# A New Approach for Optimal Capacitor Placement in Distribution Systems

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*Abstract:* - This paper presents a new approach for optimization and automatic control of reactive power in distribution feeders and substations. An optimal capacitor placement, sizing, and controlling problem is formulated objecting to improve voltage regulation and reduce power losses. Characterize and compose the main contribution of the work, a formulation and methodology to a concomitant search for optimal placement, optimal placement policy, and optimal control scheme of capacitor banks in distribution systems. These optimal solutions, provide decision support to reactive power compensation planning in large scale energy companies. Based on reinforcement learning concepts and sensitivity analysis, the proposed method has been tested in a Brazilian Central Region real system with preliminary but promising results.

*Key-Words:* - Capacitor placement, Loss minimization, Distribution planning, Voltage regulation, Reinforcement learning, Machine learning.

## 1  Introduction

Electrical power losses in distribution systems correspond to about 70% of total losses in electric power systems [1]. These electrical losses can be considerably reduced through the installation and control of reactive compensation equipments, such as capacitor banks, reducing reactive currents in distribution feeders. Furthermore, voltage profiles, power-factor and feeder capability of distribution substations are also significantly improved.

Computational techniques for capacitor placement in distribution systems, have been extensively researched since the 60's, with several available technical publications in this research area [2]. Published literature describes several approaches and techniques to the problem, standing out the analytic methods, heuristic methods, numerical programming, fuzzy logic, ant colony optimization, tabu search [3], neural networks, genetic algorithms [1] and hybrid methods [4].

Compelled to identify the location, number, size, type and control scheme for each capacitor to be in-

stalled in a distribution system, the problem is usually formulated in terms of a combinatorial optimization problem, where conflicting objectives are considered as purchase and installation cost minimization of capacitors banks and electrical losses reduction.

Despite quality and quantity of works on the issue, established a final outcome and, due lack of human and financial resources, electric utilities usually implement gradually intermediate non-optimal solutions to the problem. In addition, it's a common practice, especially in companies with large concession areas and long feeders, to apply these algorithms in planning scenarios studies, regarding different financial constraints represented by number (or size) limits to capacitors banks at buses, feeders and/or distribution substations. Nevertheless, each budget constraints defines a new combinatorial optimization problem and, not rarely, these solutions might demand reactive power compensation equipments, unfeasible on a strict technical optimal placement solution without budgets constraints.

This paper presents a new approach for optimization

and automatic control of reactive power in electric power distribution systems. Characterize and compose the main contribution of the work, a formulation and methodology to a concomitant search for optimal placement, optimal placement policy, and optimal control scheme of capacitor banks in distribution systems. The methodology uses reinforcement learning concepts and algorithms, as well as bus sensitivity-based analysis with respect to reactive power injections. These optimal solutions, provide decision support to reactive power compensation planning in large scale energy companies. The proposed method has been tested in a Brazilian Central Region real system with preliminary but promising results.

The paper is divided into five sections as followings. Sections 2 and 3 present, a brief introduction to reinforcement learning paradigm and a description of proposed approach, respectively. Section 4 shows preliminary numerical results. Finally in section 5, conclusions and future research perspectives are outlined by the authors.

## 2  Reinforcement Learning

Reinforcement learning (RL) [5] can be described as a computational approach to learning through interaction with an environment. In a sequential decision task, an agent interacts with an environment, by selecting actions that affect state transitions to optimize some reward function. Formally, at any given time $t$, an agent perceives its state $s_t$ and selects an action $a_t$. A dynamic system responds by giving the agent some numerical reward $r(s_t)$ and changing into state $s_{t+1} = \delta(s_t, a_t)$ [6]. The agent's aim is to find (or to learn) a policy $\pi : S \rightarrow A$, mapping states to actions, that maximizes some long-run measure of reinforcement.

$$\pi^* = argmax_\pi V^\pi(s), \forall s \qquad (1)$$

where $V^\pi(s)$ is the cumulative reward received from state $s$ using policy $\pi$, called value-function.

