Effect of Carrier Frequency Offset and Phase Noise on WFMT Systems

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Abstract: - This paper presents an analysis of the effect of the carrier frequency offset and phase noise on a performance of Wavelet based Filtered Multi-Tone (WFMT) systems for a general AWGN channel. It is well known that the carrier frequency offset in OFDM System yields to significant degradation of the signal-to-noise ratio (SNR). Below I show that a novel WFMT modulation is less sensitive to the frequency offset and phase noise in communication channel than OFDM /DMT modulation. The simple expression for SNR degradation in WFMT Systems is developed on base of preceding papers, those examine the carrier frequency offset effect on OFDM performance. The better performance of WFMT system versus OFDM is illustrated by calculation of SNR degradation in OFDM and WFMT systems with the same bandwidth and number of FFT points.

Key Words: - FMT, WFMT, filter-bank, OFDM, carrier, frequency offset, phase noise.

1. Introduction

Orthogonal Frequency-Division Multiplexing (OFDM) Modulation was recently proposed for local area mobile wireless networks, such as WiFi and HIPERLAN-2. By implementing an Inverse Fast Fourier transform (IFFT) at the transmitter and a Fast Fourier Transform (FFT) at the receiver, OFDM transmits data over number of ISI free sub-channels placed on orthogonal frequency carriers [1].

Although OFDM enables simple equalization, it introduces the following well-known problems.

- The peak-to-average power ratio (PAR) of the transmitted multicarrier signal is large (about 15 dB) that requires a using of components with very low nonlinear distortions.
- Because information symbols are transmitted on orthogonal sub-carriers, OFDM is very sensitive to frequency offset between transmitter and receiver oscillators, to the Doppler effect, and to phase noise.

The SNR degradation due to the carrier frequency offset in OFDM system has been extensively investigated in [11], [13], [14]. In [11] were proposed approximate expressions for SNR degradation for the additive white Gaussian noise in AWGN channel. The exact analysis of this effect for the case of multi-path fading channel was proposed in [16]. The effect of carrier phase noise on performance of OFDM and MC-CDMA systems has been depicted in [13]. Several works have been dedicated to effect of carrier frequency offset and phase noise in Filtered Multi-Tone (FMT) modulated Systems [17], [18], [20].

The Filtered Multi-Tone (FMT) Modulation was proposed by group of researches from IBM as alternative technology for xDSL [2]. This technology is based on Wavelet theory and uses complex filter-banks for synthesis and analysis of multi-channel signal. The theoretical aspects of FMT Modulation for Wireless Application were developed by Cambridge University [3] and Udine University [4]. It was shown that in many cases FMT modulation guarantees better performance than OFDM.

Effect of Phase Noise in FMT system was analyzed in [18]. An Interference Analysis of FMT over Time-varying Fading Channel was provided in [20].

A major drawback of FMT is a high realization complexity. To improve this disadvantage a Wavelet based Filtered Multi-Tone Modulation (WFMT) was developed by Data-JCE Electronics Ltd in 2002 –2003 years [5]. A new DSP algorithm realizes a low complexly WFMT core that can be implemented in a small silicon chip. The new algorithm was published in [6], [7]. In this paper, we investigate an effect of carrier frequency offset and phase noise on performance of the WFMT modulation. In next paragraph, we describe the novel

WFMT modulation, and analyze spectrum of the WFMT Transmitter. Than we compare ISI and ICI distortions, of the WFMT and OFDM systems, that appear as result of the carrier frequency offset and phase noise. The simple approximate expressions for SNR degradation in the WFMT System are presented further, followed by conclusion.

2. WFMT Modulation.

A Wavelet based Filtered Multi-Tone Modulation is a version of Filter-bank modulated communication system detail described in [], [], [], in which a synthesis and analysis of sub-channel wavelets is corresponding provided by IFFT and FFT cores. Unlike OFDM, the number M of sub-channels in WFMT system is significantly less than N - the number of IFFT/FFT points. Each sub-channel wavelet is generated in this case from K harmonics (IFFT/FFT points) so there is a simply expression for number of sub-channels:

$$M \le \frac{N}{K} \qquad (1).$$

As was shown in [], the number of harmonics K defines a quality of generated wavelets, in particular the Inter-Symbol Interference (ISI) between wavelets and a bandwidth of sub-channels. We characterize bandwidth losses of WFMT sub-channel by the roll-off factor β :

$$\beta = 2\frac{\Delta f}{R_S} , \qquad (2)$$

where Δf - is an excess bandwidth and R_S is a symbol rate. As was shown in [] the roll-off factor for WFMT system may be calculated by simple formula: $\beta = \frac{3}{K}$.

