Adding Fractal Dimension as Textural Feature for Content Based Image Retrieval

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Abstract: The objective of the present work is to evaluate the potential of the use of the image fractal dimension as textural feature in a content-based image retrieval system. In order to compare and classify the regions of an image, we have used two distances between two partitions of the image: (1) a classic distance computed with some features (contrast, energy, entropy, homogeneity, correlation) extracted from co-occurrence matrix and (2) a distance between the fractal dimensions of the two regions. The experiments proved that the degree of confidence is increased when adding the fractal dimension to the other textural features.

Keywords: content based image retrieval, textural features, co-occurrence matrices, fractal dimension, histograms

1 Introduction

Advances in modern computer and telecommunication technologies have led to huge archives of multimedia data in diverse application areas such as medicine, remote sensing, entertainment, education and on-line information services. To use this widely available multimedia information effectively, efficient methods for storage, browsing, indexing, and retrieval must be developed. Different multimedia data types may require specific indexing and retrieval tools and methodologies. Due to the emergence of large-scale image collections, content-based image retrieval (CBIR) was proposed as a way to overcome database access difficulties. In CBIR, images are automatically indexed by summarizing their visual contents through automatically extracted quantities, or features, such as color, texture or shape. Thus, low-level numerical features, extracted by a computer, are substituted for higher-level, text-based, manual annotations or keywords. Since the inception of CBIR, many techniques have been developed along this direction and many retrieval systems, both research and commercial, have been built. Low-level features such as colors, textures and shapes of objects are widely used for CBIR. In specific applications, such as medical imaging, low-level (textural) features play a substantial role in defining the content of the data. The aim of this paper was to improve the confidence in these textural features by adding another indicator – the fractal dimension. An original method to determine the distance between two images based on the fractal dimensions histograms is presented and the convergence with the classic indicators is underlined.

2 Texture information as integrated image features

In computer vision, texture is defined as all what is left after color and local shape have been considered or it is defined by such terms as structure and randomness. Many common textures are composed of small textons (or texels) usually too great in number to be perceived as isolated objects. The elements can be placed more or less regularly or randomly. They can be almost identical or subject to large variations in their appearance and pose. Texture based approaches in extracting relevant feature sets from images in databases have been shown to give very encouraging results in addressing this problem [1], [2]. Textures are
replications, symmetries and combinations of various basic patterns or local functions, usually with some random variation. Textures have the implicit strength that they are based on intuitive notions of visual similarity. This means that they are particularly useful for searching visual databases and other human computer interaction applications. However, since the notion of texture is tied to the human semantic meaning, computational descriptions have been broad, vague and something conflicting.

Many methods have been proposed to extract texture features either directly from the image statistics, e.g., cooccurrence matrix, or from the spatial frequency domain. The statistical methods rely on the moments of the grey level histogram: mean, standard deviation, flatness etc [3], [4]. These can give interesting information about the image but have the drawback that there is no information about the relative position of the pixels. Structural methods look for a basic pattern in the image, a texture element, and then describe the region according to the repetition of the pattern [5]. In spectral approaches, the textured image is transformed into frequency domain. Then, the extraction of texture features can be done by analyzing the power spectrum [6], [7].

In our researches [8], [9], [10] we have studied the performance of four types of features: Markov Random Fields parameters, Gabor multi-channel features, fractal-based features and co-occurrence features. Experience shows that the use of a single class of descriptors to index an image database does not generally produce results that are adequate for real applications, and retrieval results are often unsatisfactory even for a research prototype. A strategy to potentially improve image retrieval, both in terms of speed and quality of results, is to combine multiple heterogeneous features. In this paper the retrieval technique is based on the analysis of given regions in an image, using both co-occurrence matrix and fractal dimension estimation of a lot of grey-level images. In order to classify the regions of an image, we have used two distances between two partitions of the image. On one hand, we used a classic distance computed with some features (contrast, energy, entropy, homogeneity, correlation) extracted form co-occurrence matrix and, on the other hand, we used an original method to compute distance between the fractal dimension of the two regions.

3 The feature integration approach

The main limitation of feature integration in most existing CBIR systems is the heavy involvement of the user, who not only must select the features to be used for each individual query, but also must specify their relative weights. An interactive CBIR system designed to simplify this problem will be discussed in the final section of the paper. This system uses the concept of integrated (or cumulative) features. The general class of accumulating features aggregates the spatial information of a partitioning irrespective of the image data. Special types of accumulating features are the global features which are calculated from the entire image. Accumulating features are symbolized by:

\[ F_j = \sum_{T} h \circ f(x) \]

where \( \sum \) represents an aggregations operation (the sum in this case, but it may be a more complex operator), \( F_j \) is the set of accumulative features or a set of accumulative features ranked in a histogram. \( F_j \) is part of feature space \( F \). \( T \) is the partitioning over which the value of \( F_j \) is computed. The operator \( h \) may hold relative weights, for example, to compute transform coefficients. A simple but very effective approach to accumulating features is to use the histogram, that is, the set of features \( F(m) \) ordered by histogram index \( m \). Joint histograms add local texture or local shape, directed edges and local higher order structures.

