Guaranteeing Cost Minimax Strategies for Uncertain Discrete-Time Systems

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Abstract: The paper gives a general definition of guaranteeing cost strategy for uncertain dynamical systems. The strategy is constructed by a parametrized Riccati-inequality for a fairly general system with linear nominal part and with uncertainty of linear fractional form. The strategy is determined by an LMI system and by the full-block multiplier technique, too. The methods are compared. Also a fictitious system without uncertainty is shown, for which the minimax strategy is also the solution of the examined problem.

Key–Words: systems and control, optimal control, dynamical systems, guaranteeing cost control, robust state feedback

1 Introduction

The research of guaranteeing cost controls has been one of the focuses of control theory for a decade (see e.g. [8], [11], [12], [13] and the references therein). This type of controls are preferred for systems, where the system performance is influenced by uncertain effects, and the control should assure both stability and a certain performance level. In these systems both system uncertainties and an exogenous disturbance are present, and the control has to compensate any realization of these effects. A similar problem occurs in dynamic games, where several players give inputs for the uncertain system, and each player wishes to reach as low cost as possible according to their specific cost functions (see e.g. [4], [7]).

There are several approaches to construct guaranteeing cost strategies. If the control guaranteeing the lowest cost cannot be given analytically, a possible set of such controls can be determined by solving linear matrix inequalities (LMIs). These controls may result in higher cost values, but they can effectively be solved by standard software. These LMIs contain a positive scalar variable, since the determination of the guaranteeing cost control is based on different non-standard ϵ-inequalities (see e.g. [11], [13]). The full-block multiplier technique (see e.g. [2], [9]) can be considered as an extension of this approach. Instead of one additional variable, the LMI to be solved contains a whole parameter matrix. This method is based on Lemma A.1 of [9]. However, the main disadvantage is that one has to give further constraints for the uncertainty, which may considerably reduce the set of feasible solutions. A special approach for dynamical games is followed by [4] and [7], where a fictitious system is assigned to the original one, and the fictitious system performance is also appropriately modified. The fictitious system performance is not influenced by the system uncertainty, therefore a minimax strategy can be given, and these inputs will guarantee the corresponding level of cost for the player also in the original system.

In this paper discrete-time linear systems with parametric uncertainties of linear fractional form are examined. The purpose is to establish guaranteeing cost strategies by the above methods. The different approaches are compared. In section 2, a general definition for the guaranteeing cost minimax strategy is given, then the problem will be stated. In section 3, the controls are constructed by the different methods. In section 4 we compare the methods by numerical examples. Finally, section 5 concludes the paper.

In the paper standard notation is applied. The transpose of matrix A is denoted by A^T, and $P > 0$ (≥ 0) denotes the positive (semi-) definiteness of P. Notation $\mathbf{u}$ is used for the vector series $u_0, u_1, ...$, and I_n denotes the identity matrix of dimension n. $\mathcal{K}$ and $\mathcal{K}_\infty$ denote the set of usual class-$\mathcal{K}$ and class-$\mathcal{K}_\infty$ functions (definitions are given e.g. in [5]). The notation of time-dependence is omitted, if it does not cause any confusion.

2 Statement of the problem

Consider the following discrete-time uncertain system:

$$\begin{aligned} x^+ &= A_0x + B_0u + E_0w + H_0p, \\ q &= A_qx + B_qu + G_qp, \\ p &= \Delta(t)q, \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^{m_1}$ is the control, $w \in \mathbf{R}^{m_2}$ is the exogenous disturbance and $p \in \mathbf{R}^k$ describes the time-varying uncertainty, where $\Delta(t) \in \mathbf{R}^{k\times l}$ and

$$\Delta^T(t)\Delta(t) < I_l. \tag{2}$$

The initial condition isgiven by $x(0) = x_0$.

