Stochastic Delay Estimation and Adaptive Control of Networked Control Systems

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Abstract: - An adaptive algorithm is introduced in this paper to estimate random communication time delay in Networked Control Systems (NCS). The algorithm updates an adaptive delay estimate using gradient descent method, and identifies plant parameters by a modified RLS. By using this algorithm, future control sequences are generated to compensate for the communication time delay when control signals are delayed or missed. To illustrate the improvement in control performance using the proposed method, simulation results are presented to show that the method is superior to the standard networked predictive control.

Key-Words: - Networked control systems; Adaptive control; Time delay estimation; Least squares

1 Introduction

Recently there has been much interest in networked control systems (NCS), that is, control systems with a feed-back loop closed through a communications network or a field bus. Traditional point-to-point controller architectures are being replaced with those based on a serial communication channel because with the drop in price of microcontrollers such an architecture is less costly, more reliable, easier to maintain, and more flexible. The presence of the communication network, however, complicates the application of standard results and algorithms of control theory. The problem lies with the communication channel restricting access to the data held by the sensors; decisions must be made on old or on partial data and the intervals between updates are not regular as the network is used for many purposes besides the routing of feedback data. So there will inevitably be time delays in the communication net. As long as the sampling periods are long compared with these delays there is no need to consider the influence of the delays. As the demand on the control system increases it will be more and more important to take the delays into account in the analysis and the design of the control system. While inaccuracies, disturbances, etc., have been extensively studied in the control literature the timing problems in real-time systems have just recently attracted attention, and in the communication literature the feedback control aspect has not been treated to any larger extent. This is thus an area where much can be gained by combining ideas from the fields of control, real-time systems, and communication networks.

A general setup of NCS would involve a distributed control structure, where communication between different control system nodes is achieved over a communication network. Often a centralized controller is used and the actuators and sensors are communication with the controller(s) via a bus. The effect of communication delays is discussed, for instance, in Ray[1], Shin[2], Krtolica *et al.*[3]. The optimal state estimation and LQG control scheme of NCS is proposed by Nilsson *et al* [4]. Two types of predictors for NCS are proposed by Walsh *et al* [5]. Some other control schemes for time delays prediction and estimation of the NCS are also discussed in [6], [7].

A useful method to solve parameter time-varying problem is parameter identification. But as for time-varying delay, common identification method is invalid. Reference[8] proposes an incorporate estimator for the process parameters and time delay, and this method has to solve the Diophantine equation. Reference[9] presents a multi-step regressive prediction, but this method should know the time delay first.

In this paper, a directly regressive predictive adaptive control algorithm based on time-varying delay identification is proposed. Using a time delay identification method, the adaptive control can be obtained in simple form by regressive prediction from the system model directly. The method avoids solving the Diophantine equation and is easy to compute. Simulation results show that the approach is effective.

2 Problem Formulation

We will analyze a simple structure NCS with just one adaptive controller and one process connected as in Fig.1. There essentially three kinds of computer delays in such a system:
(1)Communication delay between the sensor and the controller τ^{sc}.
(2)Computational delay in the controller τ^{c}.
(3)Communication delay between the controller and the actuator τ^{ca}.

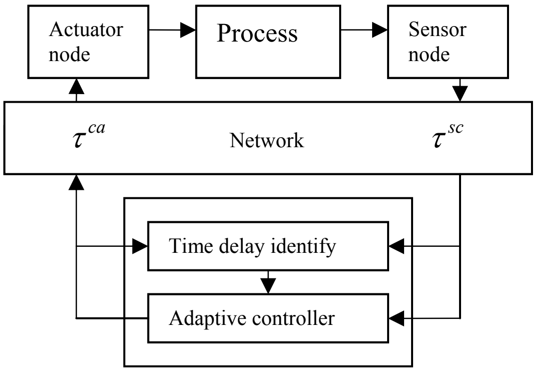

Fig.1 Structure of networked control systems

The control delay for the NCS system, in principle, equals the sum of these delays, which can be considered as a time-varying delay *d(k)* [4].So the system can be described as the following ARMAX model:

$$A(q^{-1})y(k) = q^{-d(k)}B(q^{-1})u(k) + C(q^{-1})v(k) \quad (1)$$

where

$$A(q^{-1}) = 1 - a_1(k)q^{-1} - \cdots - a_n(k)q^{-n}$$
$$B(q^{-1}) = b_0(k) + b_1(k)q^{-1} + \cdots + b_m(k)q^{-m}$$
$$C(q^{-1}) = 1 + c_1(k)q^{-1} + \cdots + c_l(k)q^{-l}$$

where $a_1(k),\cdots a_n(k); b_1(k),\cdots b_m(k); c_1(k),\cdots c_l(k)$ are the time-varying parameters of the system and $d_{\min} \le d(k) \le d_{\max}$ is the time-varying delay. $u(k), y(k)$ and $\{v(k)\}$ is controller output, system output and white noise serial respectively

