Adaptive Control and Data Fusion using EKF for Wheeled Robots with Parametric Uncertainties

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Abstract: - This paper presents a new adaptive motion control system including on-line Extended Kalman's filter (EKF) for wheeled robots with nonholonomic constraints on the motion. The presence of uncertainties both in the kinematics and in the dynamics is treated using adaptation laws where the Lyapunov's stability of the motion errors is proved. Now, if data from incremental encoders are used for the feedback directly, errors can damage the performance of the motion control. Therefore an EKF is inserted in the adaptive control system suitably. The filter above, through recursive predictions and corrections, fuses data provided by multiple proprioceptive sensors to obtain good estimations of the feedback signals in terms of cartesian positions and orientation. The control algorithm efficiency is confirmed through simulation experiments.

Key-Words: - Adaptive control, Extended Kalman's filter, Lyapunov's stability, Nonholonomic robots, On-line data fusion, Parametric uncertainties.

1 Introduction

In recent years much attention has been focused upon the control of nonholonomic mechanical systems [2]. A mobile wheeled robot is usually studied as a typical nonholonomic system, where nonholonomic constraints arise under the no-slip constraints. Our approach is about motion control of nonholonomic robots. The problem of motion control is to design a controller such that all the closed loop signal are bounded and the motion errors (longitudinal, lateral and orientation errors) converge to zero. Must research effort has been oriented to solving the problem above using only kinematical controllers [1], where the main idea is to define velocity control inputs which stabilize the closed loop system. Other control researchers have target the problem of motion control of nonholonomic robots using a *backstepping* approach [6], [9] which allows many of the steering system commands to be converted to torques, tacking into account dynamic parameters (mass, inertia, friction etc…). Now the main issues are: *stability* and good *localization*. About the *stability* issue, in [6] the backstepping approach has been proposed and a theorem for the asymptotical stability of the tracking errors has been developed. The fundamental problem of the backstepping is the uncertainty of the parameters of the robot. So in [4] an adaptive control scheme for nonholonomic robots is presented. In the works above the problem of on line localization of the robot has not been treated. Really, at each sampling instant the position of the robot is estimated on the basis of the encoders increment along the sampling interval. A drawback of this method is that the errors of each measure caused by the encoder are summed up. Therefore in [5] and [7] the problem of localization, i.e. an optimal estimation of the robot's position, has been solved by an off-line sensors data fusion based on EKF. An interesting approach has been developed in [3], where a conventional PID control strategy with a Kalman based active observer controller has been used to solve a problem of path following for nonholonomic robots.

In this paper an adaptive motion control system with on-line EKF for nonholonomic wheeled robots is presented. The contributions of this work include:

a) merging of adaptive kinematic and dynamic controllers where stability and convergence analysis is built on Lyapunov's theory. So the problem of kinematical and dynamical parametric uncertainties of the robot is solved;

b) a discrete time state space representation to apply EKF and a methodology for solving the on-line sensors data fusion problem through filters above. In a motion control system, if data from encoders are used for the feedback only, then noises can damage the motion control performances in terms of position and orientation errors. Therefore an EKF has to be introduced in the adaptive control system above to fuse data from multiple proprioceptive sensors (i.e. encoders, vector compass and sensor position) and to estimate the filtered feedback signals, i.e. the actual position of the robot, by on-line recursive predictions and corrections.

This paper is organized as it follows. Section 2 presents the kinematic and dynamic continuous time models of wheeled robots for our control strategy. Section 3 presents a discrete time state space representation to apply EKF. In Section 4 a method for adaptive kinematic and dynamic control of the system above is developed. Section 5 presents a strategy for on line sensors data fusion based on EKF positioned in the feedback of the adaptive control of the previous section. Section 6 shows results of experimental simulations to confirm the validity of the proposed control algorithm.

2 Time continuous models for wheeled robots

Let us consider a mobile vehicle of Fig. 1 with generalized coordinates $\mathbf{q} \in \Re^n$, subject to m constraints. The well known dynamic model in generalized coordinates is [6], [9] :