The coefficient matrices are of appropriate dimensions. If the system is well-posed, i.e. $I - \Delta G_q$ is invertible for all possible Δ satisfying (2), then (1) can be rewritten in a standard way as

$$x^+ = (A_0 + \delta A)x + (B_0 + \delta B)u + E_0w, \tag{3}$$

where

$$\delta A = H_0(I - \Delta G_q)^{-1}\Delta A_q,$$

$$\delta B = H_0(I - \Delta G_q)^{-1}\Delta B_q.$$

It can be shown that $I - \Delta G_q$ is invertible for all possible Δ satisfying (2), if and only if $(I_k - G_q^T G_q) > 0$, which is equivalant to $(I_l - G_q G_q^T) > 0$. This will be assumed throughout the paper. For system (3), consider the cost function

$$J(x_0, \mathbf{u}, \mathbf{w}) = \sum_{k=0}^{\infty}(x_k^T Q x_k + u_k^T R u_k - w_k^T S w_k), \tag{4}$$

where Q, R, and S are symmetric positive definite matrices of appropriate dimensions. The control has to be determined in such a way that the lowest possible cost is reached for any realization of the uncertainty and for any w.

Definition 1 Consider the uncertain system

$$x^+ = f_\Delta(x, u, w) \tag{5}$$

with cost function

$$J(x_0, \mathbf{u}, \mathbf{w}) = \sum_{k=0}^{\infty} L(x_k, u_k, w_k), \tag{6}$$

where f_Δ is not known, but it belongs to a known class of functions $\mathcal{F}$. Let $\mathcal{V} : \mathbf{R}^n \to \mathbf{R}$ denote a positive definite continuous function. The state-feedback $u = k(x)$ is a guaranteeing cost minimax strategy with a guaranteed cost $\mathcal{V}(x_0)$ if for all possible f_Δ and for all $x \neq 0$

$$\max_w\{\mathcal{V}(f_\Delta(x, k(x), w)) - \mathcal{V}(x) + L(x, k(x), w)\} < 0 \tag{7}$$

holds.

The problem is to establish a necessary and sufficient condition for the existence of a state-feedback guaranteeing cost minimax strategy for system (3)-(4), and to construct the control.

Proposition 1 Suppose that there exist a $\mathcal{K}_\infty$-function φ and a $\mathcal{K}$-function σ such that

$$L(x, u, w) \geq \varphi(\|x\| + \|u\|) - \sigma(\|w\|) \tag{8}$$

holds, and $u = k(x)$ is a guaranteeing cost minimax strategy. Then the closed-loop system is input-to-state stable (ISS) for every $f_\Delta \in \mathcal{F}$.

Proof. See [4].

Since the cost function (4) satisfies (8), it follows from Proposition 1 that a linear guaranteeing cost minimax strategy $u = Kx$ renders the system (3) ISS.

The problem stated above can be considered as a generalization of several ones known from recently published papers. If $S = I$ in (4), the problem is equivalent to that examined by [11]. If $S = 0$, $B_0 = B_q = 0$ and $E_0 = 0$, the problem is identical with the first one considered by [8], while one gets the second problem, if $G_q = 0$, $E_0 = 0$. If $S = 0$, $H_0 = B_0\widetilde{H}_1$, $G_q = 0$ and $E_0 = 0$, one arrives at the problems examined in [12]. The case of additive perturbation corresponds to $B_q = 0$, while at the choice of $A_q = 0$, the case of multiplicative perturbation is considered. Finally, the problem examined by [7] is also a special case of ours. That system can be rewritten in the form of (3) with $G_q = 0$.