3 Adaptive Time Delay Estimate and Control of Networked Control Systems

In this section we solve the adaptive control problem of NCS show in Fig.1. Suppose the identification model of NCS is:

$$\hat{A}(q^{-1})\hat{y}(k) = q^{-\hat{d}(k)}\hat{B}(q^{-1})u(k) + \hat{C}(q^{-1})v(k) \quad (2)$$

where

$$\hat{A}(q^{-1}) = 1 - \hat{a}_1(k)q^{-1} - \cdots - \hat{a}_n(k)q^{-n}$$
$$\hat{B}(q^{-1}) = \hat{b}_0(k) + \hat{b}_1(k)q^{-1} + \cdots + \hat{b}_m(k)q^{-m}$$
$$\hat{C}(q^{-1}) = 1 + \hat{c}_1(k)q^{-1} + \cdots + \hat{c}_l(k)q^{-l}$$

$\hat{\theta}^T(k) = \left[\hat{a}_1(k),\cdots,\hat{a}_n(k),\hat{b}_0(k),\cdots,\hat{b}_m(k),\hat{c}_1(k),\cdots,\hat{c}_l(k)\right]$ is the estimate of the parameters.

$\theta^T(k) = [a_1(k),\cdots,a_n(k),b_0(k),\cdots,b_m(k),c_1(k),\cdots,c_l(k)]$ at time *k*, $\hat{d}(k)$ is the estimate of *d(k)*.

3.1 Adaptive Identification Algorithm of Time-varying Delay

The error between the actual observational values output and the estimate values of the NCS are defined by:

$$e(k) = y(k) - \hat{y}(k)$$
$$= u(k)\left[\frac{B(q^{-1})}{A(q^{-1})}q^{-d(k)} - \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})}q^{-\hat{d}(k)}\right] \quad (3)$$
$$+ v(k)\left[\frac{C(q^{-1})}{A(q^{-1})} - \frac{\hat{C}(q^{-1})}{\hat{A}(q^{-1})}\right]$$

When the identification model is different from the actual NCS, $e(k) \ne 0$. Define the cost function as following:

$$J(k) = \frac{1}{2}\sum_{i=1}^{k} e^2(i) \quad (4)$$

Suppose $\hat{\theta}(k) = \theta(k), \hat{d}(k) \ne d(k)$ then according to gradient method, the time delay $\hat{d}(k)$, which minimizes J(k) should be given by

$$\hat{d}(k) = \hat{d}(k-1) - \lambda(k)\left.\frac{\partial J(k)}{\partial \hat{d}(k)}\right|_{\hat{d}=\hat{d}(k)} \quad (5)$$

where $\lambda(k)$ is the optimum step length.

$$\frac{\partial J}{\partial \hat{d}(k)} = \sum_{i=1}^{k} e(i)u(i)\frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})}q^{-d(i)}\ln q$$
$$= \sum_{i=1}^{k} e(i)\hat{y}(i)\,sT\Big|_{s=\frac{1-q^{-1}}{T}} \quad (6)$$
$$= \sum_{i=1}^{k} e(i)[\hat{y}(i) - \hat{y}(i-1)]$$

Hence, the identification algorithm of the time delay is:

$$\hat{d}(k) = \hat{d}(k-1) - \Delta\hat{d}(k)$$
$$= \hat{d}(k-1) - \lambda(k)\sum_{i=1}^{k} e(i)[\hat{y}(i) - \hat{y}(i-1)] \quad (7)$$

when

$$\lambda(k) = \frac{1-\alpha}{[\sum_{i=1}^{k}[\hat{y}(i) - \hat{y}(i-1)]]^2}, 0 \le \alpha \le 1$$

The time delay parameter estimate $\hat{d}(k)$ is globally asymptotical converge [9]. In (7), $\Delta\hat{d}(k)$ should be converted into an integer so that $\hat{d}(k)$ can be an integer.

3.2 Identification Algorithm of Time-varying Parameters

After time delay has been obtained, the parameter vector $\theta(k)$ can be identified on-line by least-square method.

