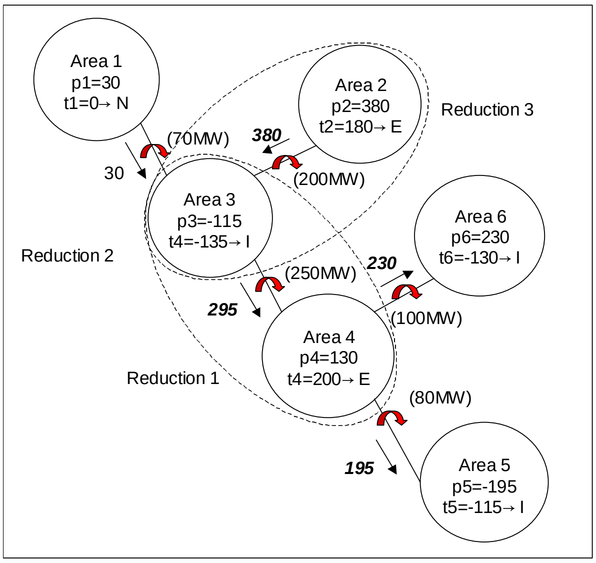


Fig. 11: Power Flow and Area Type

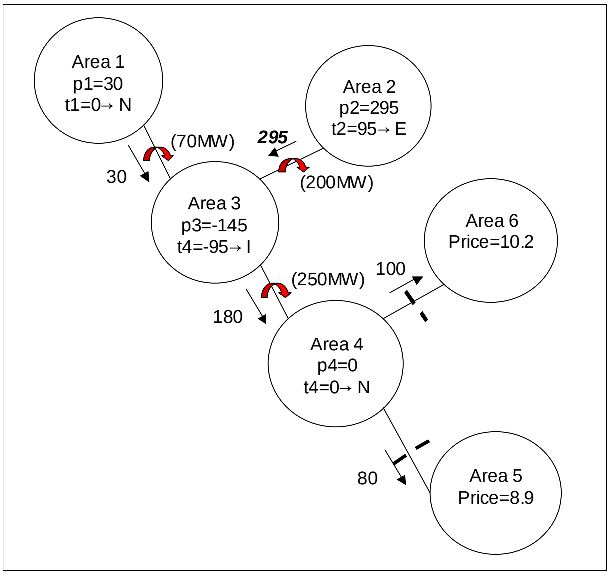
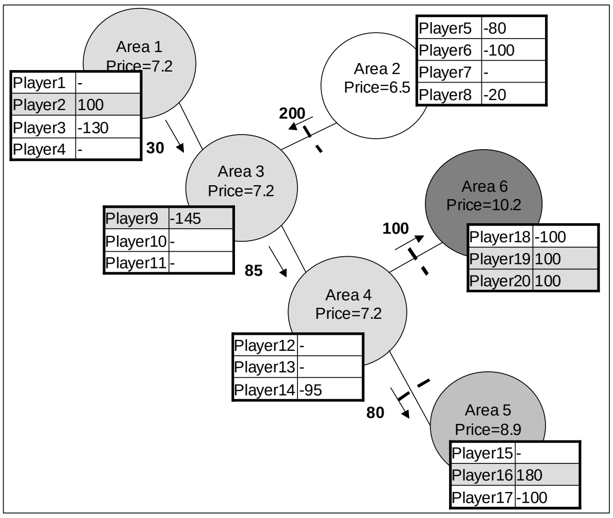


Fig. 12: Solution after the first split



| | |
|---|---|
| Player1 | - |
| Player2 | 100 |
| Player3 | -130 |
| Player4 | - |

| | |
|---|---|
| Player5 | -80 |
| Player6 | -100 |
| Player7 | - |
| Player8 | -20 |

| | |
|---|---|
| Player9 | -145 |
| Player10 | - |
| Player11 | - |

| | |
|---|---|
| Player12 | - |
| Player13 | - |
| Player14 | -95 |

| | |
|---|---|
| Player15 | - |
| Player16 | 180 |
| Player17 | -100 |

| | |
|---|---|
| Player18 | -100 |
| Player19 | 100 |
| Player20 | 100 |

Fig. 13: Final Solution

## 6 Tests and Performances

The algorithm was translated in a Fortran program. The program was tested both on small problems and large problems. Using an Itanium 1GHz CPU, the calculation for a power system with 16 areas, 15 interconnections and 80,000 orders was performed in 0.01 sec.

## 7 Conclusions

In this paper a new algorithm to identify the location where the market has to be split due to the transmission congestion was introduced. The method is simple, reproducible and always converges to a unique solution. The algorithm was tested on complex problems and the test results demonstrated the high performance of the algorithm.

The authors developed and implemented the method for the Japanese Power Exchange that is in operation from April 2005[5].

The proposed algorithm is only applicable to radial networks. This constraint fits with the Japanese interconnection topology but can be an obstacle in the implementation of a similar market in other countries. The authors are now investigating the possibility to solve mesh networks with the same method.

*Reference:*

[1] JEPX ‘Trading operation guide’

[2] Sawa et al. T. “Market Splitting Single Auction in Wholesale Electric Power Exchange” F408-A1 6-044 IEEJ General Meeting 2004 March 13-17 Tokyo (in Japanese)

[3]Wester E. “Transmission pricing in Norway” in *Electricity Pricing and Technology*, 1995 Kluwer Academic Publisher

[4] ASX. “Calculating opening and closing prices” http://www.asx.com.au/investor/education/basics/open_Close.htm

[5] Tanimoto M. et al “Japan Electric Power Exchange System” Mitsubishi Giho Vol.80 No. 3 2006 (in Japanese)