Simulation Analysis for the Bullwhip Effect in Supply Chain Model

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Abstract: - This paper introduces the confidence interval estimate for measuring the bullwhip effect, which has been observed across most industries. Calculating a confidence interval usually needs the assumption about the underlying distribution. Bootstrapping is a non-parametric, but computer intensive, estimation method. In this paper, a simulation study on the behavior of the 95% bootstrap confidence interval for estimating bullwhip effect is made. Effects of sample size, autocorrelation coefficient of customer demand, lead time, and bootstrap methods on the 95% bootstrap confidence interval of bullwhip effect are presented and discussed.

Key-Words: - Bullwhip Effect, Bootstrap, Autocorrelation, Supply Chain

1 Introduction

The bullwhip phenomenon (or effect) referring to increase of demand variability further upstream in the supply chain has been observed or recognized in industry for a long time. The phenomenon can potentially cause instability in the supply chain and increase the cost of supplying goods to customer demand. The first recognition of this phenomenon can be traced back to Forrester (1958). Other earlier papers making a major contribution to understanding of bullwhip phenomenon include Blanchard (1983), Blinder (1982, 1986), Kahn (1987), and Stermn (1989). Recently, Lee et al. (1997a, b) popularized the term “bullwhip effect”, and analyzed four potential sources of the bullwhip effect: demand signal processing, rationing game, order batching, and price variations through simple mathematical models, which focuses on the retailer-supplier relationship and considers a first-order autoregressive (abbreviated as AR (1)) demand process. Lee et al. (1997a, 2004) identified, moreover, the bullwhip effect as a natural consequence of demand signal processing, which refers to the situation where demand is non-stationary and one uses past demand information to update forecasts. Chen et al. (1999, 2000a, b) are early papers that link forecasting method with the bullwhip effect. Using an AR (1) demand process similar to Lee et al. (1997a), they quantify the magnitudes of the bullwhip effect resulting from moving averages, exponential smoothing, and other forecasting methods. Zhang (2004) continued the study of Chen et al. and derived the bullwhip effect measure for minimum mean-squared error forecasting method.

The Bullwhip effect is generally regarded as a performance index to respond to the instability in a serial supply chain. When applying the proposed measures to measuring the bullwhip effect in actual practice, lead time and autocorrelation coefficient of demand process should be known. Lead time is the elapsed time between releasing an order and receiving it. In many literatures, lead time is regards as a controllable decision variable and can be decomposed into several components, each having a crashing cost for the respective reduced lead time (Pan and Hsiao, 2005). However, the exact autocorrelation coefficient of demand process is usually unknown because the number of observations collected from customer demand process is finite during a limited time horizon As a result, it is replaced by a sample autocorrelation coefficient, and this gives the measured bullwhip effect a point estimate of the exact bullwhip effect.

In addition to the point estimator, interval estimation is important for the statistical inference on bullwhip effect of a particular supply chain. In this paper, we focus the investigation on the confidence interval of bullwhip effect. Calculation of the confidence interval for the bullwhip effect usually needs aware of the underlying distribution, but it could be difficult to know or obtain. Thus, we develop the confidence interval based on the bootstrap principle. Bootstrapping introduced by Efron (1979, 1981, 1985) is a statistical method, which is non-parametric or free from assumptions of distribution.

2 A Simple Supply Chain Model

2.1 Replenishment policy

Assume a retailer-supplier system, where a single item and order-up-to S inventory policy are considered. To simplify the model, excess inventory can be returned without cost, and excess demand is backlogged. The timing of events during a replenishment period is as follows: At the beginning of each period t, the retailer order a single item of quantity q_t from the supplier. There is a lead time of L periods between ordering and receiving the goods. After that, the goods ordered L periods ago arrived. Finally, demand is realized and the available stock is used to meet the demand. A serially correlated demand process the retailer faces is assumed to follow the AR (1) model as,

$$D_t = d + \rho D_{t-1} + \varepsilon_t \tag{1}$$

where D_t is the demand in period t, ρ and d are constants such that $d > 0$ and $-1 < \rho < 1$, and ε_t is normally distributed with zero mean and variance σ^2 .(Negative demands are negligible when σ significantly smaller than d .)