3.1 Co-occurrence matrices

The A co-occurrence matrix is a two-dimensional array \( C \) in which both the rows and the columns represent a set of possible image values \( V \). For example, for gray-tone images \( V \) can be the set of possible gray tones and for color images \( V \) can be the set of possible colors. The value of \( C(i,j) \) indicates how many times value \( i \) co-occurs with value \( j \) in some designated spatial relationship. For example, the spatial relationship might be that value \( i \) occurs immediately to the right of value \( j \). To be more precise, we will look specifically at the case where the set \( V \) is a set of gray tones and the spatial relationship is given by a vector \( d \) that specifies the displacement between the pixel having value \( i \) and the pixel having value \( j \). Let \( d \) be a displacement vector \((d_x, d_y)\) where \( d_x \) is a displacement in rows (downward) and \( d_y \) is a displacement in columns (to the right). Let \( V \) be a set of gray tones. The gray-
tone co-occurrence matrix $C_d$ for image 1 is defined by:

$$C_d(i, j) = |(r, c) |I(r, c) = i \text{ and } I(r + d, c + d) = j|$$

There are two important variations of the standard gray-tone co-occurrence matrix. The first is the normalized gray-tone co-occurrence matrix $N_d$ defined by:

$$N_d(i, j) = \frac{C_d(i, j)}{\sum_i \sum_j C_d(i, j)}$$

which normalizes the co-occurrence values to lie between zero and one and allows them to be thought of as probabilities in a large matrix. The second is the symmetric gray-tone co-occurrence matrix $S_d(i, j)$ defined by:

$$S_d(i, j) = C_d(i, j) + C_d(j, i)$$

which groups pairs of symmetric adjacencies.

Co-occurrence matrices capture properties of a texture, but they are not directly useful for further analysis, such as comparing two textures. Instead, numeric features are computed from the co-occurrence matrix that can be used to represent the texture more compactly. The following are standard features derivable from a normalized co-occurrence matrix.

$$Energy = \sum_i \sum_j N_d^2(i, j)$$  \hspace{1cm} (1)

$$Entropy = -\sum_i \sum_j N_d(i, j) \log_2 N_d(i, j)$$  \hspace{1cm} (2)

$$Contrast = \sum_i \sum_j (i - j)^2 N_d(i, j)$$  \hspace{1cm} (3)

$$Homogeneity = \sum_i \sum_j \frac{N_d(i, j)}{1 + |i - j|}$$  \hspace{1cm} (4)

$$Correlation = \frac{\sum_i \sum_j (i - \mu_i)(j - \mu_j)N_d(i, j)}{\sigma_i \sigma_j}$$  \hspace{1cm} (5)

where $\mu_i$, $\mu_j$ are the means and $\sigma_i$, $\sigma_j$ are the standard deviations of the row and column sums $N_d(i)$ and $N_d(j)$ defined by

$$N_d(i) = \sum_j N_d(i, j) \; ; \; N_d(j) = \sum_i N_d(i, j)$$

For the textural images the color and the texture are more important of perceptual point of view because there are not group of objects. The regions of textural images tend to spea in whole image, in time that the non-textural images are usual partition in group regions.

### 3.2 The distance between two images

There will be associated five characteristics to each image. Let consider two images $I_1$ and $I_2$ characterized by $Con1$, $En1$, $Ent1$, $Omo1$, $Cor1$ and $Con2$, $En2$, $Ent2$, $Omo2$, $Cor2$. Then the distance between $I_1$ and $I_2$ is:

$$d(I_1, I_2) = \sqrt{(Col1 - Col2)^2 + (En1 - En2)^2 + (Ent1 - Ent2)^2 + (Omo1 - Omo2)^2 + (Cor1 - Cor2)^2}$$  \hspace{1cm} (6)

For example, if we consider the test images in fig.2, having the characteristics features shown in table 1, then the distance between $I_1$ and $I_2$ will be: $d(I_1, I_2) = 18659$.

![Fig. 1. Test images (I1-left, I2-right)](image)

**Table 1.** The textural features of the test images

<table>
<thead>
<tr>
<th>Features of $I1$</th>
<th>Features of $I2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrast: 8909</td>
<td>Contrast: 1061</td>
</tr>
<tr>
<td>Energy: 2493</td>
<td>Energy: 3320</td>
</tr>
<tr>
<td>Entropy: 107</td>
<td>Entropy: 1325</td>
</tr>
<tr>
<td>Homogeneity: 1163</td>
<td>Homogeneity: 1507</td>
</tr>
<tr>
<td>Correlation: 3422</td>
<td>Correlation: 4876</td>
</tr>
</tbody>
</table>

Because is working directly with the pixels value from square blocks of images, the calculations are not complicated, the characteristics are directly extracted. One advantage of this method is that is not need to make a redetection to a very small number of gray levels. The co-occurrence matrix is a square matrix with the lines number equal with gray-tone. The method make a good distinction between the images with a fine texture and a roughly texture.