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\dot{q} = E(q)\tau - A^T(q)\lambda} \tag{1}$$

where $\mathbf{M(q)} \in \Re^{n\times n}$ is a symmetric, positive definite matrix; $\mathbf{C(q,\dot{q})} \in \Re^{n\times n}$ is the centripetal Coriolis matrix; $\boldsymbol{\tau} \in \Re^{n\times 1}$ is a vector including torques applied to right and left wheels; $\mathbf{A(q)} \in \Re^{m\times n}$ is the matrix of nonholonomic constraints and $\boldsymbol{\lambda} \in \Re^{m\times 1}$ is a vector of lagrange multipliers. Supposing that the m constraints are time invariant leads to:

$$\mathbf{A(q)\dot{q} = 0} \tag{2}$$

Let $\mathbf{S(q)} \in \Re^{n\times(n-m)}$ be a full rank matrix made up by a set of smooth and linearly independent vectors spanning the null space of **A(q),** i.e.,

$$\mathbf{A(q)S(q) = 0} \tag{3}$$

It is possible to find a $\mathbf{v} \in \Re^{n-m}$ vector as it follows:

$$\mathbf{v^T} = (u,\omega) \tag{4}$$

where u and ω are respectively the linear and angular body-fixed (X,Y) velocities. We indicate with (x_0, y_0) the P_0 coordinates (see Fig. 1) in an inertial cartesian frame (x,y) and with ϕ the robot orientation with respect to the inertial basis.We indicate with r the ray of the wheels and with b the distance from the wheels to the longitudinal axis. Let P_c be the mass center of the vehicle, which is on the X-axis, and let d be the distance from P_0 to P_c. For the later description, m_c is the mass of the vehicle without the driving wheels, m_w is the mass of each driving wheels, I_c, I_w and I_m are the inertia moments of the body around a vertical axis through P_0, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. We indicate with $\dot{\theta}_r$ and $\dot{\theta}_l$ the angular velocities of right and left wheels respectively. The relation ship between (u,ω) and $(\dot{\theta}_r, \dot{\theta}_l)$ is the following:

$$\begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & \frac{-b}{r} \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} \tag{5}$$

Equations (5) can be rewritten as it follows :

$$\boldsymbol{\eta} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & \frac{-b}{r} \end{bmatrix} \mathbf{v} \tag{6}$$

where:

$\boldsymbol{\eta}^\mathbf{T} = [\ \dot{\theta}_r\ \ \dot{\theta}_l\ ]$ $\quad \mathbf{v^T} = [\ u\ \omega\ ]$

Now we consider the following vector:

$$\mathbf{q^T} = [\ x_0\ \ y_0\ \ \phi\ ] \tag{7}$$

We can write the following kinematic model:

$$\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} cb\cos\phi & cb\cos\phi \\ cb\sin\phi & cb\sin\phi \\ c & -c \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \mathbf{S(q)}\boldsymbol{\eta} \tag{8}$$

where:

$c = r/2b$.

About the dynamic model in body fixed coordinates (X, Y), differentiating (8), replacing it into (1) and performing additional operations with **S(q)** lead to:

$$\overline{\mathbf{M}}\dot{\boldsymbol{\eta}} + \overline{\mathbf{V}}_\mathbf{m}(\boldsymbol{\eta})\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\tau} \tag{9}$$

$$\overline{\mathbf{M}} = \begin{bmatrix} b^2c^2m + c^2I + I_w & b^2c^2m - c^2I \\ b^2c^2m - c^2I & b^2c^2m + c^2I + I_w \end{bmatrix}$$

$$\overline{\mathbf{V}}_\mathbf{m}(\boldsymbol{\eta}) = \begin{bmatrix} 0 & 2bc^3dm_c(\dot{\theta}_r - \dot{\theta}_l) \\ -2bc^3dm_c(\dot{\theta}_r - \dot{\theta}_l) & 0 \end{bmatrix}$$

$$\mathbf{B} = diag[1\ \ 1]; \quad \boldsymbol{\tau}^\mathbf{T} = [\tau_r\ \ \tau_l]$$

where:

$m = m_c + 2m_w;\ \ I = m_c d^2 + I_c + 2m_w b^2 + 2I_m$

The $\overline{\mathbf{M}}$ and $\overline{\mathbf{V}}_\mathbf{m}$ matrices are respectively the Inertia and Coriolis matrices in body-fixed (X,Y) coordinates system (see Fig. 1). The vector τ has in his components the torques applied to the right and left wheels respectively.