The exact bullwhip effect

Given the unit holding cost, unit shortage penalty cost, and the unit ordering cost, Lee et al. (1997a) formulated the cost minimum problem to optimize retailer's order q_t and the order-up-to level S_t, which is the amount in stock plus on order (including those in transit) after the decision q_t has been made. The optimal order-up-to level S^*_t resulting from the cost minimum problem was given by

$$S^*_t = d\sum_{k=1}^{L+1}\frac{1-\rho^k}{1-\rho} + \frac{\rho(1-\rho^{L+1})}{1-\rho}D_{t-1} + K\sigma\sqrt{\sum_{k=1}^{L+1}\sum_{i=1}^{k}\rho^{2(k-i)}} \tag{2}$$

where K is the level of customer service. From (2), the optimal order amount q^*_t was given by

$$\begin{aligned} q^*_t &= S^*_t - S^*_{t-1} + D_{t-1} \\ &= \frac{\rho(1-\rho^{L+1})}{1-\rho}(D_{t-1} - D_{t-2}) + D_{t-1}. \end{aligned} \tag{3}$$

Expression (3) implies if the demand surge happen in period $(t-1)$, then in period t the retailer will order a quantity to bring the inventory back to the original level S_{t-1}, plus an additional quantity to reflect the update of the further demands. Symmetrically, lower demand observed at the retailer leads to a lower order quantity than original lower demand. As a result, the variance of customer demands amplifies when passed upstream to the supplier. Such a phenomenon, called the bullwhip effect, can be measured by he ratio of the variance of retailer demand, q^*_t , to the variance of customer demand, D_{t-1} :

$$E_B = \mathrm{Var}(q^*_t)\big/\mathrm{Var}(D_{t-1}), \tag{4}$$

and can be simply derived as,

$$E_B = 1 + \frac{2\rho(1-\rho^{L+1})(1-\rho^{L+2})}{1-\rho}. \tag{5}$$

For a detailed derivation of expression, see Lee et al. (1997a). The value for this measure greater than one indicates amplified order variability.

3 Proposed Procedure

Procedure of building up a confidence interval for E_B

Li and Maddala (1996) has discussed the issue of how applying the bootstrap technique in an autoregressive (AR) context. One commonly used approach is to resample residuals, which implies first differencing the observed and then applying a bootstrap scheme to their residuals to generate the pseudo-series. In this paper, we utilize this approach to repeatedly generate the pseudo-series of customer demand, each of which would be used to calculate the $\hat{E}^*_B$ for estimating the exact bullwhip effect E_B. The procedure to establish a bootstrap confidence interval of E_B is summarized as follows.

Step1. Let $D_{-1}, D_0, D_1, \cdots, D_n$ be the successive observations from customer demand process, satisfying the AR (1) model (see expression (1)), and $\Delta D_t = D_t - D_{t-1}$, $t = 0, 1, 2, \cdots, n$ be their differences.

Step2. Calculate the residuals e_t , $t = 1, 2, \cdots, n$ by fitting a first order autoregression to ΔD_t , that is,

$$e_t = \Delta D_t - \hat{\rho}\,\Delta D_{t-1}, \qquad (6)$$

where $\hat{\rho}$ is the least squares estimator of the regression of ΔD_t on ΔD_{t-1}.

Step3.Generate a pseudo-series of customer demand $D_1^*, \cdots, D_n^*$ as $D_1^* = D_0 + \Delta D_1^*$ and $D_t^* = \sum_{j=1}^{t} \Delta D_j^*$, for $t = 2, 3, \cdots, n$,

where $\Delta D_j^* = \hat{\rho}\,\Delta D_{j-1} + e_j^*$. Here e_j^*, $j = 1, 2, \cdots, n$, obtained by the bootstrap scheme is an iid sequence with $e_j^* \sim \hat{F}_e$, where $\hat{F}_e$ is the empirical distribution of e_t's.