### 4 The fractal dimension of an image

Many natural surfaces have a statistical quality of roughness and self-similarity at different scales. Fractals are very useful and have become popular in modeling these properties in image processing. A deterministic fractal is defined using this concept of self-similarity as follows. Given a bounded set $A$ in a Euclidean n-space, the set $A$ is said to be self-similar when $A$ is the union of $N$ distinct (non-
overlapping) copies of itself, each of which has been scaled down by a ratio of $r$. The fractal dimension $D$ is related to the number $N$ and the ratio $r$ as follows:

$$D = \frac{\log N(r)}{\log(1/r)}$$

The fractal dimension gives a measure of the roughness of a surface. Intuitively, the larger the fractal dimension, the rougher the texture is. There are a number of methods proposed for estimating the fractal dimension $D$. One of most used is the box-counting method. Such an algorithm was implemented as an original solution [11] and will be presented in the following.

4.1 The software for fractal dimension computing

The software system we used in our fractal analysis process 24-bit color images, following the steps: First, the area of interest is selected, using a mobile cursor. The size area can be 64x64, 128x128, 256x256 or 512x512. Second, the true color image is converted into 256-gray levels image, using the formula: $I=0.299R+0.587G+0.114B$, where $R/G/B$ are the red/green/blue components which defines the color of every pixel. Then, the image is binarized using a threshold between 1-255 gray levels: all pixels whose gray level is greater or equal to the threshold will be transformed in white, the rest will become black. At this point, the forms inside the image are white on a black background. Once the image is binarized, the next step is to trace an outline of the white areas: all the white pixels which have at least one neighbor black will become part of the contour. The rest of pixels will be transformed in black. The resulted outline can now be analyzed by estimating its global fractal dimension, using the box-counting algorithm. Fig.2. presents two stages of the fractal analysis process.

4.2 The use of histograms in comparing images

Fig.3 depicts two histograms that can be obtained in the fractal analysis process. At the right, the histogram presents the fractal dimension computed for the outline in the selected area, corresponding to every gray level used as threshold in the binarization process. At the left the histograms present the fractal dimensions of two test images shown in fig.2. As the histogram shows, the image at the left contains more gray level that the image at the right.

We evaluate the distance between the fractal dimensions of two images using the least square method:

$$d = \sqrt{\frac{1}{|G|} \sum_{Df \in \mathbb{R}} (Df \text{ Im}\{\text{thres}) - Df \text{ Im}\{\text{thres})])^2}$$

where $G$ is the set of gray levels used in both images (whose corresponding fractal dimensions are different form zero) and $|G|$ is the cardinal of that set. For a higher accuracy of the result, the two images have to contain the same gray levels. For the two image set we considered above, the distance will be $d=0.2620$.

5 Experimental results

For our analysis we used 20 sets of images, four of them are shown below in fig. 4. For each set we computed the distance between two images value and the distance between fractal dimension histograms. In order to confirm the convergence of the two indicators, extracted from co-occurrence matrix and respectively the histograms of fractal dimensions, table 2 presents the results obtained by comparing four different images with a reference image (the first comparison is made directly with the original. For each pair we computed the distance between the images value computed according to Eq.6 and the distance between fractal dimensions histograms computed according to Eq.7. We obtained the results listed below:

**Fig. 2.** The dependencies of the box-counting fractal dimension on the threshold used for binarization: (a) The selected area inside an 256 gray levels image. (b) The outline of the selected area, after its binarization with the threshold 90.

**Fig. 3.** Examples of histograms obtained by fractal analysis
Table 2. Difference computed by comparing four sets of images

<table>
<thead>
<tr>
<th>Image set</th>
<th>Distance (eq.6)</th>
<th>Distance (eq.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0041</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.0296</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>0.0591</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
<td>0.1203</td>
</tr>
</tbody>
</table>

As the values indicate, the closer images are, the lower are the classic distance and the fractal distance. So, the fractal distance may be used for classifying (regions of) images composed with similar gray levels, as an alternative method to other classic method, such as distance extracted from co-occurrence matrix, or can improve the confidence when using both methods for comparison.

Fig. 4. Four of the 20 sets of images used for our investigation

Fig. 5 shows the histograms that express the fractal dimension of the images in the four sets. One can observe that the histogram of the original image remains the same.

- set 1 -

- set 2 -

- set 3 -

- set 4 -

Fig. 5. Four of the 20 sets of images used for our investigation and histograms of fractal dimension
6 Conclusions

In this paper an integrate feature vector is proposed to be utilized in comparing gray-level images, adding an indicator of the image fractality. The advantage of using the fractal distance consists in a significant decrease of the number of calculations. On the other hand, the histograms may be used for further analysis; they offer information about the complexity of the outline for every gray level in the image. The experiments proved that a classification of images into classes according to the image relevance allows effective high dimensional indexing and are attractively for applications where image collection sizes continue to expand rapidly. Results of the classification effectiveness tests showed that the algorithm assigned 80% of the sub-image pairs we were sure were relevant to the relevance class correctly. The obtained results encourage a further investigation involving larger databases.

References:


