

# VarMet - A Novel Method for Detection of Image Singularities

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**Abstract:** The aim of this study is to detect singularities in digital images based on a novel method for feature extraction; VarMet. The multiscale median transform (MMT) is known as a tool for extracting singularities in an image. In this work an extension of the MMT is proposed. The VarMet scheme relies on computing statistical features based on the coefficients obtained from the MMT of the input image. Some experiments are presented from which it can be concluded that the VarMet scheme provides very good enhancement of singularities in the test images.

**Key-Words:** image singularities, feature extraction, object recognition, multiscale median transform (MMT), VarMet, microcalcifications

## 1 Introduction

Object recognition is a fundamental task in most image analysis applications and is also the focus of the present work. In general, objects in a digital image can be described by certain characteristics such as shape, size, orientation, and texture. In this paper, we especially focus on objects appearing in an image as singularities. As an example, astronomical images often have properties which make them different from images in industrial vision and remote sensing [1]. Astronomical images mostly contain point sources, i.e. singularities, and extended objects. Many digital X-ray mammograms exhibit some of the same characteristics as astronomical images. In some of the mammograms *microcalcifications* occur as small bright spots surrounded by normal breast tissue of varying density and complexity. For a radiologist analyzing the mammograms it is important to detect these microcalcifications at an early stage [2].

It is well known that the human visual system recognizes objects of interest at different scales. Thus, image processing techniques making use of multiscale or multiresolution representations have received considerable attention for more than a decade. Several techniques are available, such as quadrees and pyramid representations, scale space filtering, and the wavelet transform (WT). Among these, the WT has become very popular. The main advantage of this transform is its ability to separate the objects contained in an image according to their size.

Among the different types of two-dimensional (2-

D) WT algorithms the à trous algorithm [3] is one of the most well known. This algorithm produces, at each scale  $j$ , a coefficient set  $\{w_j\}$  having the same number of pixels as the original image,  $c_0$ . A pixel at position  $(x, y)$  in the original image can be expressed as the sum of all the wavelet coefficients at this position, plus the smoothed array,  $c_p(x, y)$ :

$$c_0(x, y) = c_p(x, y) + \sum_{j=1}^p w_j(x, y).$$

A transform which is quite similar to the à trous algorithm is the multiscale median transform (MMT) [4]. The main difference between the MMT and the à trous algorithm is that the MMT is a nonlinear transform. In short, the MMT is based on a series of smoothings of the input image, with successively larger kernels for the applied median filter. Consequently, each successive smoothing provides a new resolution scale. The main advantage of the MMT is that the transform is very robust to strong singularities. These singularities can be due to non-white noise, or from the observed data themselves. The MMT can be a useful tool to separate these singularities from the rest of the signal.

In this work we propose an extension to the MMT algorithm. The aim of this extension is to further enhance singularities in an image. In short our proposed scheme relies on computing statistical features (local variances) based on the coefficients obtained from the MMT of the input image. In order to demonstrate the performance of our extended version, from now on

called *VarMet*, we apply it to digital images containing singularities. An example of such an image is a digital mammogram containing microcalcifications, representing the singularities to be separated from the rest of the image. As will be demonstrated, the *VarMet* scheme generates feature images in which the singularities are further enhanced compared to the original MMT scheme.

## 2 Methods

In this section a description of the feature extraction method, *VarMet*, is presented. A block-diagram of the scheme is shown in Figure 1.

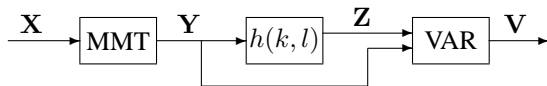


Figure 1: Block-diagram of the *VarMet* scheme for feature extraction.

In the first block, MMT, the input image  $\mathbf{X}$  is decomposed into a set of images (or scales). The number of scales depends on the given application and is determined by the user. It is a key point that each scale contains only structures of a given size. We denote the output from the MMT block as  $\mathbf{Y}$ . The purpose of the next block,  $h(k, l)$ , is to compute an estimate of the local mean at position  $(m, n)$  in the image  $\mathbf{Y}$ , resulting in the image  $\mathbf{Z}$ . In the last block, VAR, the output image from the MMT block,  $\mathbf{Y}$ , is used together with the image  $\mathbf{Z}$  to create an image of local variances,  $\mathbf{V}$ . The variance image,  $\mathbf{V}$ , can be looked upon as a feature image in which strong singularities are enhanced.

