A Novel Optimal Power Flow for Real-time Application

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Abstract: - Under the competitive environments, power companies may calculate optimal power flow (OPF) in many occasions: especially for such real time applications as contingency analysis and congestion management. However, caused by congestions, etc., it is not guaranteed that OPF has the feasible solution in every calculation case. This paper proposes a new optimal power flow calculation model applicable to the infeasible power flow cases. Namely, a concept of virtual generator is introduced to OPF to make it applicable to the infeasible power flow cases. Fuel costs of the virtual generators are set very large compared with those of the real generators. Therefore, a virtual generator generates electricity only when constraint violations must be resolved through its output. Then, if a virtual generator is connected to every node, any congestion is expected to be resolved through hypothetical generators. To demonstrate the method, several numerical examples are shown.

Key-Words: - Optimal power flow, Virtual generator, Tabu search, Congestion management, Feasible solution

1 Introduction

Recently, deregulation of electric power industry is explodes in world wide. Under such an environment, so-called “Independent Power Producer (IPP)” and “Power Producer and Supplier (PPS)” can supply power to power network or to customers. Sometimes, they construct power stations at unexpected locations for power system operators. Caused by unexpected power flows from such IPPs and PPSs, the risk of congestions in power system becomes higher if a number of IPPs or PPSs has increased. Therefore, under the competitive environments, OPF is often used to estimate the nodal prices[1-4] or to analyze congestions of the power system, and the need for OPF has been increasing.

OPF has been studied since 1960's [5][6], and has been solved by various techniques (i.e. non-linear programming, quadratic programming, linear programming, interior point methods, etc.). Under the competitive circumstances, power companies must calculate OPF in many occasions for various purposes. However, caused by congestions, etc., it is not guaranteed that OPF has the feasible solution in any case. If the problem dose not have the feasible solution, normally, conventional OPF cannot converge, and cannot find an acceptable solution. Since OPF is often used as a part of the online calculation in many cases under the competitive environment, we must find the usable converged solution even under such cases.

This paper proposes a new optimal power flow calculation model applicable to the infeasible power flow cases. Namely, a concept of virtual generator is introduced to OPF to make it applicable to the infeasible power flow cases. Fuel costs of the virtual generators are set very large compared with those of the real generators. Therefore, a virtual generator generates electricity only when constraint violations must be resolved through its output. Then, if a virtual generator is connected to all the nodes, any kind of congestions is expected to be solved. To demonstrate the method, several numerical examples are shown.

In this paper, in Chapter 2, OPF model with virtual generators is precisely explained. In chapter 3, the OPF with virtual generators is formulated mathematically. The numerical examples are shown in Chapter 4.

2 OPF model with virtual generators

If congestions exist in a power flow calculation case, the feasible solution of the OPF can not be found. However, if we assume generators that can generate unlimited power are installed in all the buses, then OPF always can converge to the computationally feasible solution because the power flow equations and all of the constraints can be satisfied at each bus by using the generators’ outputs. The above fact can easily be understood if we consider that each generator connected to every bus generates just the amount of the load of the corresponding bus.

From the above discussions, we may make the solution of OPF always feasible if we connect the hypothetical (virtual) generators to all the buses as shown in Fig.1.

If fuel cost of a virtual generator is set very large compared with that of the real generator, normally, the virtual generator's output is zero. Then, the virtual generators generate power only to resolve the constraint violations when the problem is infeasible.

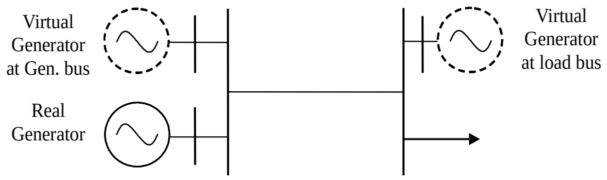
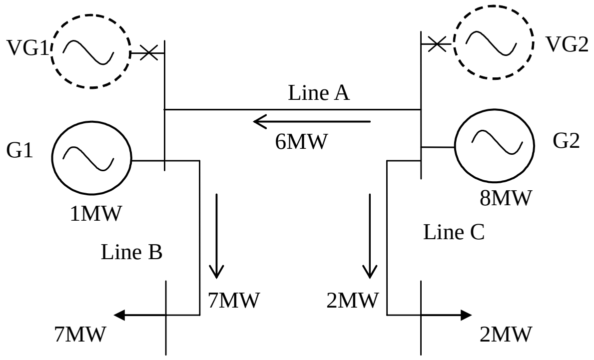