Substituting (6) into (8) leads to:

$$\dot{\mathbf{q}} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} = \mathbf{S}_2\mathbf{(q)v} \tag{10}$$

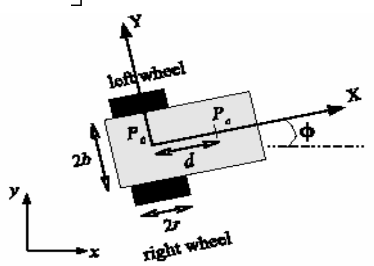

Fig.1 . Constrained wheeled robots with references

3 A discrete time kinematic model

Preliminarily we consider the following change of coordinates:

$$[\xi_1(t)\ \xi_2(t)\ \xi_3(t)]^{\mathbf{T}} = \mathbf{R}(\phi)[x_0(t)\ y_0(t)\ \phi(t)]^{\mathbf{T}} \quad (11)$$

where:

$$\mathbf{R}(\phi) = \begin{bmatrix} 0 & 0 & -1 \\ \cos\phi(t) & \sin\phi(t) & 0 \\ -\sin\phi(t) & \cos\phi(t) & 0 \end{bmatrix} \quad (12)$$

Applying the transformation (11) to model (10) lead to a chained form model [8]. An analogical to digital converter (ADC) provides to obtain samples of the chained form model. We assume constant sampling period $\Delta t_k = T$ and denote $k+1 = (k+1)T, k \in Z$. So, after some algebra, the perturbed sampled state space model in the new coordinates yields:

$$\boldsymbol{\xi}(k+1) = \mathbf{A}(k)\boldsymbol{\xi}(k) + \boldsymbol{\chi}(k) + \mathbf{w}(k); \quad (13)$$
$$\mathbf{w}^{\mathbf{T}} = [w_1\ w_2\ w_3];\ \boldsymbol{\xi}^{\mathbf{T}} = [\xi_1\ \xi_2\ \xi_3];\ k \in Z$$

where $\mathbf{w}$(k) is the process noise with Gaussian statistical distribution. Statistical mean and variance of the noise above are the following:

$$E\{\mathbf{w}(k)\} = \mathbf{0}; E\{\mathbf{w}(i)\mathbf{w}^{\mathbf{T}}(j)\} = \mathbf{0}\ \ for\ \ i \neq j$$
$$E\{\mathbf{w}(k)\mathbf{w}^{\mathbf{T}}(k)\} = \mathbf{Q} \quad (14)$$

where $\mathbf{Q}$ is the diagonal covariance matrix. Also it is:

$$\mathbf{A}(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega(k)T) & \sin(\omega(k)T) \\ 0 & -\sin(\omega(k)T) & \cos(\omega(k)T) \end{bmatrix} \quad (15)$$

$$\boldsymbol{\chi}^{\mathbf{T}}(k) = \begin{bmatrix} -\omega(k)T & u(k)\dfrac{\sin(\omega(k)T)}{\omega(k)} & u(k)\dfrac{\cos(\omega(k)T)-1}{\omega(k)} \end{bmatrix}$$

Remark 1. Note that the state space representation (13) is linear.

Remark 2. Really noised data on $\dot{\theta}_r$ and $\dot{\theta}_l$ are provided by encoders. They have to be processed by equations (6) and (10) to obtain informations on position and orientation of the robot. So an additional noise $\mathbf{w}$ has been added to model (13).

About the outputs, orientation and cartesian positions provided by a vector compass and position proprioceptive sensor respectively have been considered. So, if $\mathbf{q}$ coordinates are used, it yields:

$$\mathbf{z}(k) = \mathbf{C}\mathbf{q}(k) \quad (16)$$

where $\mathbf{z}$ is the output vector and $\mathbf{C}$ is an identity matrix. Applying the transformation (11) and considering additive measurement noise $\mathbf{n}$(k) lead to:

$$\mathbf{z}(k) = \mathbf{C}\mathbf{R}^{-1}(\phi)\boldsymbol{\xi}(k) + \mathbf{n}(k) = \mathbf{g}(\boldsymbol{\xi}(k)) + \mathbf{n}(k) \quad (17)$$

where the statistical parameters of the noise are:

$$E\{\mathbf{n}(k)\} = \mathbf{0}; E\{\mathbf{n}(i)\mathbf{n}^{\mathbf{T}}(j)\} = \mathbf{0}\ \ for\ \ i \neq j$$
$$E\{\mathbf{n}(k)\mathbf{n}^{\mathbf{T}}(k)\} = \mathbf{R} \quad (18)$$

and $\mathbf{R}$ is the diagonal covariance matrix. Note that, in consequence of the rotation (11), the $\mathbf{g}$ function of (17) is naturally nonlinear. Also $\mathbf{w}(k)$ and $\mathbf{n}$(k) noises are independent. We may write a new governing equation that linearizes the measurement process. It yields:

$$\boldsymbol{\xi}(k+1) = \mathbf{A}(k)\boldsymbol{\xi}(k) + \boldsymbol{\chi}(k) + \mathbf{w}(k);$$
$$\Delta\mathbf{y}(k) = \mathbf{H}^{\mathbf{g}}_{\xi}(k)\Delta\boldsymbol{\xi}(k) + \mathbf{n}(k) \quad (19)$$

$$\mathbf{H}^{\mathbf{g}}_{\xi}(k) = \begin{bmatrix} -(\xi_2\sin\xi_1 - \xi_3\cos\xi_1) & \cos\xi_1 & \sin\xi_1 \\ -(\xi_2\cos\xi_1 + \xi_3\sin\xi_1) & -\sin\xi_1 & \cos\xi_1 \\ -1 & 0 & 0 \end{bmatrix}_{\boldsymbol{\xi}=\boldsymbol{\xi}^*(k)} \quad (20)$$

where $\boldsymbol{\xi}^*(k)$ is solution of the process model (13) where the noise $\mathbf{w}$ is assumed to be zero.

Remark 3. With respect to $\mathbf{q}$ coordinates, the new reference in $\boldsymbol{\xi}$ coordinates is with the same origin of the world frame but rotated so as to align the axis with the robot orientation. Therefore the noises of the new ξ_i (i=1,2,3) variables are expressed in a frame attached to the robot body.

4 Adaptive Control of wheeled robots with parametric uncertainties

Let the reference trajectory of the robot be:

$$\dot{x}_r = u_r\cos\phi_r;\ \dot{y}_r = u_r\sin\phi_r;\ \dot{\phi}_r = \omega_r \quad (21)$$

were u_r>0 for all t and ω_r are the reference linear and angular velocities. The motion errors between the reference position $[\ x_r, y_r, \phi_r\ ]^{\mathbf{T}}$ and the actual position $[\ x, y, \phi\ ]^{\mathbf{T}}$ can be expressed in the vehicle local frame (*X, Y*) as [8]:

$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_r - x \\ y_r - y \\ \phi_r - \phi \end{bmatrix} \quad (22)$$

Consider the following kinematic control law :

$$u_c = u_r\cos e_\phi + k_x e_x$$
$$\omega_c = \omega_r + u_r(k_y e_y + k_\phi\sin e_\phi) \quad (23)$$

After some computations, the closed loop model yields:

$$\dot{\mathbf{e}} = [\dot{e}_x\dot{e}_y\dot{e}_\phi]^T =$$
$$\begin{bmatrix} (\omega_r + u_r(k_y e_y + k_\phi\sin e_\phi))e_y - k_x e_x \\ -(\omega_r + u_r(k_y e_y + k_\phi\sin e_\phi))e_x + u_r\sin e_\phi \\ -u_r(k_y e_y + k_\phi\sin e_\phi) \end{bmatrix} \quad (24)$$

In [6] the asymptotical stability of the equilibrium point of the closed loop model (24) has been proved. Now suppose that the values of the kinematical parameters b and r of the model (8) are not known precisely. Preliminarily one consider the following position:

$$\alpha = 1/r; \ \beta = b/r \qquad (25)$$

The estimation errors of the kinematical parameters can be defined as it follows:

$$\hat{\alpha} = \overline{\alpha} - \alpha; \ \hat{\beta} = \overline{\beta} - \beta \qquad (26)$$

where $\overline{\alpha}$ and $\overline{\beta}$ are the estimated values. So, from (5), (8) and (24), after some calculations, it results:

$$\frac{d}{dt}\begin{bmatrix} e_x \\ e_y \\ e_\phi \end{bmatrix} = \left(1 + \frac{\hat{\alpha}}{\alpha}\right) u_c \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \left(1 + \frac{\hat{\beta}}{\beta}\right) \omega_c \begin{bmatrix} e_y \\ -e_x \\ -1 \end{bmatrix} + \qquad (27)$$
$$+ \begin{bmatrix} u_r \cos e_\phi & u_r \sin e_\phi & \omega_r \end{bmatrix}^{\mathrm{T}}$$

It is possible to formulate the following theorem.

Theorem. Let the kinematic model and the control law be (10) and (23) respectively. If the linear and the angular velocities are bounded functions and the angular velocity reference converges to zero, by choosing of the following adaptive parametric laws:

$$\dot{\overline{\alpha}} = \gamma e_x u_c \quad \dot{\overline{\beta}} = (\delta \omega_c \sin e_\phi)/k_y \quad \gamma, \delta > 0 \qquad (28)$$

the solutions of the differential equations (27), i.e. longitudinal position error e_x, lateral position error e_y and orientation error e_ϕ are bounded and converge to zero.