In the following we first present the basic MMT algorithm. Thereafter, the extension is described.

### 2.1 The MMT algorithm

The median, say  $\xi$ , of a set of pixel values is such that half the values in the set are less or equal to  $\xi$ , and half are greater than or equal to  $\xi$ . In order to compute the median of pixels contained within a given image area we apply a nonlinear spatial filter. The filter response is based on ranking the pixels (gray levels) contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the median of the ranking result. The principal function of the median filter is to force points with peaked gray levels to be more like their neighbors. As an example, an isolated group of pixels that are bright with respect to

their neighbors, is eliminated by a  $N \times N$  median filter given that the area of the group is less than  $N^2/2$ .

In the following we denote the median transform of the image  $\mathbf{X}$ , using a square filter kernel of dimensions  $N \times N$ , as  $med_N\{\mathbf{X}\}$ . The iteration index is denoted as  $j$ , and  $J$  is the number of resolution levels or scales. The MMT algorithm is then given as:

1. Let  $N = 2s + 1, s = 1$ .
2. Let  $\mathbf{M}_j = \mathbf{X}$  for  $j = 0$ .
3. Compute  $\mathbf{M}_{j+1} = med_{2s+1}(\mathbf{M}_j)$ .
4. Compute the multiresolution coefficients  $\mathbf{W}_{j+1} = \mathbf{M}_j - \mathbf{M}_{j+1}$ .
5. Let  $j \leftarrow j + 1; s \leftarrow 2s$ . Go to Step 3 if  $j < J$ .

The MMT algorithm produces at each scale,  $j$ , a set  $\{\mathbf{W}_j\}$  having the same number of pixels as the original image,  $\mathbf{X}$ . A series expansion of the original image,  $\mathbf{X}$ , in terms of the multiresolution coefficients,  $\mathbf{W}_j$ , is now given as follows. The original image is expressed as the sum of all the scales and the smoothed (residual) image  $\mathbf{X}_J$ :

$$\mathbf{X} = \mathbf{X}_J + \sum_{j=1}^J \mathbf{W}_j. \tag{1}$$

Equation (1) provides a reconstruction formula for the original image. Note that the smoothed image,  $\mathbf{X}_J$ , is equivalent to  $\mathbf{M}_J$ .

### 2.2 The *VarMet* scheme

As shown in Section 2.1 the output from the original MMT algorithm is  $\{\mathbf{W}_j\}_{j=1}^J$ , and singularities in the images are often found using  $\mathbf{W}_J$ . However, for our new scheme we want to be able to apply a more general output variable, say  $\mathbf{Y}$ , which may differ from  $\mathbf{W}_J$ . As an example, for the detection of microcalcifications (see Section 3) we found it beneficial to proceed from the MMT algorithm with a variable  $\mathbf{Y} = \mathbf{M}_1 - \mathbf{M}_3 = \mathbf{W}_2 + \mathbf{W}_3$ . Thus, in the following we assume that the output from the MMT step is the image  $\mathbf{Y} = f(\{\mathbf{W}_j\}_{j=1}^J)$ . Let  $\mathbf{h}$  represent a vector with  $N^2$  elements with value  $h(i) = \frac{1}{N^2}$  for all  $i$ . In addition, we define

$$\mathbf{y}(m, n) = \begin{bmatrix} y(m - K, n - K) \\ y(m - K, n - K + 1) \\ \vdots \\ y(m, n) \\ \vdots \\ y(m + K, n + K) \end{bmatrix}, \tag{2}$$