3 Construction of the guaranteeing cost strategy

3.1 Solution by Riccati-inequality

Firstly, the best state-feedback $u = Kx$ is given analytically in such sense that the maximum on the left-hand side of (7) is as low as possible. Let $\epsilon > 0$ be a constant and introduce the following notations:

$$\begin{aligned} A_1 &= A_0 + H_0(I - G_q^T G_q)^{-1}G_q^T A_q, \\ B_1 &= B_0 + H_0(I - G_q^T G_q)^{-1}G_q^T B_q, \\ \mathcal{A} &= A_0 + B_0K, \\ \mathcal{A}_1 &= A_1 + B_1K, \\ \mathcal{A}_q &= A_q + B_qK, \\ \mathcal{A}_\Delta &= H_0(I - \Delta G_q)^{-1}\Delta(A_q + B_qK), \end{aligned}$$

$$Q_\epsilon = \frac{1}{\epsilon^2} A_q^T (I - G_q G_q^T)^{-1} A_q,$$
$$N_\epsilon = \frac{1}{\epsilon^2} A_q^T (I - G_q G_q^T)^{-1} B_q,$$
$$R_\epsilon = \frac{1}{\epsilon^2} B_q^T (I - G_q G_q^T)^{-1} B_q.$$

Theorem 1 Consider the uncertain system (3) with the cost function (4). Let P denote a positive definite symmetric matrix. Function $x \to \mathcal{V}(x) = x^T P x$ satisfies (7) with $k(x) = Kx$ for all possible Δ and for all $x \neq 0$ if and only if there exists an $\epsilon > 0$ such that

$$P_2 = (P_1^{-1} - E_0 S^{-1} E_0^T)^{-1} > 0 \tag{9}$$

with

$$P_1 = (P^{-1} - \epsilon^2 H_0 (I - G_q^T G_q)^{-1} H_0)^{-1} > 0, \tag{10}$$

and

$$\mathcal{A}_1^T P_2 \mathcal{A}_1 - P + Q + Q_\epsilon + K^T R K < 0. \tag{11}$$

If P and ϵ satisfy (9)-(10) and

$$0 > A_1^T P_2 A_1 + Q + Q_\epsilon - P - (N_\epsilon + A_1^T P_2 B_1) \times (R + R_\epsilon + B_1^T P_2 B_1)^{-1} (B_1^T P_2 A_1 + N_\epsilon^T), \tag{12}$$

then for

$$K = -(R + R_\epsilon + B_1^T P_2 B_1)^{-1} (B_1^T P_2 A_1 + N_\epsilon^T) \tag{13}$$

the state feedback $u = Kx$ is the best guaranteeing cost minimax strategy with the guaranteed cost $\mathcal{V}(x_0)$.

Proof. Consider $V(x) = x^T P x$ and set $\widetilde{\Delta} = (I - \Delta G_q)^{-1} \Delta$. Then the following inequality should be fulfilled for every w:

$$0 > \left(x^T \; w^T\right) \left[\begin{pmatrix} \mathcal{A}^T + \mathcal{A}_\Delta^T \\ E_0^T \end{pmatrix} P \left(\mathcal{A} + \mathcal{A}_\Delta \; E_0\right) + \begin{pmatrix} Q - P + K^T R K & 0 \\ 0 & -S \end{pmatrix} \right] \begin{pmatrix} x \\ w \end{pmatrix}. \tag{14}$$

By Schur-complement (14) is equivalent to

$$0 > \begin{pmatrix} Q + K^T R K - P & 0 & -\mathcal{A}^T \\ 0 & -S & -E_0^T \\ -\mathcal{A} & -E_0 & -P^{-1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -H_0^T \end{pmatrix} \widetilde{\Delta} \left(\mathcal{A}_q \; 0 \; 0\right) + \begin{pmatrix} \mathcal{A}_q^T \\ 0 \\ 0 \end{pmatrix} \widetilde{\Delta} \left(0 \; 0 \; -H_0\right).$$

By lemma 2.6. of [11], this is equivalent to the existence of a scalar $\epsilon > 0$ for which

$$\begin{pmatrix} P - (Q + Q_\epsilon + K^T R K) & 0 & \mathcal{A}_1^T \\ 0 & S & E_0^T \\ \mathcal{A}_1 & E_0 & P_1^{-1} \end{pmatrix} > 0. \tag{15}$$