Transform the system model into the least-square structure

$$y(k) = \phi^T(k)\theta(k) + v(k) \quad (8)$$

where

$$\phi^T(k) = [y(k-1),\cdots,y(k-n),u(k-\hat{d}(k)),\cdots ,u(k-\hat{d}(k)-m),v(k-1),\cdots,v(k-l)]$$

$$\theta^T(k) = [a_1(k),\cdots,a_n(k),b_0(k),\cdots,b_m(k),c_1(k),\cdots,c_l(k)]$$

In time-varying parameters identification, many approaches based on least-squares have been proposed. In this paper, we adopt an improved least-squares method with covariance modification as follow

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\phi(k-1)}{r + \phi^T(k-1)P(k-1)\phi(k-1)} \times [y(k) - \phi^T(k-1)\hat{\theta}(k-1)] \quad (9)$$

$$P(k) = P(k-1) - \frac{P(k-1)\phi(k-1)\phi(k-1)^T P(k-1)}{r + \phi(k-1)^T P(k-1)\phi(k-1)} \quad (10)$$

where r is oblivious factor, and $0 \le r \le 1$, suppose the initial values $P(0) = \sigma^2 I, \theta(0) = 0$, where σ^2 is a sufficient numeral, and I is an identity matrix.

3.3 Multi-step Regressive Generalized Predictive Algorithm of Networked control systems

After the parameter estimate, the system becomes as follow

$$\hat{A}(q^{-1})\hat{y}(k) = q^{-\hat{d}(k)}\hat{B}(q^{-1})u(k) + \hat{C}(q^{-1})v(k) \quad (11)$$

The optimal prediction of the plant output at time $k+d$, defined as $\hat{y}(k+d|k)$, is the conditional mathematical expectation under the known conditions as following:

$$y(k), y(k-1),\cdots; u(k-1), u(k-2),\cdots; v(k), v(k-1),\cdots$$

That is $\hat{y}(k+d|k) = E\{y(k+d|k)\}$, where E denotes mathematical expectation. Because *v(t)* can't be observed directly, we can use its prediction $\hat{v}(t)$ to replace it:

$$\hat{v}(k) = y(k) - \hat{y}(k|k-1)$$

Changing k to $k+d$ in equation (11), and taking the mathematical expectation on both sides, then the optimal regressive prediction of the system output is

$$\hat{y}(k+d|k) = \sum_{i=1}^{n}\hat{a}_i\hat{y}(k+d-i|k) + \sum_{i=0}^{m}\hat{b}_i u(k+d-\hat{d}(k)) + \delta\sum_{i=k}^{l}\hat{c}_i\hat{v}(k+d-i) \quad (12)$$

When $1 \le d \le l, \delta = 1$ and when $d \ge l, \delta = 0$, we can decompose $\hat{y}(k+d|k)$ into two parts: one related to *u(k), u(k+1),…u(k+d-* $\hat{d}$ *(k))*, the other related to $U(\tau), \tau \le k$. Let *u(k+j)*=0, *j*=0,1,... $d-\hat{d}(k)$, and denote $\hat{y}^*(k+d|k) = \hat{y}(k+d|k)$ at this time, then the optimal prediction correlative to the *u(k)* before time *k* is:

$$\hat{y}^*(k+d|k) = \sum_{i=1}^{n}a_i\hat{y}^*(k+d-i|k) + \sum_{i=d-\hat{d}(k)+1}^{m} b_i u(k+d-\hat{d}(k)-i) + \delta\sum_{i=d}^{l}c_i\hat{v}(k+d-i) \quad (13)$$

The optimal regressive prediction is obtained from the plant model directly, which avoids solving the Diophantine equation, and can be realized easily.

3.4 Adaptive Control Based on Multi-step Regressive Prediction

Define the cost function as follow:

$$J = E\{\sum_{i=1}^{N}[\hat{y}(k+i|k) - y_r(k+i)]^2 + \sum_{i=1}^{N}\mu u^2(k+i-1)\} \quad (14)$$

where E denotes mathematical expectation with respect to the noise process acting on the NCS, $\hat{y}(k+i|k)$ is the optimal prediction of the plant output at time $k+i$, and $y_r(k+i)$ is the desired reference output sequence, N is the length of the control sequence.