Proof Track. We define an extended state vector:

$$\overline{\mathbf{e}}^{\mathrm{T}} = \begin{bmatrix} e_x & e_y & e_\phi & \hat{\alpha} & \hat{\beta} \end{bmatrix}$$

A Lyapunov function can be chosen as it follows:

$$V(\mathbf{e}) = \frac{1}{2}(e_x^2 + e_y^2) + [(1 - \cos e_\phi)/k_y] +$$
$$+ \hat{\alpha}^2/2\gamma\alpha + \hat{\beta}^2/2\delta\beta \quad k_y, \gamma, \delta > 0 \qquad (29)$$

The function above is positive definite. Calculating the time derivative of (29) and substituting (27) into it lead to:

$$\dot{V} = -k_x e_x^2 - (u_r k_\phi \sin^2 e_\phi)/k_y + (\hat{\alpha}/\gamma\alpha)(\dot{\overline{\alpha}} - \gamma e_x u_c) +$$
$$+ (\hat{\beta}/\delta\beta)[\dot{\overline{\beta}} - (\delta\omega_c \sin e_\phi)/k_y] \qquad (30)$$

Now considering adaptive laws (28) and substituting them into (29) lead to the stability of the motion errors (22). By applying Barbalat's Lemma one concludes on convergence to zero of e_x and e_ϕ. Considering the third equation of (27) and substituting (23) into it, one concludes on the convergence of e_y if ω_r converges to zero (Q.E.D).

That done, the adaptive kinematic control law yields:

$$\boldsymbol{\eta}_\mathbf{c} = \begin{bmatrix} \dot{\theta}_{rc} \\ \dot{\theta}_{lc} \end{bmatrix} = \begin{bmatrix} \overline{\alpha} & \overline{\beta} \\ \overline{\alpha} & -\overline{\beta} \end{bmatrix} \begin{bmatrix} u_c \\ \omega_c \end{bmatrix} \qquad (31)$$

where u_c and ω_c are given by (23). Now suppose that the values of the dynamical parameters of model (9) are not known. One consider a property of the dynamical model so that:

$$\overline{\mathbf{M}}\dot{\boldsymbol{\eta}} + \overline{\mathbf{V}}_\mathbf{m}(\boldsymbol{\eta})\boldsymbol{\eta} = \mathbf{Y}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\mathbf{p}$$

where:

$$\mathbf{p} = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}^T = \begin{bmatrix} b^2c^2m + c^2I + I_w \\ b^2c^2m - c^2I \\ 2bc^3dm_c \end{bmatrix} \qquad (32)$$

$$\mathbf{Y}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \begin{bmatrix} \ddot{\theta}_r & \ddot{\theta}_l & (\dot{\theta}_r - \dot{\theta}_l)\dot{\theta}_l \\ \ddot{\theta}_l & \ddot{\theta}_r & -(\dot{\theta}_r - \dot{\theta}_l)\dot{\theta}_r \end{bmatrix} \qquad (33)$$

Also the kinematical model (8) appears as:

$$\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r}{2} \\ \frac{r}{2b} \end{bmatrix} \dot{\theta}_r +$$
$$+ \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{r}{2} \\ \frac{r}{2b} \end{bmatrix} \dot{\theta}_l = \boldsymbol{\Sigma}_\mathbf{1}\boldsymbol{\theta}_\mathbf{1}\dot{\theta}_r + \boldsymbol{\Sigma}_\mathbf{2}\boldsymbol{\theta}_\mathbf{2}\dot{\theta}_l \qquad (34)$$

Based on adaptive backstepping approach [4], we use the following torques control law:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_l \\ \tau_r \end{bmatrix} = \overline{\mathbf{B}}^{-1}\left(-\mathbf{K}_\mathbf{d}\tilde{\boldsymbol{\eta}} + \mathbf{Y}\hat{\mathbf{p}} - \left(\frac{\partial V}{\partial \mathbf{q}}\hat{\mathbf{S}}\right)^{\mathrm{T}}\right)$$
$$\dot{\hat{\boldsymbol{\theta}}}_\mathbf{i} = \boldsymbol{\Lambda}_i\left(\frac{\partial V}{\partial \mathbf{q}}\boldsymbol{\Sigma}_i\right)\tilde{\eta}_i \quad \mathrm{i} = 1...2 \ ; \dot{\hat{\mathbf{p}}} = -\boldsymbol{\Psi}\mathbf{Y}^T\tilde{\boldsymbol{\eta}} \qquad (35)$$