The most common approach to learning value functions is the temporal difference (TD) methods. These methods can learn directly by experience without any explicit model of environment's dynamics. Furthermore, they update estimates based on previous learned estimates, without waiting for a final outcome. Defining the action-value function

$$Q(s_t, a_t) = r(s_t) + V^\pi(\delta(s_t, a_t)), \qquad (2)$$

it's possible to set up the update rule of the off-policy RL algorithm *Q-learning* [5], on its simplest form.

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \Delta_Q(s_t, a_t) \qquad (3)$$

$$\Delta_Q(s_t, a_t) = U(s_{t+1}, a_{t+1}) - Q(s_t, a_t) + r(s_t) \qquad (4)$$

$$U(s_{t+1}, a_{t+1}) = \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1}) \qquad (5)$$

where $\gamma$ and $\alpha$ denote, respectively, the learning rate and discount rate of reinforcements along time. In equation (3), the function $Q(s_t, a_t)$ is updated based on its current value, immediate reward $r(s_t)$, and the difference between the maximum action-value at the next state (finding and selecting the action at the next state that maximizes it) minus the action-value function in the current time.

Differing from supervised learning techniques, the environment is explicitly considered on a trade-off between exploration and exploitation. The agent must learn which actions maximize gains in time, but also how to act to reach this maximization, looking for actions still not selected or regions not considered in a state space. As both directives bring, in specific moments, benefits to problem solutions, the exploration and exploitation modules are usually mixed.

Let the $\epsilon$-*greedy* policy, a policy where the parameter $\epsilon$ indicates the probability of choosing a random action, and $(1 - \epsilon)$ the probability of choosing the action of larger expected long-run value of return. Then, the greedy action $a_t^*$ for state $s_t$ can be obtained according to equations below [7].

$$a_t^* = argmax_{a_t \in A(s_t)} Q(s_t, a_t) \qquad (6)$$

$$\pi(s_t, a_t^*) = 1 - \epsilon + \frac{\epsilon}{|A(s)|} \qquad (7)$$

$$\pi(s_t, a_t) = \frac{\epsilon}{|A(s)|}, \forall a \in A(s) - \{a_t^*\} \qquad (8)$$

Fundamental for TD methods convergence, the trade-off between exploration and exploitation as well as, policy functions $\pi$, value functions $V$, action-value functions $Q$, and agent/enviroment interactions, are reinforcement learning paradigm elements, and have been used in the development of proposed approach.

## 3  Proposed Approach

## 3.1  Problem Formulation

The capacitor placement problem consists of determining optimal location, number, size, type and control scheme of capacitor banks, such that minimum yearly cost due to power losses and cost of capacitors are achieved, while operational and power supply quality constraints are respected.

Mathematically, the problem can be formulated initially as a combinatorial optimization problem, with search space size given by $\Theta(\Lambda + 1)^N$, where $\Lambda$ is the number capacitor sizes, $\Theta$ is the number of load levels under analysis and, $N$ is the number of network buses.

$$min\ f\left(v, z\right) = f_C\left(v, z\right) + f_L\left(v, z\right) \quad (9)$$

subjected to

$$g\left(v^m, z^m\right) = 0 \quad (10)$$

$$V^{min} \leq \left(v_k^m\right) \leq V^{max} \quad (11)$$

where

$$f_C(v, z) = \sum_m^\Theta \sum_k^N \kappa_C^{f,s} z_k^m C_k^m \quad (12)$$

$$f_L(v, z) = \sum_m^\Theta \kappa_L^m T^m P_L^m(v^m, z^m) \quad (13)$$

In equations (9), (12) and (13), the objective function $f$ is divided into costs associated to capacitors banks (purchase and installation) $f_C$ and costs associated to electrical losses $f_L$ (obtained through the cost coefficient $\kappa_L^m$, for $\Theta$ load levels, and electrical losses $P_L^m$). The variable $z_k^m$ denotes shunt capacitance existence at node $k$ for load level $m$. Equations (10) and (11) correspond to load flow and voltage magnitudes constraints, respectively. This last constraint has been considered through the addition of a penalty factor $\beta$ to the voltage deviation as followings:

$$f\left(v, z\right) = f_C\left(v, z\right) + f_L\left(v, z\right) + \beta \sum_{k,m}^{N,\Theta} \phi_{v_k^m} \quad (14)$$

subjected to

$$g\left(v^m, z^m\right) = 0 \quad (15)$$

where

$$\phi_{v_k^m} = \begin{cases} 0.01, \text{ if } V^{min} \leq v_k^m \leq V^{max} \\ 0.5\ abs\left(1 - \left(v_k^m\right)^2\right), \text{otherwise} \end{cases} \quad (16)$$

Pointing out policy search, financial constraints like-wise number (or size) limits to capacitors banks at buses, feeders and/or distribution substations, are not directly included in such object function formulation. Conversely, these constraints are considered in aiming for the most profitable ordination for capacitors placement. In fact, although equation (14) is suited to optimal location search and control scheme search of capacitors banks in distribution system, an optimal capacitor placement policy search is also proposed in this approach. For this purpose, let $s$ be the *placement state*, a bi-univocal function of $z^m C^m$, $\forall m$

$$\overrightarrow{s} = [\ ...\ \overbrace{\|}^{m-1}\ \underbrace{... s_{k-1}^m s_k^m s_{k+1}^m ...}_{N}\ \overbrace{\|}^{m}\ ...]_{\Theta N} \quad (17)$$

$$s = h(\overrightarrow{s}), \text{ such that } \exists h^{-1}, h : \mathbb{N}^{\Theta N} \rightarrow \mathbb{N} \quad (18)$$

and let $a \in A$ be the action of installing a capacitor bank of size and type $C_k^{t,\Theta}$ at bus $k$, $\forall k$. Hence, let **Q** a function representing the expected value of allocating each capacitors bank, whereas the placement state $s$, the optimal placement policy is conveniently summarized through the equation (19) bellow.

$$\pi^*\left(s\right) = argmax_a Q\left(s, a\right) \quad (19)$$

## 3.2  Methodology

The proposed methodology is based on modeling an agent objecting to learn and discover the optimal location, placement policy, and control scheme of capacitor banks, by means of try-and-error interactions in an environment. This environment is defined as the electric network where the installation of each capacitor bank is an agent's action with this environment. Hence, the optimal placement policy is referred to actions, in each network status, which maximize a future reward function obtained until the optimal placement state. Considering different load levels in problem formulation, this optimal placement state includes also the optimal capacitor control scheme.

Throughout iterative learning, the optimal placement state search (that is, the state associated to the optimal immediate return)

$$r\left(s_t\right) \equiv \frac{1}{f\left(v, z\right)} \quad (20)$$

is self–managed by the learning technique, sensitivity-based analysis, and the use of potential heuristic rules developed and/or already available for the problem.

At this point, two heuristic rules are described below.

### 3.2.1 Immediate Reward Estimation

During the first visit of a placement state, it's specified as a directive of the methodology, the choose of the action associated with the larger expected immediate reward. Common in applications to the problem, this procedure requires extensive object function numerical evaluations. Aiming to reduce the computational burden required, it's performed an immediate reward estimation for each possible action given a placement state, as followings.

Let $s_j$ be the state obtained in time $j$ of agents' iterative learning. Also let $a_{j_u}$ be the action representing a capacitor bank installation at bus $u$ such that $Q_{j_u}^m = C_{j_u}^m z_{j_u}^m$, for $\Theta$ load levels. Moreover, consider now $v_{a_i}^m$ and $v_{b_i}^m$, the initial and final bus voltages at line $i$ for load level $m$. Then the object function obtained for state $s_{j+1}$ can be estimated through sensitivity voltage [8] evaluations, related to reactive power injection, as shown below.