Step4.Calculate the least squares estimator of the regression of D_t^* on D_{t-1}^* (denoting by $\hat{\rho}^*$), and produce an point estimate $\hat{E}_B^* = 1 + \dfrac{2\hat{\rho}^*(1-\hat{\rho}^{*L+1})(1-\hat{\rho}^{*L+2})}{1-\hat{\rho}^*}$ for E_B.

Step5.Repeatedly do Step 3 to Step 4 until a total of f point estimate values, $\hat{E}_B^*(1),\ \hat{E}_B^*(1),\ \cdots,\ \hat{E}_B^*(f)$, are acquired. Note that Ffron and Tibshirani (1986) indicated that a rough minimum of $f = 1000$ is usually necessary to compute reasonably accurate confidence interval estimates.

4 Illustrative Example

A simulation study on the behavior of the bootstrap confidence interval at 95% confidence level for estimating bullwhip effect is presented. Table 1 illustrates 20 various combinations of lead time intervals $(L = 1, 2, \cdots, 5)$ and lag-one autocorrelations ($\rho = 0.2, 0.4, 0.6, \text{and } 0.8)$.

The exact bullwhip effects corresponding to each combination were calculated by expression (4). For each combination, a samples of size n =25, 50, or 100 was drawn from an AR (1) demand process with constant $d = 1000$ and variance $\sigma^2 = 1$ (Note that the values of d and σ didn't affect the simulation results), and $f = 1000$ values of E_B^*'s were produced by Steps 1 to 5. Thus, a 95% bootstrap confidence interval was able to be constructed by each of the three methods (SB, PB, and BCPB) for each of sample size, and then determined if the bullwhip effect is covered by this bootstrap confidence interval. This single simulation was replicated $N = 1000$ times, and run by a computer program coded by MATLAB 6.0. One result from the simulation is the percentage of times that the exact bullwhip effect is covered by the 95% bootstrap confidence interval, which is called the "coverage percentage". Another result based on the $N = 1000$ trials is the average width of the 95% bootstrap confidence interval.

Table 1.Combinations of model parameters and their corresponding bullwhip effects

No.	ρ	L	E_B
1	0.2	1	1.4762
2	0.2	2	1.4952
3	0.2	3	1.4990
4	0.2	4	1.4998
5	0.2	5	1.5000
6	0.4	1	2.0483
7	0.4	2	2.2161
8	0.4	3	2.2859
9	0.4	4	2.3143
10	0.4	5	2.3257
11	0.6	1	2.5053
12	0.6	2	3.0472
13	0.6	3	3.4082
14	0.6	4	3.6376
15	0.6	5	3.7800
16	0.8	1	2.4054
17	0.8	2	3.3049
18	0.8	3	4.1755
19	0.8	4	4.9686
20	0.8	5	5.6649

On the average length of confidence interval

Usually, the coverage percentage is the most important assessment of a confidence interval method, but the width of the confidence interval may be also important.