Fig.1 Example of virtual generator which is connected to generator bus and load bus

This means the following facts. If the virtual generator which is connected to the load-bus generates P (and Q), then a load of the bus must be curtailed by the amount of virtual generator's output to make the system feasible. On the other hand, if the virtual generator which is connected to the generator-bus generates P (and Q), this means that the generator must increase its power by the amount of virtual generator's output to make the system feasible.



note： G1：Max. 1MW
G2：Max. 10MW
VG：Max. 10MW
Line A：Capacity 5MW
Line B：Capacity 8MW
Line C：Capacity 5MW

Fig. 2 Simple example

For example, let us assume that congestion exists in a power system with two generators and two loads as shown in Fig.2. VG1 and VG2 are virtual generators which are connected to generator buses, and G1 and G2 are real generators. The rated output of generator G1 and G2 are 1MW and 8MW respectively. Two loads are 7MW and 2MW, respectively. A capacity of line-A is 5MW, and 8MW and 5MW for line B and C. If a transmission loss is neglected, for the case where the virtual generator is not connected, power flow of line-A exceeds its capacity by 1MW as shown in Fig.2. This congestion cannot be resolved by controlling the outputs of generators G1 & G2. Therefore, we cannot get the feasible solution for this case.

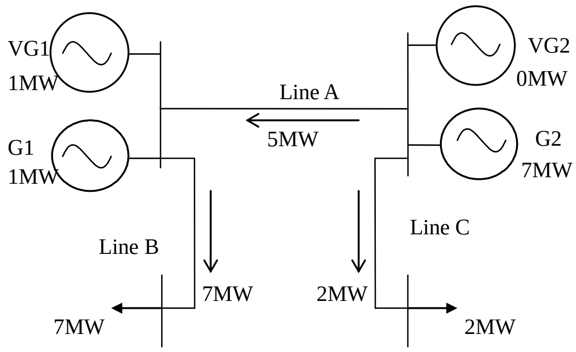


Fig. 3 Simple example with virtual generator

Here, let us assume the virtual generators, VG1 and VG2, which are connected to the generator-buses as shown in Fig. 3. We can find a feasible solution when the output of virtual generator VG1 is 1MW. Namely, the power flow of line A is 5MW, and no violation exists in the system as shown in Fig.3.

3 Formulation of OPF with virtual generators

The mathematical formulation of the OPF with virtual generators which minimizes the cost of power generation is as follows:

[Objective function]

Minimize

$$F = \sum_i f_i(P_i) + \sum_j f_j(P_j^{virtual}) \quad (i = 1,2,\cdots,I,\ j = 1,2\cdots J) \tag{1}$$

[Constraints]

(Power flow equations)

$$\begin{cases} P_k = \sum_{l=1}^{K} V_k V_l \{G_{kl}\cos(\theta_k - \theta_l) + B_{kl}\sin(\theta_k - \theta_l)\} \\ Q_k = \sum_{l=1}^{K} V_k V_l \{G_{kl}\sin(\theta_k - \theta_l) - B_{kl}\cos(\theta_k - \theta_l)\} \quad (k = 1,2,\cdots,K) \end{cases} \tag{2}$$

(Max. / Min. P output of generators)

$$\begin{cases} P_{i,\min} - P_i + \delta_{P_{i,\min}} = 0 \\ P_i - P_{i,\max} + \delta_{P_{i,\max}} = 0 \quad (i = 1,2,\cdots,I) \end{cases} \tag{3}$$