$$
\begin{aligned}
f_{j+1} &\approx f_j + \kappa_C^{f,s} + \sum_m^{\Theta} \kappa_L^m T^m \frac{\partial P_L^m}{\partial Q_u^m} \Delta Q_u^m \\
&+ \beta \sum_{k,m}^{N,\Theta} \phi_{v_{k+1}^m}
\end{aligned}
\tag{21}
$$

where

$$
\frac{\partial P_L^m}{\partial Q_u^m} = \sum_i^{N_L} \frac{2\left(v_{a_i}^m - v_{b_i}^m\right)}{Z_{ab}^i}\left(\frac{\partial v_{a_i}^m}{\partial Q_u^m} - \frac{\partial v_{b_i}^m}{\partial Q_u^m}\right)
\tag{22}
$$

$$
\frac{\partial P_L^m}{\partial Q_u^m} = \sum_i^{N_L} \frac{2\left(v_{a_i}^m - v_{b_i}^m\right)}{Z_{ab}^i}\left(\frac{\partial v_{a_i}^m}{\partial Q_u^m} - \frac{\partial v_{b_i}^m}{\partial Q_u^m}\right)
\tag{23}
$$

and

$$
\phi_{v_{k+1}^m} = \begin{cases} 0.01, & \text{if } V^{min} \leq v_{k+1}^m \leq V^{max} \\ 0.5\ abs\left(1 - \left(v_{k+1}^m\right)^2\right), & \text{otherwise} \end{cases}
\tag{24}
$$

$$
v_{k+1}^m \approx v_k^m + \frac{\partial v_k^m}{\partial Q_u^m}\Delta Q_u^m
\tag{25}
$$

### 3.2.2 Reduction of the Search Space

Sensitivity-based analysis is also performed for reducing the search space, limiting the search to the $\eta(\%)$ most sensitive buses in relation to objective function.

### 3.2.3 Algorithm

Based on TD method Q-learning, the proposed algorithm is outlined below.

1) Read input data (line and bus data) and initialize parameters and variables.
2) Define search space considering the $\eta(\%)$ most sensitive buses with respect to objective function.
3) Start with state representing the uncompensated reactive power status of the electric network.
4) If the placement state is visited for the first time, choose the action associated with the larger expected immediate reward. This procedure can be performed through immediate reward estimation of all possible actions for the placement state through equation (21). Otherwise, choose an action under $\epsilon$-greedy policy defined by the equations (6), (7) and (8).
5) Obtain next placement state from current state and current action. States not visited and respective immediate reward (optionally) are stored in memory.
6) Update action-value function using equations (3) and (5).
7) If the immediate reward of next state is smaller than the immediate return of current state under greedy action, return to Step 4. Otherwise, go to Step 8.
8) Update learning rate $\alpha$ and probabilities $\epsilon$

$$
\alpha^{iter+1} = \max\left(0.95\alpha^{iter}, \alpha_{final}\right)
\tag{26}
$$

$$
\epsilon^{iter+1} = \max\left(0.95\epsilon^{iter}, \epsilon_{final}\right)
\tag{27}
$$

9) If the $\pi$ policy convergence is characterized, go to Step 10. Otherwise, return to Step 3.

$$
\sum \left|Q^{iter}\left(s_j, a_j\right) - Q^{iter-1}\left(s_j, a_j\right)\right| < \Psi
\tag{28}
$$

10) Computation and impression of numerical results.

## 4 Simulation Results

### 4.1 Case study description

The proposed approach for optimization and automatic control of reactive power has been tested in a 13.8 kV, 29-bus Brazilian Central Region real system. The annual load curves were segmented in three load levels (light, intermediate, peak) of demand factors and duration $T$, specified in Table 1.

Fixed and switched capacitor purchase and installation costs are shown in Table 2 [9]. Simulations considered energy losses costs by the coefficient $\kappa_L =$

|  | Light | Intermediate | Peak |
|---|---|---|---|
| Demand factor | 0.30 | 1.67 | 2.00 |
| $T$ (hours/year) | 3102.50 | 4562.50 | 1095.00 |

Table 1.  Load levels.