As usual, the control objective is to get an adaptive control law that minimizes the cost function (14). To obtain the control sequence to achieve output tracking, by comparing formula (12) and (13), we can get

$$\hat{y}(k+d|k) = \hat{y}^*(k+d|k) + \sum_{i=1}^{d}g_i u(k+d-i), \quad d = 1,2,\cdots N \quad (15)$$

where N (let $N \ge d_{\max}$) is the length of the control sequence, and

$$g_1 = b_0$$
$$g_i = \sum_{j=1}^{i-1}a_j g_{i-1} + b_{i-1} - \hat{d}(t), i = 2,\cdots,N \quad (16)$$

Write (15) in matrix form

$$Y = GU + Y_N \qquad (17)$$

where

$$Y = [\hat{y}(k+1|k), \hat{y}(k+2|k), \cdots, \hat{y}(k+N|k)]^T$$

$$U = [u(k), u(k+1), \cdots, u(k+N-1)]^T$$

$$Y_N = [\hat{y}^*(k+1|k), \hat{y}^*(k+2|k), \cdots, \hat{y}^*(k+N|k)]^T$$

$$G = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ g_2 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ g_N & g_{N-1} & \cdots & g_1 \end{bmatrix}_{N\times N}$$

Define the reference vector as

$$Y_r = [y_r(k+1), y_r(k+2), \cdots, y_r(k+N)]^T$$

Then the cost function (14) can be written as:

$$J = E\{(Y - Y_r)^T (Y - Y_r) + \mu U^T U\}$$

From $\frac{\partial J}{\partial U} = 0$, we can obtain the adaptive control law that minimizes the cost function (14) as follow:

$$U = (G^T G + \mu I)^{-1} G^T (Y_r - Y_N) \qquad (18)$$

The first element of U is the current control quantity.

4 Simulation

To illustrate the properties of the proposed adaptive predictive control algorithm, we use a DC-servo process with transfer function as follow [11].

$$G(s) = \frac{1000}{s(s+1)} \qquad (19)$$

The simulation example contains four computer nodes, each represented by a TrueTime kernel block. A time-driven sensor node samples the process periodically and sends the samples over the network to the controller node. The adaptive control algorithm in this node calculates the control signal and sends the result to the actuator node, where it is subsequently actuated. The simulation also involves an interfering node send disturbing traffic over the network.

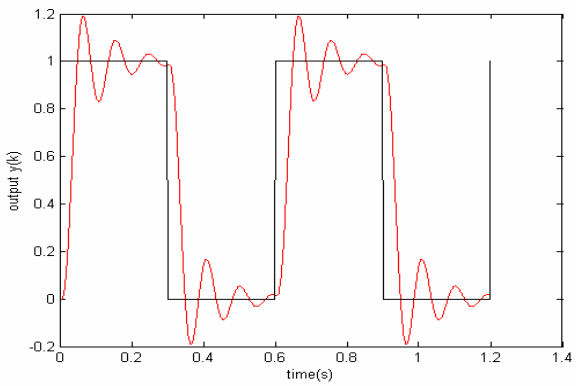

Fig.2 Response of the output without time-delay compensation

The response of the NCS without time-delay compensation is show in Figure 2.

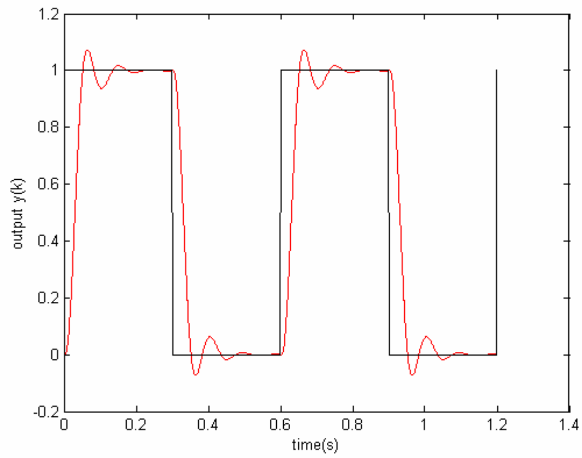

Fig.3 Response of the output with time-delay compensation

The proposed scheme in this paper is used to control the same process so that its output sequence *y(k)* should follow the desired reference inputs. The result is show in Fig. 3. It can be observed that, the adaptive control algorithm has a superior performance to NCS.

5 Conclusion

In this paper we have proposed an adaptive control algorithm to estimate and compensate for randomly varying time delays of networked control systems. By using gradient descent method, an adaptive identification algorithm for delay estimate is obtained. And then, an adaptive control algorithm based on multi-step regressive prediction has been presented in a simple form. Simulation results show that the approach is an effective method for networked predictive control.

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