(Max. / Min. Q output of generators)

$$\begin{cases} Q_{i,\min} - Q_i + \delta_{Q_{i,\min}} = 0 \\ Q_i - Q_{i,\max} + \delta_{Q_{i,\max}} = 0 \quad (i = 1,2,\cdots,I) \end{cases} \tag{4}$$

(Max. / Min. P output of virtual generators)

$$\begin{cases} P_{j,\min}^{virtual} - P_j^{virtual} + \delta_{P_{j,\min}^{virtual}} = 0 \\ P_j^{virtual} - P_{j,\max}^{virtual} + \delta_{P_{j,\max}^{virtual}} = 0 \quad (j = 1,2,\cdots,J) \end{cases} \tag{5}$$

(Max. / Min. Q output of virtual generators)

$$\begin{cases} Q_{j,\min}^{virtual} - Q_j^{virtual} + \delta_{Q_{j,\min}^{virtual}} = 0 \\ Q_j^{virtual} - Q_{j,\max}^{virtual} + \delta_{Q_{j,\max}^{virtual}} = 0 \quad (j = 1,2,\cdots,J) \end{cases} \tag{6}$$

(Max. / Min. of line capacities)

$$\begin{cases} P_{ij,\min} - P_{ij} + \delta_{P_{ij,\min}} = 0 \\ P_{ij} - P_{ij,\max} + \delta_{P_{ij,\max}} = 0 \quad (i, j : all\ Lines) \end{cases} \tag{7}$$

(Max. / Min. of bus voltage)

$$\begin{cases} V_{k,\min} - V_k + \delta_{V_{k,\min}} = 0 \\ V_k - V_{k,\max} + \delta_{V_{k,\max}} = 0 \quad (k = 1,2,\cdots,K) \end{cases} \tag{8}$$

(Max. / Min. of transformer's tap positions)

$$\begin{cases} t_{kl,\min} - t_{kl} + \delta_{t_{kl,\min}} = 0 \\ t_{kl} - t_{kl,\max} + \delta_{t_{kl,\max}} = 0 \quad (kl = 1,2,\cdots,T) \end{cases} \tag{9}$$

(Max. / Min. of capacitor banks)

$$\begin{cases} SC_{k,\min} - SC_k + \delta_{SC_{k,\min}} = 0 \\ SC_k - SC_{k,\max} + \delta_{SC_{k,\max}} = 0 \quad (k = 1,2,\cdots,K) \end{cases} \tag{10}$$

(Max. / Min. of shunt reactor banks)

$$\begin{cases} ShR_{k,\min} - ShR_k + \delta_{ShR_{k,\min}} = 0 \\ ShR_k - ShR_{k,\max} + \delta_{ShR_{k,\max}} = 0 \quad (k = 1,2,\cdots,K) \end{cases} \tag{11}$$

(Non-negative constraints of slack variables)

$$\delta_{P_{i,\min}}, \delta_{P_{i,\max}}, \delta_{Q_{i,\min}}, \delta_{Q_{i,\max}}, \delta_{P_{ij,\min}}, \delta_{P_{ij,\max}}, \delta_{V_{k,\min}}, \delta_{V_{k,\max}}, \delta_{t_{kl,\min}}, \delta_{t_{kl,\max}},$$
$$\delta_{SC_{k,\min}}, \delta_{SC_{k,\max}}, \delta_{ShR_{k,\min}}, \delta_{ShR_{k,\max}}, \delta_{P_{j,\max}^{virtual}}, \delta_{P_{j,\min}^{virtual}}, \delta_{Q_{j,\max}^{virtual}}, \delta_{Q_{j,\min}^{virtual}} \geq 0 \tag{12}$$

(Integer variables)

$$t_{kl}, SC_k, ShR_k \in Integer \tag{13}$$

where,

(Generator's fuel cost)

$$f_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{14}$$

(Virtual generator's cost)

$$f_j(|P_j^{virtual} + jQ_j^{virtual}|) = a_j |P_j^{virtual} + jQ_j^{virtual}|^2 + b_j |P_j^{virtual} + jQ_j^{virtual}| + c_j \tag{15}$$

$f_i(P_i)$: Fuel cost of generator i, P_i : Active power of i-th generator, Q_i : Reactive power of i-th generator, $P_j^{virtual}$: active power output of j-th virtual generator, $Q_j^{virtual}$: reactive power output of j-th virtual generator, P_{ij} : Line power flow between i-th bus and j-th bus, V_k : Voltage magnitude of k-th bus G_{kl}, B_{kl} : Real part and imaginary part of an admittance of line kl, θ_{kl} : Voltage angle between k-th bus and l-th bus, SC_k, ShR_k : Shunt capacitor and inductor connected to k-th bus (integer variable), I : Total number of actual generators, J : Total number of virtual generators, K : Number of buses, a_i, b_i, c_i : coefficient for i-th generator's fuel cost, t_{kl} : klth Transformer's tap position (Integer variable), T : Number of transformers.

In the objective function (eq. (1)), the first term represents sum of fuel costs of the real generators; the second term is sum of fuel costs of the virtual generators. Eq. (2) shows the power flow equations. Eq. (3) - Eq. (11) show the upper and lower limits of active power outputs of real generators, reactive power outputs of real generators, active power outputs of virtual generators, reactive power outputs of virtual generators, line capacities, bus voltages, transformer tap positions, capacitor banks, shunt reactor banks, respectively. The variables of Eq. (9) - Eq. (11) are the discrete variables. Eq. (12) shows the non-negative constraints of slack variables. Eq. (14) and eq. (15) show the fuel cost functions for every real generator and virtual generator respectively.

4 Solution algorithm

The OPF problem formulated as above is a nonlinear mixed-integer optimization problem. The general flow chart of the solution algorithm for the problem is shown in Fig. 4 [8].

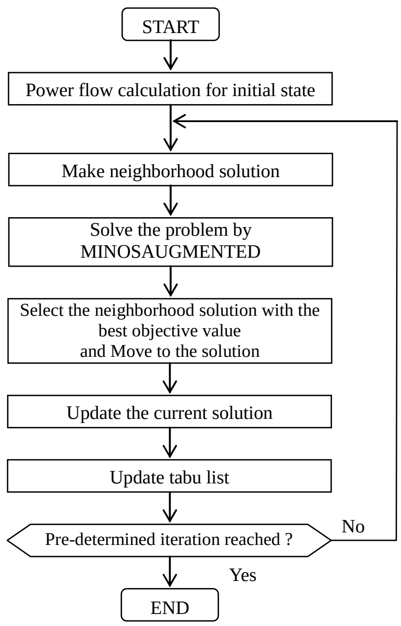


Fig. 4 Flowchart of solution algorithm

As shown in Fig. 4, first, discrete variables are fixed to create several neighborhood solutions. Since the resulting neighborhood solutions show continuous problems, they can be solved through MINOS AUGMENTED [7]. The neighborhood solution with the best objective value is selected, and the solution moves to the best one from the current solution. Then, tabu list is updated and the best solution so far is memorized. The above processes are repeated until the maximum iteration number reaches.

5 Numerical examples

The proposed OPF model is tested by IEEJ EAST-10[9] system as shown in Fig.4. The EAST-10 system has 10 generators and 12 loads. It is assumed that virtual generators are connected to all the generator buses or to all the load buses. The line capacity is assumed sufficiently large except the thick line indicated in Fig.5. The line capacity of thick line (bus24-bus27) is 2.0 P.U. Generation costs of the virtual generators are the same for all the virtual generators. Upper and lower limits of generators' output are shown in Table 1. The fuel costs for real and virtual generators are shown in Table 2. A load of each node is shown in Table 3.

Calculation parameters for tabu search are shown in Table 4.