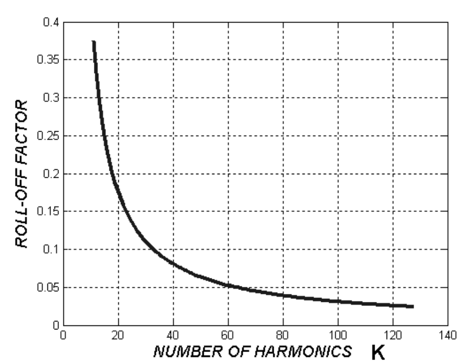

Figure 1. The Roll-off factor of the WFMT sub-channel

A level of the Inter-Symbol Interference between wavelets decreases very rapidly with an increasing of the number of the wavelet components, as it illustrates by Figure 2.

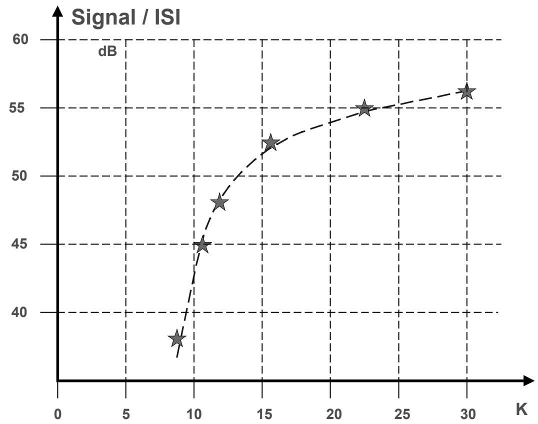

Figure 2. Inter-Symbol Interference between wavelets in WFMT Sun-channel

The principle of a WFMT modulation was described in [5] and may be illustrated by block diagram of the WFMT transmitter shown in Figure 3.

The WFMT transmitter uses an N point core IFFT and transmits M data streams a_i each at rate $1/T_0 = L/(NT)$. As shown in Fig. 2, the data streams modulate groups of K inputs of IFFT core to provide the frequency shifted wavelet coefficients. Some of the outermost data streams can be set to zero for spectral containment reasons. Finally, the coefficients are passed through the IFFT, P/S converted and finally an overlap and add operation takes place to obtain the synthesized multicarrier signal.

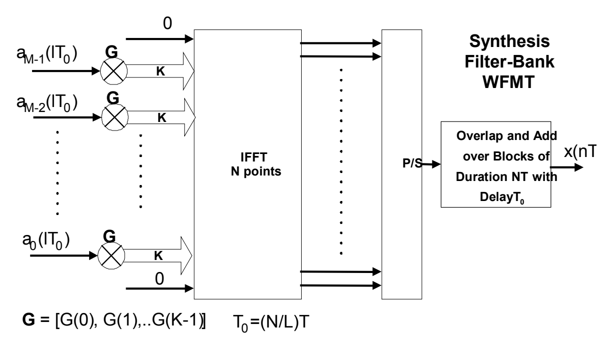

Figure 3. WFMT Transmitter

At the receiver side, after amplification, and A/D conversion, the received WFMT signal is processed by the analyses filter-bank that implements demodulation of the sub-channels wavelets. The analysis of the sub-channel wavelets is provided by the analysis of its frequency components by an FFT. To compensate the distortions introduced by the frequency selective fading channel authors propose to use a simple equalization scheme.

3. Inter-Channel Interference in WFMT System and SNR degradation

One of major advantages of WFMT technology is low level of Inter-Channel Interference (ICI) between sub-channels, because wavelets of different sub-channels are constructed from non-overlapped frequency components. Figure 4 shows how a carrier frequency offset produces an additional Inter-Channel Interference (ICI) in WFMT system.

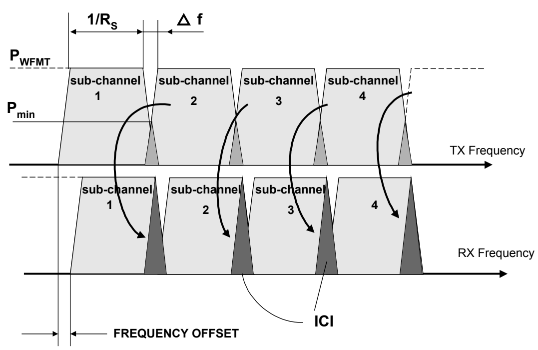

Figure 4. Inter-Channel Interference in WFMT System

We can define a power of Inter-Channel Interference P_{ICI} (in sub-channel) as function of a power of the sub-channel signal P_{WFMT} and frequency offset F_{OFF}.

$$P_{ICI} = \lambda P_{\min}\frac{\Delta f}{W} + \alpha P_{WFMT}\frac{F_{OFF}}{W}, \quad (3)$$

where $W = \frac{K}{T}$ - sub-channel bandwidth, $P_{\min}$ - a minimum power of signal on the overlapping point, α, λ - interference coefficients:

$$\alpha = \int_{-\infty}^{\infty} S(\omega)S(\omega + W + F_{OFF})\partial\omega,$$

$$\lambda = \int_{-\infty}^{\infty} S(\omega)S(\omega + W)\partial\omega,$$

where $S(\omega)$ is a spectrum of the sub-channel.

The signal to ICI ratio SNR_{ICI} in sub-channel is:

$$SNR_{ICI} = 10\log\frac{P_{WFMT}}{\lambda P_{\min}\Delta f + \alpha P_{WFMT}F_{OFF}}, \quad (4)$$

The SNR_{ICI} degradation D_{ICI} in the case of carrier frequency offset may be written as:

$$D_{ICI} = 10\log(1 + \frac{\alpha}{\lambda}\frac{P_{WFMT}}{P_{\min}}\frac{F_{OFF}}{\Delta f}), \quad (5)$$

Let insert a function ε - normalized carrier frequency offset:

$$\varepsilon = F_{OFF}T \quad (6)$$

and note that $\Delta f = 3/T$. Then (5) get an expression:

$$D_{ICI} = 10\log(1 + \frac{\alpha}{\lambda}\frac{P_{WFMT}}{P_{\min}}\frac{\varepsilon}{3}). \quad (7)$$

For small value of ε (several percent) $\alpha \cong \lambda$ and a ratio $\alpha/\lambda \cong 1$. The minimal level of $P_{\min}$ depends from form of prototype wavelet and for practical system is about 6-10 dB below P_{WFMT} so we can write an approximation expression:

$$D_{ICI} \le 10\log(1 + \frac{10}{3}\varepsilon); \quad (8)$$

4. Inter-Symbol Interference and SNR degradation

We will investigate effect of carriers frequency offset on Inter-Symbol Interference (ISI) of sub-channel wavelets. The ISI appears as result of breaking of orthogonality between K frequency components of the wavelet in the same way that appears ICI in OFDM system, which comprises only K carriers. In [11] were proposed simple expressions for degradation D of SNR for OFDM and Single carrier modulation (SC):

$$D \approx \frac{10}{\ln 10}\frac{1}{3}(\pi N\varepsilon)^2 SNR_0 \quad \text{- for OFDM,}$$

$$(9)$$

$$D \approx \frac{10}{\ln 10}\frac{1}{3}(\pi\varepsilon)^2 \quad \text{- for SC,}$$

where SNR_0 is a value of SNR in absence of the carrier frequency offset $(\varepsilon = 0)$. For sub-channel of WFMT system we can write formula for value of D_{ISI} -a SNR degradation as result of frequency shift of wavelet components:

$$D_{ISI} = \frac{10}{\ln 10} \frac{1}{3} (\pi K \varepsilon)^2 SNR_0 \quad , \qquad (10)$$

Let compare the SNR degradation in OFDM and WFMT system:

$$D_{ISI} / D = (\frac{K}{N})^2 = \frac{1}{M^2}, \qquad (11)$$

where M - is the number of WFMT sub-channels.

As we may see from (11) the WFMT system has significant less sensitivity to carrier frequency offset than OFDM system.