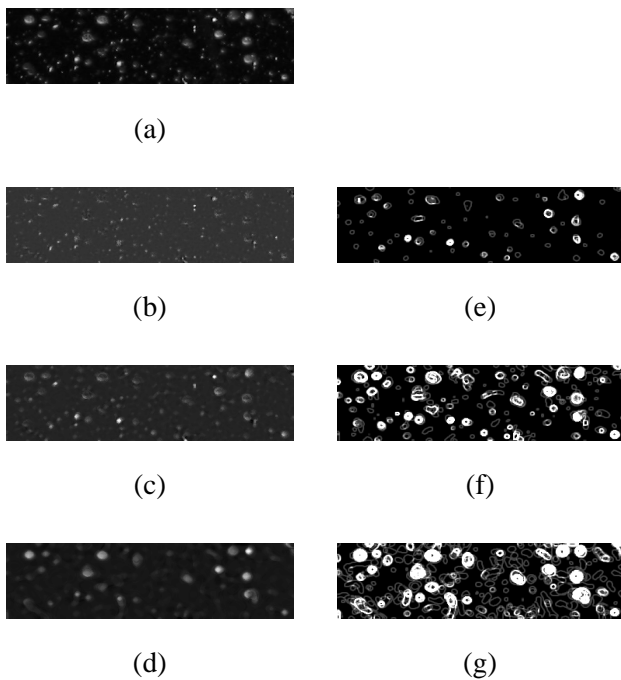


Figure 2: Figure(a): The input image "snowflakes". Figure(b): The MMT image at the second scale ( $\mathbf{W}_2$ ). Figure(c): The MMT image at the third scale ( $\mathbf{W}_3$ ). Figure(d): The MMT image at the fourth scale ( $\mathbf{W}_4$ ). Figure(e): The VarMet image at the second scale ( $\mathbf{Y} = \mathbf{W}_2$ ). Figure(f): The VarMet image at the third scale ( $\mathbf{Y} = \mathbf{W}_3$ ). Figure(g): The VarMet image at the fourth scale ( $\mathbf{Y} = \mathbf{W}_4$ ).

where  $K = \frac{N}{2}$ . Then, an estimate of the local mean at position  $(m, n)$  in  $\mathbf{Y}$  may be written as the vector expression  $\mathbf{h}^T \mathbf{y}(m, n)$ . Finally, we can estimate an image of local variances,  $\mathbf{V}$ , which is the output from the VarMet algorithm. The variance at position  $v(n, m)$  is estimated as:

$$v(m, n) = \frac{1}{N^2 - 1} \|\mathbf{y}(m, n) - \mathbf{h}^T \mathbf{y}(m, n)\|^2. \quad (3)$$

### 3 Experiments and Results

In this section we want to evaluate the performance of the feature extraction scheme, VarMet, in the detection of singularities. Figure 2 illustrates the difference between the basic MMT algorithm and the VarMet algorithm. Both algorithms are applied to the image "snowflakes", shown in Figure 2(a). For the VarMet algorithm the size of the local neighborhood is  $3 \times 3$ . Figure 2(b), (c), and (d) show the classical MMT algorithm at different scales, while in Figure 2(e), (f), and (g) the effect of applying our new scheme is il-

lustrated. By comparing Figure 2(b), (c), and (d) with Figure 2(e), (f), and (g), respectively, it can be observed that the VarMet scheme performs better than the classical MMT algorithm concerning the enhancement of the objects (i.e. snowflakes) contained in the original image. Note that the snowflakes in the original image are of different sizes and that each scale contains enhanced snowflakes of a given size.

As previously mentioned, microcalcifications may be considered as singularities in a mammogram. Several methods for the detection of microcalcifications have been presented in the literature and two examples can be found in [2, 5]. In Figure 3(a) a subimage containing a rather distinct (for the purpose of illustration) microcalcification is shown. The goal is to enhance the microcalcification as much as possible relative to the surrounding, normal tissue. Figure 3(b) shows a surface plot of the subimage in Figure 3(a). Note that the distinct microcalcification corresponds to the high peak in the surface plot. Also note the relatively variable nature of the surface surrounding this peak. Figure 3(c) shows the results of applying the MMT algorithm ( $\mathbf{W}_2 + \mathbf{W}_3$ ) to the subimage in Figure 3(a). The surface plot indicates that the normal tissue appears slightly more homogeneous than in the original subimage. In Figure 3(d) the effect of applying the VarMet algorithm ( $\mathbf{Y} = \mathbf{W}_2 + \mathbf{W}_3$ ) to the subimage is demonstrated. As can be observed, the surface corresponding to the normal tissue is almost completely flat, or homogeneous. Consequently, the corresponding VarMet feature image can easily be segmented into two classes - normal tissue and microcalcification - by simple gray level thresholding.

Experiments performed on direct digital mammograms are presented in Figure 4 and 5. The mammograms were supplied by the Breast Diagnostic Center at Stavanger University Hospital (SUS) in Stavanger, Norway. The images have a resolution of 100 microns (0.1 mm/pixel), 14 bits/pixel, and are recorded on a GE Senograph DS as a part of the daily clinical work at SUS. From the images we have extracted regions of interest (ROI) containing a cluster of microcalcifications. The clusters are biopsy proven to be malignant by expert radiologists at SUS.

Subimages containing malignant clusters of microcalcifications are shown in Figure 4(a) and 5(a). In Figure 4(b) and 5(b) the corresponding VarMet feature image is shown. As in the example above, we have proceeded from the MMT algorithm with the image  $\mathbf{Y} = \mathbf{W}_2 + \mathbf{W}_3$ . Figure 4(c) and 5(c) show a thresholded version of the feature image in Figure 4(b) and 5(b), respectively. For these two cases the threshold value is manually chosen. The white objects correspond to microcalcifications in the original subimage. Obviously, the VarMet algorithm could be

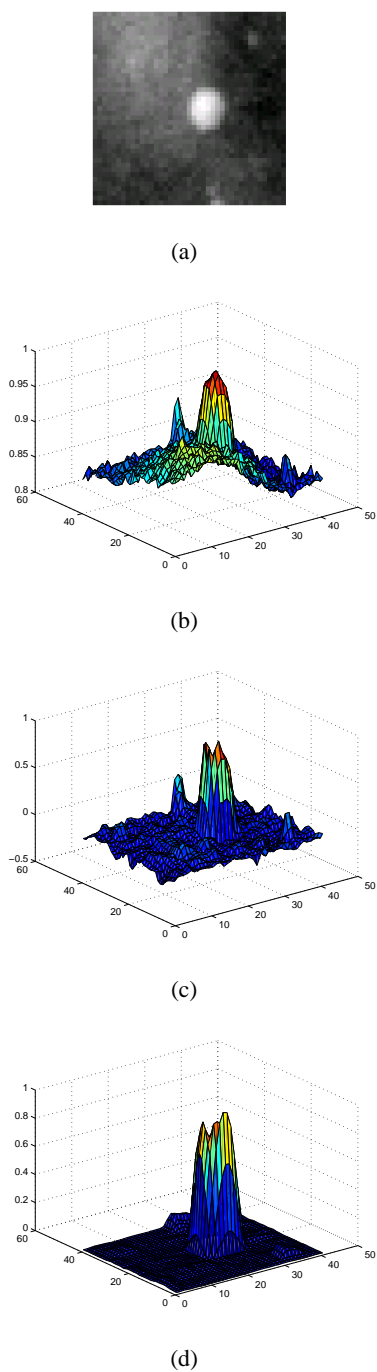


Figure 3: Figure(a): Subimage of a mammogram containing a microcalcification. Figure(b): Surface plot of the image in Figure(a). Figure(c): Surface plot of the MMT version of the image in Figure(a). Figure(d): Surface plot of the VarMet version of the image in Figure(a).