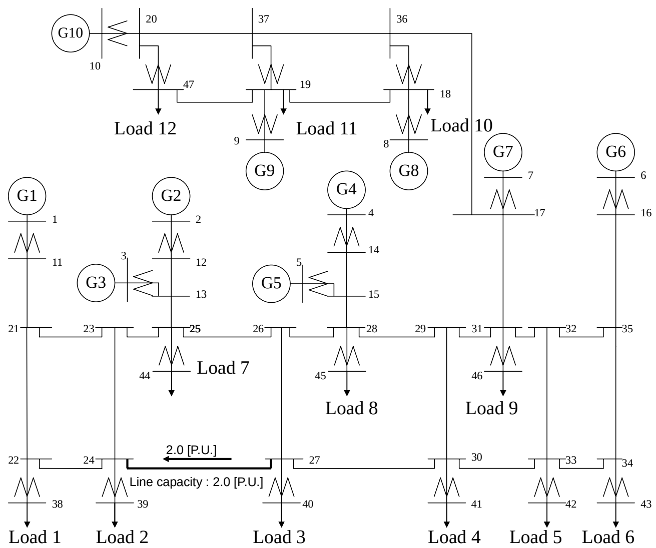


Fig. 5 IEEJ EAST-10 system

Table 1 Operational limits of generators

Generator	Active power		Reactive power	
	Upper	Lower	Upper	Lower
Generator 1	5.000	1.400	2.500	0.560
Generator 2	11.000	11.000	4.950	-1.100
Generator 3	1.700	1.500	0.805	-0.300
Generator 4	6.500	3.300	3.250	-1.100
Generator 5	1.600	1.500	0.800	-0.300
Generator 6	6.500	3.300	3.250	-1.100
Generator 7	11.000	11.000	4.950	-1.100
Generator 8	5.000	4.800	1.050	-1.050
Generator 9	5.000	4.800	1.050	-1.050
Generator 10	3.500	3.300	1.750	-0.250

Table 2 Constants of fuel costs for generators

Generator number	a_i	b_i	c_i
Generator 1	400.0	2400.0	117.0
Generator 2	0.0	0.0	0.0
Generator 3	0.0	0.0	0.0
Generator 4	700.0	400.0	550.0
Generator 5	0.0	0.0	0.0
Generator 6	700.0	400.0	550.0
Generator 7	0.0	0.0	0.0
Generator 8	380.0	500.0	260.0
Generator 9	380.0	500.0	260.0
Generator 10	50.0	500.0	200.0
Virtual Generator	0.0	10000.0	0.0

Table 3 Load of each node

Load number	Active power	Reactive power
Load 1	3.50	0.986
Load 2	7.00	1.972
Load 3	7.00	1.972
Load 4	7.00	1.972
Load 5	7.00	1.972
Load 6	3.50	0.986
Load 7	3.50	0.100
Load 8	3.50	0.100
Load 9	3.50	0.100
Load 10	3.85	1.205
Load 11	3.85	1.205
Load 12	2.80	0.806

Table 4 Calculation parameters for tabu search

Tabu Length	20
Iteration number	200

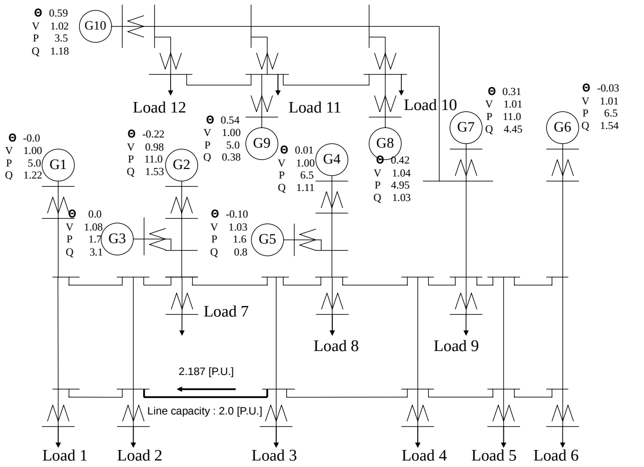


Fig. 6 Initial state for EAST-10 system (A case where line capacity violation exists)

The calculated results are shown in Fig.6, Fig.7 and Fig.8. Fig.6 shows the power flow result of the initial state of the numerical example where no virtual generator exists. From Fig.6, in the initial state, the thick line has over-load (by 0.19 P.U.) since the capacity of this line is 2.00 P.U.