where τ_l and τ_r are the control torques applied to the left and right wheels; $\hat{\boldsymbol{\theta}}_i$ is the estimation of $\boldsymbol{\theta}_i$,(cf. eq. 36); $\mathbf{Y}$ and $\mathbf{p}$ are given by (32) and (33); $\hat{\mathbf{p}}$ is the estimation of the dynamical parameters of $\mathbf{p}$ vector (cf. eq. 32); $\boldsymbol{\Sigma}_i$ (i=1,2) matrices are given by (34); V is given by (29); $\mathbf{S}$ is the Jacobian matrix (cf eq. 8) and it depends on estimated kinematic parameters $\hat{\boldsymbol{\theta}}_\mathbf{i}$; $\tilde{\boldsymbol{\eta}}$ is the following velocity error:

$$\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta}_\mathbf{c} - \boldsymbol{\eta} = \begin{bmatrix} \tilde{\eta}_1 & \tilde{\eta}_2 \end{bmatrix}^T \qquad (36)$$

where $\boldsymbol{\eta}_\mathbf{c}$ is given by (31) and $\boldsymbol{\eta}$ is the dynamical velocity (cf. eq. 9) ; $\mathbf{K}_\mathbf{d}, \boldsymbol{\Psi}$ and $\boldsymbol{\Lambda}_i$ are simmetric and positive definite matrices with appropriate dimensions. In this way the velocity error $\tilde{\boldsymbol{\eta}}$ converges to zero and, based on the theorem 2, the motion errors (22) are bounded and converge to zero.

5 On-line data fusion using EKF

From output data provided by encoders, a noised information on the actual feedback position signal $\mathbf{q}$ (cfr. eq. 7) for the adaptive control system of the previous section may be obtained suitably. So an

EKF has to be introduced in the adaptive control system. From data of more sensors (i.e. data fusion with encoders, vector compass and position sensor) the filter above estimates a filtered position signal for the feedback. Consider the sample state model (13) in $\boldsymbol{\xi}$ coordinates to elaborate the encoders data. Also consider the output equations $\Delta\mathbf{y}(k)$ of (19) to have position and orientation measurements from vector compass and sensor position. We desire estimates $\hat{\xi}(k)$ of the state $\xi(k)$ based on observation of the output $\mathbf{y}(k)$ alone. The Kalman's filtering task is to determine a Kalman gain $\mathbf{K}$ to minimize the variance of the estimation error, which is denoted $\mathbf{D}(k)$:

$$\mathbf{D}(k) = E\left\{(\xi(k)-\hat{\xi}(k))(\xi(k)-\hat{\xi}(k))^{\mathbf{T}}\right\} \tag{37}$$

Let us consider a state estimate $\bar{\xi}(k)$ so that:

$$\bar{\xi}(k) = \mathbf{A}(k-1)\hat{\xi}(k-1) + \boldsymbol{\chi}(k-1) \tag{38}$$

with error variance given by:

$$\mathbf{F}(k) = E\left\{(\xi(k)-\bar{\xi}(k))(\xi(k)-\bar{\xi}(k))^{\mathbf{T}}\right\} \tag{39}$$

Consider the incremental update (cf. eqs. 19,20):

$$\Delta\hat{\xi}(k) = \Delta\bar{\xi}(k) + \mathbf{K}(k)(\Delta\mathbf{y}(k) - \mathbf{H}^{\mathbf{g}}_{\xi}(k)\Delta\bar{\xi}(k)) \tag{40}$$

where $\mathbf{K}(k)\in\Re^{3\times1}$ is the Kalman's gain. Adding $\xi^*(k)$ on both sides of (40) and considering $\xi^*(k) = \bar{\xi}(k)$ lead to:

$$\hat{\xi}(k) = \bar{\xi}(k) + \mathbf{K}(k)(\mathbf{y}(k) - \mathbf{g}(\xi(k))) \tag{41}$$

After some computations, the solution of the minimization of $\mathbf{D}$(k) (cf. eq. 37) is:

$$\begin{aligned}\mathbf{K}_o(k) &= \mathbf{F}(k)\mathbf{H}^{\mathbf{g}}_{\xi}(k)\left(\mathbf{H}^{\mathbf{g}}_{\xi}(k)\mathbf{F}(k)(\mathbf{H}^{\mathbf{g}}_{\xi}(k))^{\mathbf{T}} + \mathbf{R}\right)^{-1} \\ \mathbf{P}_o(k) &= (\mathbf{I} - \mathbf{K}_o(k)\mathbf{H}^{\mathbf{g}}_{\xi}(k))\mathbf{F}(k)\end{aligned} \tag{42}$$

The steps of the Kalman's algorithm for the sensors data fusion are the following:
-evaluate the gain factor by using the first equation of (42); -solve the equation of measurement update (41); -update the error variance by using the second equation of (42); -prediction of the future state by using (38);-prediction of the covariance error, where:

$$\mathbf{F}(k+1) = \mathbf{A}(k)\mathbf{P}(k)\mathbf{A}^{\mathbf{T}}(k) + \mathbf{Q}\ ; \tag{43}$$

-update the time and repeat the steps.