5. Phase Noise and SNR Degradation in WFMT system

In the case of WFMT modulation the dependence of SNR from phase noise may be simple calculated if we note than only K frequency wavelet components must be taken in account. Use simple expressions, which were proposed in [11] for definition of phase noise effect on performance of OFDM and SC systems we can write:

$$D \approx \frac{10}{\ln 10} \frac{11}{60} (4\pi N \psi) SNR_0, \quad \text{-for OFDM}$$

(12)

$$D \approx \frac{10}{\ln 10} \frac{1}{60} (4\pi \psi) SNR_0, \quad \text{- for SC}$$

where ψ is a relative phase noise line width: $\psi = \sigma T$ (σ - noise line width).

A SNR degradation D_{PN} as function of a phase noise in the case of WFMT modulation may be calculated by using a simple formula:

$$D_{PN} \approx \frac{10}{\ln 10} \frac{11}{60} (4\pi K \psi) SNR_0 \qquad (13)$$

We may compare the SNR degradation of OFDM and WFMT systems:

$$D_{PN} / D \approx \frac{K}{N} = \frac{1}{M} \quad , \qquad (14)$$

The WFMT system as we can see from (14) has better immunity to phase noise than OFDM system.

6. Conclusions

In the figures 5 and 6 we depict the SNR degradation for WFMT, OFDM and SC system in existence of the carrier frequency offset and phase noise. Figure 5 illustrates how performance of those systems is decreased with increasing of relative frequency offset ε. The SNR degradation for OFDM and SC system was calculated in accordance with expression (9), and for WFMT system was used formula (10).

As we can see from Fig.5 the WFMT system is significant less sensitive to carrier frequency offset than OFDM system. The Inter-Channel Interference decreases SNR of WFMT system in smaller manner than the Inter-Symbol Interference especially for high number of constellation points.

Of course, SC system demonstrates better performance than both OFDM and WFMT because of absence of Inter-Channel Interference (ICI) and shortest length of QAM symbols.

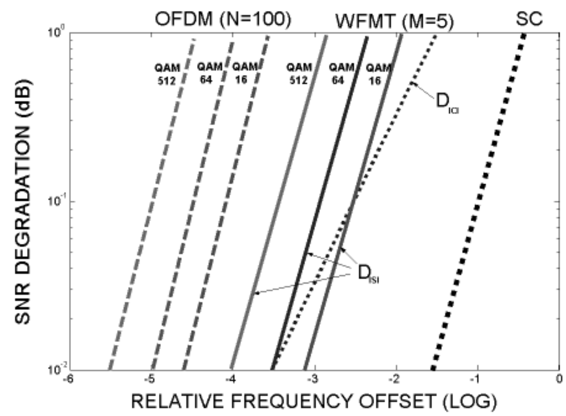

Figure 5. SNR degradation in WFMT, OFDM and SC systems as function of carrier frequency offset

The decreasing of the OFDM, WFMT, and SC performance as function of phase noise is shown in Figure 6. This graph depicts SNR degradation in dependence from relative phase noise spectrum line width ψ for different types of modulation and different numbers of constellation points. The characteristics of OFDM and SC system were defined in accordance with expressions (12). For calculation of WFMT performance we used expression (13).

As for case of carrier frequency offset, so for case of phase noise, the WFMT modulation demonstrates better performance than OFDM.

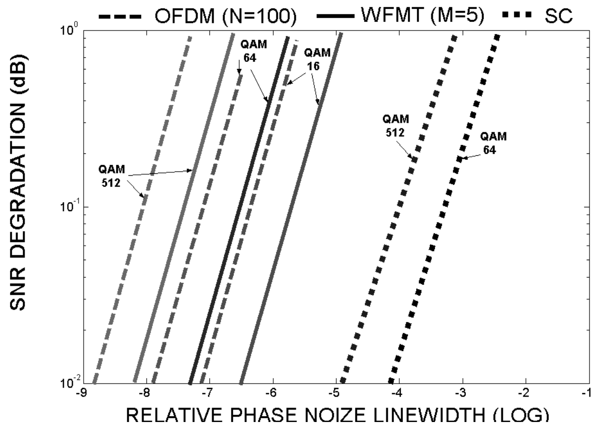

Figure 6. SNR degradation in WFMT, OFDM, and SC systems as function of phase noise line width ψ

The advantage of WFMT modulation versus OFDM increases with rising of the number of WFMT sub-channels. In practical WFMT system is used IFFT/FFT core with the number of points $128 \le N \le 1024$. The corresponding number of WFMT sub-channels is $5 \le M \le 50$. Therefore WFMT system may work properly in channel with much more phase noise level than OFDM system.

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