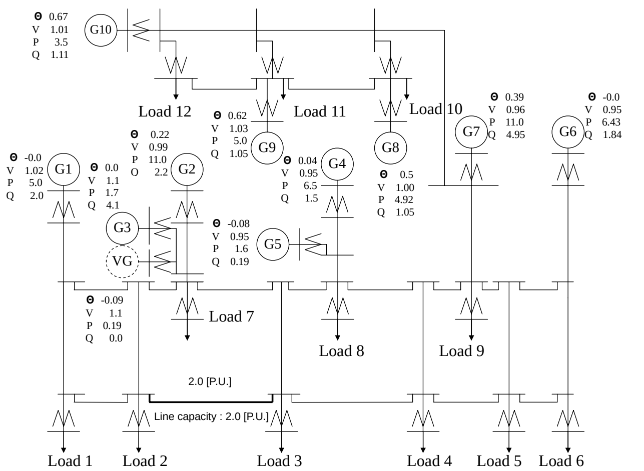


Fig. 7 Calculated power flow with virtual generator at generator-buses (Only working virtual generator is shown in Fig. 6)

For the case where virtual generators are connected to all the generator buses, the calculation result is shown in Fig.7. If the virtual generator at bus G3 increases its output by 0.19 P.U. and generator G6 decreases it output by 0.17 P.U., the overload is eliminated. For all the virtual generators other than that of bus G3, outputs are zero. Although virtual generators are installed in the all generator buses, virtual generators with output zero are not shown in Fig.7 to make the figure simple.

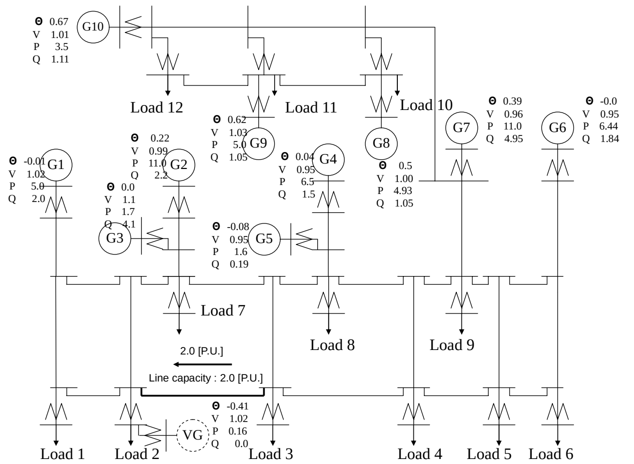


Fig. 8 Calculated power flow with virtual generator at load-buses (Only working virtual generator is shown in Fig. 7)

When a virtual generator is connected to all the load buses, the calculation result is shown in Fig.8. If the output of the virtual generator connected to the bus of load2 increases its output by 0.16 P.U. and the output of generator G6 decreases its output by 0.06 P.U., then the congestion is eliminated. Other virtual generators' outputs are zero.

The case where the virtual generators are connected to all of the buses (generator buses + load buses) is also solved. The calculation result is the same as the one shown in Fig.8. It is because the overload is eliminated at the lowest price, when virtual generators at load buses are controlled, caused by the differences of the over load elimination sensitivities.

From the above examples, it can be found that the proposed model can find the acceptable (computationally feasible) solution for an infeasible case where the normal OPF cannot be converged. Since any calculation case can be converged, the model can be applicable to the on-line calculation of the OPF. Also, the calculation result shows the minimum amount of increased generation (Fig.7) or load curtailment (Fig.8) to eliminate the overloads or to resolve the congestions.

5 Conclusion

This paper proposed a new OPF calculation model applicable to even infeasible power flow cases. In the model, the concept of the virtual generator is introduced to get feasible solutions. By introducing this model, we can obtain the optimal power flow solutions for the feasible problems, and the computationally acceptable solutions for the infeasible ones. From numerical examples, it is certified that the acceptable (computationally feasible) solutions can be found even under the infeasible power flow conditions. This model also can accurately measure the amount of electric power to resolve the congestions. Since the proposed method can solve any infeasible cases, the model can be applicable to on-line uses.

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