In conclusion one provide to reconstruct a $\hat{\mathbf{q}}(k)$ vector of position in the world frame with a DAC and zero order hold to have analogical informations and to apply the adaptive control laws (31) and (35).

6 Experimental simulations

Experimental tests are performed in a nonholonomic robot. About the hardware, a PCL 1800 card with D/A 12 bit converter is used to generate reference voltage from the control torque (35); drivers LMD 18200 provide to the current generation for the DC motors of the wheels; a PCL 833 card generates the angular velocities of the wheels from data of the encoders; a microcontroller DALLAS 89C420 receives orientation data from vector compass (resolution of 0.1 degree), data on x and y coordinates from a position sensor (maximum resolution of 800 counts/inch) and transmits them to on board PC through serial port. The PC on board communicates with a PC (host) where the adaptive control laws (31), (35) and the on-line EKF are implemented by using Matlab Simulink. The real parameters of the nonholonomic robot are:

$b = 0.75m;\ d = 0.3m;\ r = 0.15m;\ m_c = 30kg$

$m_w = 1kg;\ I_c = 15.6;\ I_w = 0.005;\ I_m = 0.0025.$

The parameters of the adaptive laws (28) and of the adaptive dynamical control law (35) are:

$\gamma = 0.005;\ \delta = 20.75;\ \mathbf{K_d} = 5*\mathbf{I}_2;\ \boldsymbol{\psi} = 5*\mathbf{I}_3$

where $\mathbf{I_2}$ and $\mathbf{I_3}$ are identity matrices (2x2) and (3x3) respectively. About the kinematic control law (23), the parameters are chosen as:

$k_x = k_y = k_\phi = 5$

The initial conditions for the reference and robot positions are the following:

$(x_r(0), y_r(0), \phi_r(0)) = (0,0,3.48rad)$

$(x(0), y(0), \phi(0)) = (-30,20,5.68rad)$

The sample time for the EKF is T=10^{-4} s.

The initial values for the EKF parameters are:

$\mathbf{F}(0) = diag(0.9\ \ 0.7\ \ 0.7);$

$\mathbf{R} = diag(0.00003\ \ 0.00003\ \ 0.00003);$

$\mathbf{Q} = diag(0.1\ \ 0.1\ \ 0.1)$

We compare two cases: a) adaptive motion control without EKF where encoders data are used for the feedback directly; b)adaptive motion control with EKF where encoders informations are fused with data of the other sensors before the feedback.

Fig. 2 shows the reference trajectory. Since it satisfies the equations (21), it is nonholonomic. Figs. 3 and 4 show the longitudinal and lateral errors of the robot by using our adaptive control strategy with and without EKF. Note that the EKF filters the measurement noise in a good way.

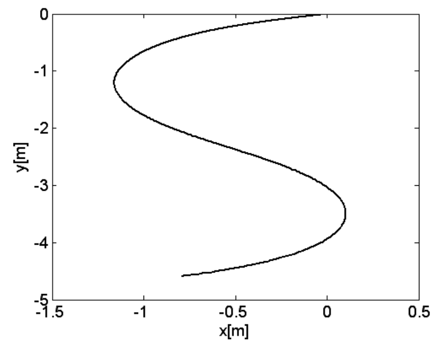

Fig.2. Reference trajectory

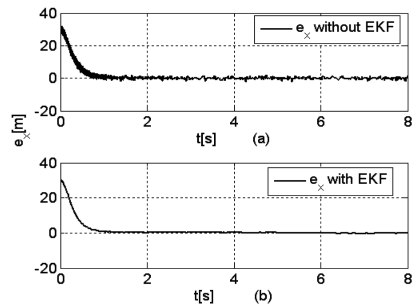

Fig. 3. Longitudinal motion errors without EKF (a) and with on-line EKF (b)

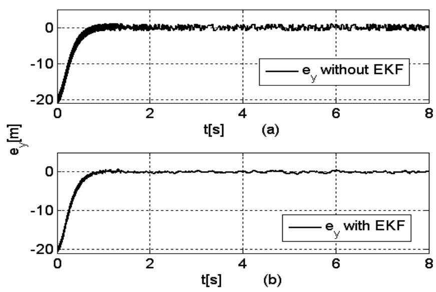

Fig.4. Lateral motion errors without EKF (a) and with on-line EKF (b)

Figs. 5 and 6 show the adaptation of the parameters. Note that the adaptive control is direct. In fact the steady state values of the parameters are constants. However they are not the physical values, but the motion errors are bounded and converge to zero suitably.