Applying the Schur-complement again, (15) is equivalent to the negative definiteness of

$$\begin{pmatrix} \mathcal{A}_1^T P_1 \mathcal{A}_1 - P + Q + Q_\epsilon + K^T R K & \mathcal{A}_1^T P_1 E_0 \\ E_0^T P_1 \mathcal{A}_1 & E_0^T P_1 E_0 - S \end{pmatrix},$$

which is fulfilled if and only if (9) holds and also

$$0 > \mathcal{A}_1^T P_1 \mathcal{A}_1 - P + Q + Q_\epsilon + K^T R K - \mathcal{A}_1^T P_1 E_0 (E_0^T P_1 E_0 - S)^{-1} E_0^T P_1 \mathcal{A}_1. \tag{16}$$

Applying the matrix inversion lemma, (16) can be rewritten as (11). By completion of square we get that

$$\begin{aligned} 0 > & A_1^T P_2 A_1 + Q + Q_\epsilon - P + \\ & [K + (R + R_\epsilon + B_1^T P_2 B_1)^{-1} (B_1^T P_2 A_1 + N_\epsilon^T)]^T \\ & \times (R + R_\epsilon + B_1^T P_2 B_1) \times \\ & [K + (R + R_\epsilon + B_1^T P_2 B_1)^{-1} (B_1^T P_2 A_1 + N_\epsilon^T)] \\ & -(N_\epsilon + A_1^T P_2 B_1)(R + R_\epsilon + B_1^T P_2 B_1)^{-1} \\ & \times (B_1^T P_2 A_1 + N_\epsilon^T). \end{aligned} \tag{17}$$

From (17) it is clear that the best choice of K is (13).

It follows from Theorem 1 that, if P and ϵ are chosen to satisfy (9)-(12), the best control is obtained analytically. In the case of $N_\epsilon = 0$, these parameters can be determined by an LMI given in [4].

3.2 Solution by LMI

In this subsection an LMI system is derived, which gives the solution of the formulated problem in terms of P, ϵ and K simultaneously. By Schur-complement it is easy to prove that (9)-(10) is satisfied if and only if

$$P^{-1} - (H_0 \; E_0) \begin{pmatrix} \epsilon^2 (I - G_q^T G_q)^{-1} & 0 \\ 0 & S^{-1} \end{pmatrix} \begin{pmatrix} H_0^T \\ E_0^T \end{pmatrix} > 0. \tag{18}$$

Introduce the notation $W = P^{-1}$, $V = KP^{-1}$ and $\mu = \epsilon^2$. By Schur-complement again and applying the congruence transformation $diag\{I, \mu I, I\}$, inequality (18) can be rewritten as

$$\begin{pmatrix} W & \mu H_0 & E_0 \\ \mu H_0^T & \mu (I - G_q^T G_q) & 0 \\ E_0^T & 0 & S \end{pmatrix} > 0. \tag{19}$$

Substituting the definition of P_2 from (9) and (10), inequality (11) can be rewritten as

$$\mathcal{A}_1^T (P^{-1} - \epsilon^2 H_0 (I - G_q G_q^T)^{-1} H_0^T - E_0 S E_0^T)^{-1} \mathcal{A}_1 - P + Q + Q_\epsilon + K^T R K > 0. \tag{20}$$

Applying the Schur-complement twice we arrive at

$$0 < \begin{pmatrix} (1,1) & \mathcal{A}_1^T & \frac{1}{\epsilon} \mathcal{A}_q^T & 0 \\ \mathcal{A}_1 & (2,2) & 0 & \epsilon H_0 \\ \frac{1}{\epsilon} \mathcal{A}_q & 0 & (3,3) & 0 \\ 0 & \epsilon H_0^T & 0 & (4,4) \end{pmatrix},$$

where $(1,1) = P - (Q + K^T RK)$, $(2,2) = P^{-1} - E_0 S^{-1} E_0^T$, $(3,3) = I - G_q G_q^T$ and $(4,4) = I - G_q^T G_q$. Now by the subsequent application of the congruence transformation $diag\{P^{-1}, I, I, I\}$, then by the Schur-complement again and by the congruence transformation $diag\{I, I, \epsilon I, \epsilon I, I, I, I\}$ the following LMI for W, V and μ is obtained:

$$0 < \begin{pmatrix} W & WA_1^T + V^T B_1^T & WA_q^T + V^T B_q^T & 0 & W & V^T & 0 \\ A_1 W + B_1 V & W & 0 & \mu H_0 & 0 & 0 & E_0 \\ A_q W + B_q V & 0 & \mu(I - G_q G_q^T) & 0 & 0 & 0 & 0 \\ 0 & \mu H_0^T & 0 & \mu(I - G_q^T G_q) & 0 & 0 & 0 \\ W & 0 & 0 & 0 & Q^{-1} & 0 & 0 \\ V & 0 & 0 & 0 & 0 & R^{-1} & 0 \\ 0 & E_0^T & 0 & 0 & 0 & 0 & S \end{pmatrix} \quad (21)$$

Certainly, the LMI system to be solved has to be completed with the requirement of

$$W > 0, \quad \mu > 0. \quad (22)$$

It is easy to see that, if there are matrices W, V and a scalar μ satisfying (21) and (22), this involves (19). Therefore, if the LMI system (21)-(22) has a feasible solution, then the control can be determined as $K = VP$. In order to reach as low guaranteed cost as possible, one has to seek matrix P that has the minimum largest eigenvalue. This can be achieved in such a way that we introduce a new variable ω and a new condition as

$$\omega I < W,$$

with the objective function $\omega \to \max$.

Remark 1 Consider the fictitious without any uncertainty

$$x^+ = A_1 x + B_1 u + \begin{pmatrix} E_0 & H_0 \end{pmatrix} \begin{pmatrix} w \\ \nu \end{pmatrix} \quad (23)$$

and introduce the objective function of type (6) parametrized by ϵ as follows.

$$L_\epsilon(x, u, \overline{w}) = \begin{pmatrix} x^T & u^T \end{pmatrix} \begin{pmatrix} Q + Q_\epsilon & N\epsilon \\ N_\epsilon^T & R + R_\epsilon \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} - \overline{w}^T \begin{pmatrix} S & 0 \\ 0 & \frac{1}{\epsilon}(I - G_q^T G_q) \end{pmatrix} \overline{w} \quad (24)$$

with $\overline{w}^T = \begin{pmatrix} w^T & \nu^T \end{pmatrix}$. It is easy to show that matrix

$$\begin{pmatrix} Q + Q_\epsilon & N\epsilon \\ N_\epsilon^T & R + R_\epsilon \end{pmatrix}$$

is positive definite. In the same way as [1], it can be shown that under appropriate stabilizability and observability conditions (9)-(12) admits a positive definite solution if and only if there exists a positive ϵ such that the fictitious game (23) with (6) and (24) has equal upper and lower values. The minimax control for this game is given by $u = Kx$, where K is defined by (13).

3.3 The full-block multiplier technique

Finally we solve the problem by the full-block multiplier technique. Let the set of admissible uncertainties be denoted by

$$D = \{\Delta \in \mathbf{R}^{k \times l}, \ \Delta^T \Delta \leq I_l\}. \quad (25)$$

Consider the original system again, for which we seek the linear state-feedback guaranteeing cost minimax strategy $u = Kx$. The closed-loop system is $x^+ = \mathcal{A}_\Delta x + E_0 w$. The inequality (7) of definition 1 is equivalent to the negative definiteness of

$$\begin{pmatrix} I & 0 \\ \mathcal{A}_\Delta & E_0 \\ I & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} -P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & Q + K^T RK & 0 \\ 0 & 0 & 0 & -S \end{pmatrix} (*)$$

for all $\Delta \in D$, where the asterisk replaces the matrix that is inferred readily by symmetry. By the full-block multiplier technique, this is equivalent to the existence of an invertible multiplier