0.13380 US$/kWh, for the three load level under analysis. Upper and Lower bound voltage magnitudes, according to voltage levels standards in [10], are set up in $V^{min} = 0.93$ pu and $V^{max} = 1.05$ pu. The parameter $\beta$ will have been calibrated to represent Brazilian regulatory penalties. For this preliminary simulations, $\beta$ was used as a penalty factor to capacitor placements associated to high voltage deviation.

| Type | Size (kVAr) | Cost (US$) |
|---|---|---|
| Fixed | 600 | 3091 |
|  | 1200 | 3909 |
| Switched | 0/600 | 5818 |
|  | 0/1200 | 6636 |
|  | 0/600/1200 | 8455 |

Table 2.  Capacitor costs.

## 4.2   Result Analysis

After several numerical simulations, penalty factor was adjusted in $\beta = 100000$. Training parameters used on final outcome are shown in Table 3. The proposed algorithm presented good convergence and adequate robustness to variations of these parameter. It's recommended initial learning rates values near to unit (typical values are 0.75 to 0.90), and low final learning rates (typical values are 0.1 to 0.3). For initial and final probabilities $\epsilon$, typical values are 0.2 to 0.3 and 0.01 to 0.1, respectively. In addition, discount rates $\gamma$ near to unit (typical values are 0.80 to 0.95), modeling high influences of long-run reinforcements on state values, increase significantly algorithm performance.

| $\alpha_{initial}$ | $\alpha_{final}$ | $\gamma$ | $\epsilon_{initial}$ | $\epsilon_{final}$ |
|---|---|---|---|---|
| 0.90 | 0.20 | 0.95 | 0.30 | 0.01 |

Table 3.  Training parameters.

Table 4 summarizes the optimal capacitor placement solution, control scheme and policy, resulted for the application of proposed methodology in the Brazilian case study described previously. For each step of the capacitor placement policy solution, are indicated capacitor type, size, bus location, and control scheme, obtained.

| Step | Bus | Type | kVAr | | |
|---|---|---|---|---|---|
|  |  |  | Light | Interm. | Peak |
| 0 | - | - | - | - | - |
| 1 | 29 | Fixed | 1200 | 1200 | 1200 |
| 2 | 28 | Fixed | 1200 | 1200 | 1200 |
| 3 | 26 | Fixed | 1200 | 1200 | 1200 |
| 4 | 24 | Fixed | 1200 | 1200 | 1200 |
| 5 | 21 | Fixed | 1200 | 1200 | 1200 |
| 6 | 20 | Switched | 600 | 1200 | 1200 |
| 7 | 18 | Fixed | 1200 | 1200 | 1200 |
| 8 | 17 | Fixed | 1200 | 1200 | 1200 |

Table 4.  Capacitor placement solution, control scheme and policy, obtained.

Table 5 shows results for each capacitor placement policy step in terms of electrical losses $P_L$, capacitor purchase and installation accumulated cost $f_c^{ac}$, and object function cost regardless voltage penalty $f'$. A comparative between uncompensated network and compensate network is shown in Table 6, considering additionally the maximum and minimum voltage magnitudes $v_{min}$, $v_{max}$, and savings $Sav$ obtained.

| Step | Bus | $f_c^{ac}$ (US$) | $P_L$ (kW) | $f'$ (US$) |
|---|---|---|---|---|
| 0 | - | - | 418.72 | 296418.66 |
| 1 | 29 | 3909 | 395.68 | 278353.58 |
| 2 | 28 | 7818 | 376.22 | 264086.35 |
| 3 | 26 | 11727 | 359.67 | 252932.30 |
| 4 | 24 | 15636 | 345.79 | 244571.42 |
| 5 | 21 | 19545 | 334.18 | 238575.62 |
| 6 | 20 | 28000 | 324.48 | 238318.58 |
| 7 | 18 | 31909 | 316.48 | 235916.05 |
| 8 | 17 | 35818 | 310.02 | 235274.34 |

Table 5.  Allocation policy results.

|  | Uncompensated | Compensated |
|---|---|---|
| $v_{min}$ (pu) | 0.929 | 0.953 |
| $v_{max}$ (pu) | 0.993 | 1.002 |
| $P_L$ (kW) | 418.72 | 310.02 |
| $f'$ (US$) | 296418.66 | 235274.34 |
| $Sav$ (US$) | 61144.32 | |

Table 6.  Reactive compensation effect.