Table 2. The 95 % confidence interval width

	95% Interval width								
		SB			PB			BCPB	
No.	n=25	n=50	n=100	n=25	n=50	n=100	n=25	n=50	n=100
1	0.4576	0.3136	0.2235	0.4596	0.3148	0.2235	0.4593	0.3150	0.2236
2	0.4727	0.3278	0.2318	0.4755	0.3288	0.2321	0.4752	0.3290	0.2322
3	0.4770	0.3307	0.2349	0.4799	0.3316	0.2353	0.4794	0.3316	0.2354
4	0.4820	0.3320	0.2346	0.4845	0.3331	0.2349	0.4838	0.3330	0.2348
5	0.4878	0.3313	0.2352	0.4902	0.3321	0.2353	0.4899	0.3324	0.2354
6	1.2356	0.8449	0.5822	1.2378	0.8456	0.5821	1.2386	0.8457	0.5823
7	1.4571	0.9737	0.6745	1.4587	0.9755	0.6757	1.4585	0.9755	0.6761
8	1.5558	1.0255	0.7151	1.5546	1.0269	0.7160	1.5543	1.0264	0.7163
9	1.5699	1.0548	0.7283	1.5741	1.0572	0.7293	1.5719	1.0575	0.7296
10	1.5893	1.0665	0.7275	1.5891	1.0682	0.7279	1.5910	1.0681	0.7281
11	2.2970	1.4721	0.9863	2.2948	1.4733	0.9875	2.2978	1.4753	0.9885
12	3.1060	1.9642	1.3345	3.1018	1.9658	1.3335	3.1089	1.9669	1.3346
13	3.6583	2.3427	1.5837	3.6443	2.3418	1.5838	3.6492	2.3451	1.5853
14	4.0222	2.5448	1.7288	4.0126	2.5432	1.7303	4.0197	2.5438	1.7322
15	4.2767	4.2646	1.8132	4.2557	4.2382	1.8111	4.2715	4.2518	1.8119
16	3.2220	3.1781	1.1575	3.2035	3.1641	1.1592	3.2112	3.1811	1.1604
17	5.1740	5.3951	1.8901	5.1312	5.3447	1.8880	5.1527	5.3649	1.8919
18	7.4011	7.3956	2.5463	7.3293	7.3234	2.5428	7.3903	7.3496	2.5438
19	8.8920	9.1467	3.1602	8.7906	9.0570	3.1561	8.8328	9.1076	3.1614
20	10.6812	110069	3.7570	10.5621	10.8878	3.7531	10.6030	10.9673	3.7568

Table 3.ANOVA for the coverage width of bootstrap confidence interval

Source of variation	df	SS	MS	F	Pr>F
Model	12	3304.483	275.374	181.643	0.000
Sample Size	2	218.577	109.288	72.089	0.000
Autocorrelation	3	1163.119	387.706	255.740	0.000
Lead Time	4	144.181	36.045	23.776	0.000
Bootstrap Methods	2	4.874E-03	2.437E-03	0.002	0.998
Error	348	527.573	1.516		
Total	360	3832.056			

R-square: 0.862

Table 2 displays the coverage percentages for 95% confidence interval under various combinations of model parameters. An analysis of variance (ANOVA) table of the width of the confidence interval shown in Table 3 is made to study the foregoing four main effects. The R^2 value of the four main effects is 0.862. According to the results of Table 3, it illustrates that sample size, autocorrelation, and lead time are significant to affect the interval width.

5 Concluding Remarks

Processing of non-stationary demand signal is one of the main causes resulting in the bullwhip effect in a supply chain. Great bullwhip effect would bring about increased cost and poor service. In reality, the approach to measure the bullwhip effect usually relies upon a sample of finite observations from the demand process, and the measured value is taken as a point estimate of the bullwhip effect. In this paper, in estimating the bullwhip effect confidence interval estimate is used instead of simple point estimate. In doing so, we first calculate the residuals obtained from a fitting auto-regression model on demand differences, and then apply the bootstrap scheme to resampling residuals to generate the pseudo series of demands. According to these series of demands, the bootstrap confidence interval can be constructed. In order to validate its usefulness, we considered a simple supply chain structure with a first-order auto-regressive AR(1) demand process in our simulation study. The simulation results indicate that the coverage percentage of bootstrap confidence interval for exact bullwhip effect is probably in a range from 70% to 95%, which didn't perform as the stated 95% confidence interval, and the width of confidence interval increases as the exact bullwhip effect increases. In exploring the effects of sample size, lag-on autocorrelation coefficient, lead time, and bootstrap method on the coverage percentage and width of bootstrap confidence interval, two observations are found:

(1)For the effect of autocorrelation coefficient, highly positive correlated demand data yields a lower percentage coverage and lead to a wider confidence interval. However, the larger confidence interval might be shortened by increasing sample size.

(2)For the effect of lead time, a longer lead time gives a wider confidence interval. This is because that a longer lead time can lead to a larger exact bullwhip effect, under which a wider interval length is employed. However, the length might be shortened when the sample size is increased.

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