$$Z = Z^T = \begin{pmatrix} Z_1 & Z_2 \\ Z_2^T & Z_4 \end{pmatrix}$$

such that the matrix

$$\Psi = \begin{pmatrix} I & 0 & 0 \\ \mathcal{A} & E_0 & H_0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ \mathcal{A}_q & 0 & G_q \end{pmatrix}^T \begin{pmatrix} -P & 0 & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 & 0 \\ 0 & 0 & (3,3) & 0 & 0 & 0 \\ 0 & 0 & 0 & -S & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_1 & Z_2 \\ 0 & 0 & 0 & 0 & Z_2^T & Z_4 \end{pmatrix} (*),$$

where $(3,3) = Q + K^T RK$, is negative definite, and

$$(\Delta^T \ I) \, Z \begin{pmatrix} \Delta \\ I \end{pmatrix} > 0, \quad (26)$$

for all $\Delta \in D$. In what follows, the inequality $\Psi < 0$ will be transformed to a linear one. Introduce the notation

$$\begin{pmatrix} Z_1 & Z_2 \\ Z_2^T & Z_4 \end{pmatrix}^{-1} = \begin{pmatrix} Y_1 & Y_2 \\ Y_2^T & Y_4 \end{pmatrix}.$$

By permutations and by the dualization lemma (see e.g. [9], Lemma A.2) we get that

$$0 > \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -\mathcal{A}^T & -I & -\mathcal{A}_q \\ -E_0 & 0 & 0 \\ -H_0^T & 0 & -G_q^T \end{pmatrix}^T \begin{pmatrix} -P & 0 & 0 & 0 & 0 & 0 \\ 0 & -(Q+K^TRK)^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_4 & 0 & 0 & -Y_2^T \\ 0 & 0 & 0 & P^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & S^{-1} & 0 \\ 0 & 0 & -Y_2 & 0 & 0 & -Y_1 \end{pmatrix} (*).$$

By the linearization lemma, and by the subsequent application of a permutation, of the multiple use of the Schur-complement and of the congruence transformation $diag\{I, Y_1, I, P^{-1}, I, I\}$ one can prove that the following LMI for W, V, Y_1, Y_2 and Y_4 is equivalent to $\Psi < 0$:

$$\begin{pmatrix} W & H_0Y_1 & -H_0Y_2 & 0 & E_0 & A_0W + B_0V & 0 \\ Y_1H_0^T & -Y_1 & Y_1G_q^T & 0 & 0 & 0 & 0 \\ -Y_2^TH_0^T & G_qY_1 & -Y_2^TG_q^T - G_qY_2 + Y_4 & 0 & 0 & A_qW + B_qV & 0 \\ 0 & 0 & 0 & R^{-1} & 0 & V & 0 \\ E_0^T & 0 & 0 & 0 & S & 0 & 0 \\ WA_0^T + V^TB_0^T & 0 & WA_q^T + V^TB_q^T & V^T & 0 & W & W \\ 0 & 0 & 0 & 0 & 0 & W & Q^{-1} \end{pmatrix} > 0. \quad (27)$$

We can summarize these results by the following theorem.

Theorem 2 Consider system (3) with the cost function (4). Assume (27) has a feasible solution W, V and Y_1, Y_2, Y_4 such that (26) is satisfied with the corresponding Z. Then the state-feedback $u = Kx = VPx$ is a guaranteeing cost minimax strategy, and $\mathcal{V}(x_0) = x_0^TPx_0$ is a guaranteed cost, where $P = W^{-1}$.

To achieve the possible minimum cost bound, a further variable ω can be introduced and an expedient objective function can be taken for the LMI (32) as discussed earlier.

Remark 2 It should be emphasized that compared with the LMI obtained by the ϵ-inequality, the full-block multiplier technique is more flexible. Namely, in this case there is not only one parameter, but a whole parameter matrix occurs in the LMI to be solved. However, it has to be assured that (26) holds for any realization of the uncertainties. Therefore further assumptions has to be imposed for the uncertainties, e.g. the matrix Δ should be block diagonal, or the admissible set of uncertainties should be a convex polyhedron, etc. In this case (26) can be substituted by uncertainty-independent LMIs for the blocks of the multiplier as it is discussed in [9].