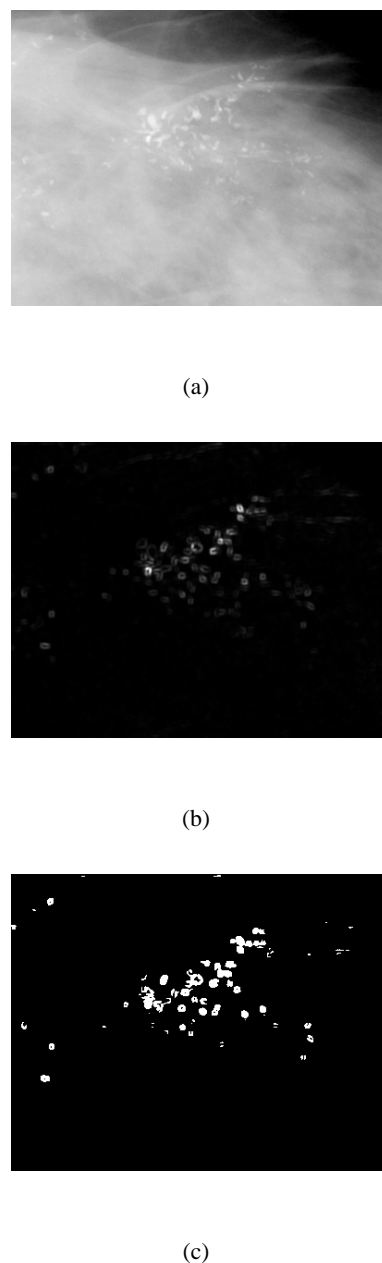
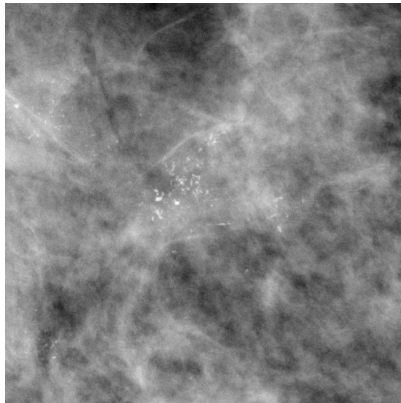
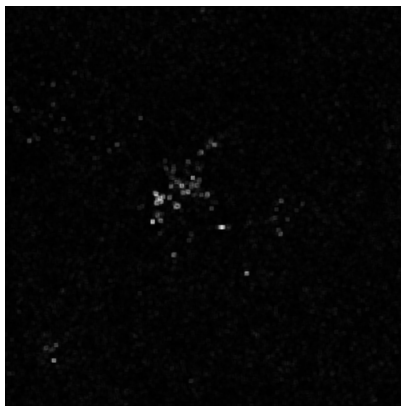


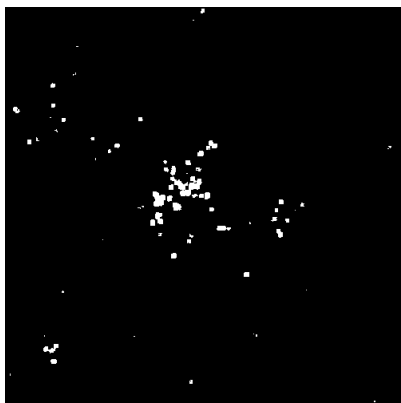
Figure 4: Figure(a): A subimage containing a malignant cluster of microcalcifications. Figure(b): The corresponding VarMet feature image. Figure(c): A thresholded version of the feature image in Figure(b).



(a)



(b)



(c)

Figure 5: Figure(a): A subimage containing a malignant cluster of microcalcifications. Figure(b): The corresponding VarMet feature image. Figure(c): A thresholded version of the feature image in Figure(b).

a useful part of a CAD system for the detection of microcalcifications. It should be mentioned that if the VarMet algorithm is to be applied in a practical Computer Aided Detection (CAD) system the feature images should be thresholded using an adaptive thresholding scheme. This is due to the fact that there may be large variations from one feature image to the next. Thus, a constant threshold value would not work very well.

#### 4 Conclusion

A novel method, VarMet, for the detection of image singularities has been presented. The method is based on an extension of the MMT algorithm. Experiments indicated that the VarMet algorithm is superior to the MMT algorithm in the enhancement of image singularities. In addition, it was demonstrated that VarMet may be a useful part of a CAD system for the detection of microcalcifications.

#### References:

- [1] J.-L. Starck, "Multiresolution Support Applied to Image Filtering and Restoration," *Graphical Models and Image Processing*, vol. 57, no. 5, 1995, pp. 420–431.
- [2] Thor Ole Gulsrud and John Håkon Husøy, "Optimal filter based detection of microcalcifications," *IEEE Transactions on Biomedical Engineering*, vol. 48, no. 11, 2001, pp. 1272–1280.
- [3] M. J. Shensa, "Discrete wavelet transform: Wedding the à trous and Mallat algorithms," *IEEE Trans. Signal Processing*, vol. 40, 1992, pp. 2464–2482.
- [4] J.-L. Starck, "Non Linear Multiscale Transforms," Springer-Verlag, 2002, pp.239–278.
- [5] Peter Heinkein and Johann Drexler and Wilfried Schneider, "Integrated Wavelets for Enhancement of Microcalcifications in Digital Mammography," *IEEE Transactions on Biomedical Engineering*, vol. 22, no. 3, 2003, pp. 402–413.