As observed in tables above, capacitor installation at this distribution feeder performs meaning electrical losses reduction and voltage profiles improvement. The solution indicated installation and purchase of seven 1200 kVAr type fixed and one 0/600/1200 kVAr

type switched-capacitor bank. Total losses without any compensation is found to be 418.72 kW. After compensation, the total losses is 310.02 kW, equivalent 26% reduction. Savings are estimated to be US$ 61144.32.

From policy placement solution, electric utilities can set up a compensation strategy, differentiated in steps, leading to optimal capacitor placement state, minimizing losses, purchase and installations costs. For example, given an annual budget for system improvements of US$ 16000, policy solution indicates the purchase and installations of four 1200 kVAr type fixed-capacitors banks (Steps 1 to 4 in Table 5), as the optimal strategy to reactive power compensation. Futhermore, policy placement solution can aid in budgetary assessments for system improvements.

Finally, preliminary results of proposed approach applied to a real system suggest effectiveness and robustness in power distribution systems optimization problems. Problem formulation and solution, provide decision support to reactive power compensation planning in large scale energy companies.

## 5    Conclusion

A new methodology for optimization and automatic control of reactive power in distribution systems is proposed in this article. The capacitor placement policy search was approached to provide decision support to planning compensation in large scale energy companies. Based on reinforcement learning concepts, a formulation to a concomitant search for optimal placement, optimal placement policy, and optimal control scheme of capacitor banks in distribution networks was presented. In fact, the reinforcement learning technique showed effectiveness in landscape combinatorial optimization problems. Sensitivity-based analysis improve the proposed method adding knowledge to the model and speeding up simulations.

The designed algorithm was applied in a Brazilian Central Region real system to improve voltage profiles and reduce electric power losses. Preliminary results pointed up good performance and robustness in power distribution systems optimization problems.

Future work needs to be carried out with regard to the following issues: nonlinear and unbalanced loads, annualized maintenance costs, meta-heuristic methods hybridism.

## References

[1] C. Lyra, C. Pissara, C. Cavellucci, A. Mendes, P. M. França.  Capacitor placement in large-sized radial distribution networks, replacement and sizing of capacitor banks in distorted distribution networks by genetic algorithms,  In *IEEE Proceedings Generation, Transmision & Distribution*, 2005, pp. 498–516.

[2] A. Y. Chikhani, H. N. Ng, M. M. A. Salama. Classification of capacitor allocation techniques, Vol.15, No.1, 2000.

[3] S. Tsunokawa, H. Mori.  Variable neighborhood tabu search for capacitor placement in distribution systems,  Vol.3, 2005, pp. 4747–4750.

[4] S. N. Kim, S. K. You, K. H. Kim, S. B. Rhee. Application of esga hybrid approach for voltage profile improvement by capacitor placement, Vol.18, No.4, 2003.

[5] A. G. Barto, R. S. Sutton. Reinforcement Learning, The MIT Press, 1998.

[6] J. J. Grefenstette, D. E. Moriarty, A. C. Shultz. Evolutionary algorithms for reinforcement learning, Vol.11, 1999, pp. 241–276.

[7] G. Bittencourt, E. Camponogara.  Genetic algoritms and reinforcement learning,  Booklet: Electrical Engineering's Courses and Lectures of Federal University of Santa Catarina, Brazil, August 2005. (In Portuguese).

[8] A. B. K. Sambaqui.  *Improvement of Voltage Profiles Methodologies in Distribution Systems*. PhD thesis, Federal University of Santa Catarina, June 2005. (In Portuguese).

[9] S. Haffner, F. A. B. Lemos, J. S. Freitas, M. V. D. Freitas.  Fixed and switched caacitor banks placement in radial distribution systems for different load levels, 2004.

[10] Resolution 505. Stead state voltage levels standards. Technical report, Brazilian Electricity Regulatory Agency, November 2001. (In Portuguese).