4 Numerical examples

In order to illustrate the application of the different methods, consider first the numerical examples of [12]. In these examples we have

$$A_0 = \begin{pmatrix} -1.0 & 0.5 \\ 1.0 & 1.5 \end{pmatrix},\ B_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ E_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$Q = I$, $R = 1$, $S = 0$.

In the case of additive perturbation we set $A_q = I_2$, $B_q = (0\ 0)^T$, $G_q = 0$ and

$$H_0 = \begin{pmatrix} 0 & 0 \\ \sqrt{0.2} & \sqrt{0.2} \end{pmatrix}.$$

If the guaranteeing cost minimax strategy is determined according to (13), and P and ϵ are determined by the LMI given in [4], the maximum eigenvalue of P is $\lambda_{\max}(P) = 47.4831$, and the eigenvalues of the closed-loop system are -0.5936 and 0.0001. If the problem is solved by the LMI (21)-(22) with the proposed objective function, we get that $\lambda_{\max}(P) = 47.4877$ and the eigenvalues are -0.5934 and 0.000045. This means that both methods give similar results, which are a bit better than those provided by the method of [12].

In the case of multiplicative perturbation we have $A_q = (0\ 0)$, $B_q = 1$, $G_q = 0$ and $H_0 = (0\ \sqrt{0.2})^T$. In this case $N_\epsilon = 0$ and $Q_\epsilon = 0$, but the LMI to be solved is non-linear in $\upsilon = \epsilon^{-2}$. Therefore, we solved (21)-(22) firstly to determine the optimal ν. This resulted in $\lambda_{\max}(P) = 117.6933$, and the eigenvalues of the closed-loop system are -0.5112 and 0.000013. If the solution is directly determined by (21)-(22), the results proved to be similar: $\lambda_{\max}(P) = 117.016$, and the corresponding eigenvalues are -0.5115 and 0.0001. In this second case we also applied the full-block multiplier technique. Here $\Delta \in \mathbf{R}$ and multipliers Y_i are scalars. In this case the fulfillment of (26) can be assured by the linear inequalities $Y_1 \pm 2Y_2 + Y_4 < 0$. In the solution the value of $\lambda_{\max}(P)$ and the eigenvalues of the closed-loop system are the same as before within the numerical accuracy. This is not surprising, since the LMIs to be solved are almost the same except for the parameter Y_2, which is close to zero in the optimal solution.

Secondly, the parameters of the multiplicative case were changed as $A_q = (0.5\ 0.25)$, $B_q = 0.5$, $G_q = 0.5$ and $E_0 = 0.1I_2$. The results showed that the guaranteed cost is far lower compared to the previous case. Here $\lambda_{\max}(P) = 34.4103$, and the eigenvalues are -0.4521 and 0.2967. If $B_q = 1$ is set again, the LMI (32) is infeasible, even if entries of A_q, G_q and E_0 are small. This shows that the guaranteed cost can relevantly be improved only if the uncertainty in the input coefficient matrix is small enough.

5 Conclusions

We determined guaranteeing cost minimax strategies in a fairly general uncertain discrete-time system with linear nominal part, where uncertainty is of linear fractional form. Firstly, the problem has been solved by a parametrized Riccati-inequality. The parameter and the strategy can simultaneously be calculated by determining a feasible solution of an LMI, too. In this case we proposed an objective function, by which one can reach the minimum cost. The full-block technique provides flexibility by more parameters in the LMI, although in this case further constraints are needed in the specific problems to capture the possible realizations of the uncertainties. The numerical examples showed that the LMI with the proposed objective function provides similar results to those resulted in the Riccati inequality. The numerical examples showed that the guaranteed cost had relevantly been reduced by the application of the proposed methods.

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