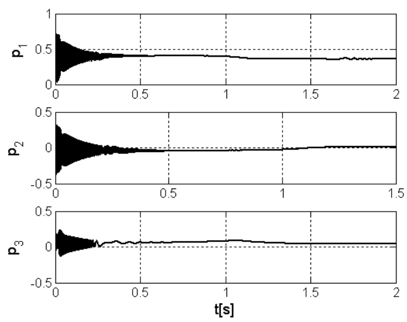

Fig.5. Adaptation of the dynamical parameters

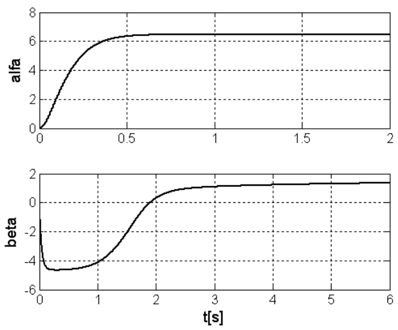

Fig. 6. Adaptation of the kinematical parameters

Also the EKF assures a good convergence of the parameters above. In fact, since the adaptive control is direct, in case of adaptive control without EKF the noised feedback signal may perturb the parameters adaptation during the transient and the steady states.

7 Conclusions

In this paper we have considered a direct adaptive EKF control for nonholonomic robots. We have shown dynamic and kinematic adaptive control laws to solve the problems of uncertainties of the robot parameters where the Lyapunov's stability has been proved. Since the adaptive control of our paper is direct, the steady state values of the parameters resulting from the adaptation are constant and are not the physical values. The values above assure the boundedness and convergence to zero of the position errors. Since data of the encoders are corrupted by noises, we have inserted an EKF in the feedback of the control system. We have shown that, based on data fusion of more measurement sensors, the EKF estimates the filtered position of the robot for the feedback. The positions errors of the experimental studies and the good parametric adaptation confirm the efficiency of the EKF and of our adaptive control strategy.

References

[1] Aicardi M., Casalino G., Bicchi A., Balestrino A., Closed Loop Steering of Unicycle-Like Vehicles via Lyapunov Techniques, *IEEE Robot. Automatic. Magazine*, vol. 2, pp. 27-35, 1995.

[2] Block A. M., Nonholonomic Mechanics and Control, *Springer Verlag*, New York inc., 2003.

[3] Coehlo P., Nunes U., Path Following Control of Mobile Robots in Presence of Uncertainties, *IEEE Transactions on Robotics*, vol. 21,pp. 252-261, 2005.

[4] Dong W., Kuhnert K.D., Robust Adaptive Control of Nonholonomic Mobile Robot with Parameter and Nonparameter Uncertainties, *IEEE Transactions on Robotics,* vol.21, No.2, 2005.

[5] Fabrizi E, Oriolo G., Panzieri S., Ulivi G., A KF based Localization Algorithm for Nonholonomic Mobile Robot, *Proceedings of 6th IEEE Mediterranean Conference on Control and Systems,* Alghero, Italy, pp. 130-135, 1998.

[6] Fierro R., Lewis F.L., Control of a Nonholomonic Mobile Robots: Backstepping Kinematics into Dynamics, *Journal of Robotic Systems,* vol. 14, no.3, pp. 149-163, 1997.

[7] Jetto L., Longhi S., Venturini G., Development and Experimental Validation of an Adaptive Extended Kalman Filter for the Localization of Mobile Robots, *IEEE Transactions on Robotics and Automation*, vol. 15, no. 2, pp. 219-229, 1999.

[8] Murray R.M. and Sastry S.S., "Nonholonomic motion planning: Steering using sinusoids", *IEEE Trans. on Aut. Control.*, vol. 38, pp. 700-716, 1993.

[9] Raimondi F.M., Melluso M., Noised Trajectory Tracking Control based on Fuzzy Lyapunov approach for Wheeled Vehicles, *Wseas Transactions on Systems*, Issue 5, vol.4, pp. 654-661, 2005.