Power Market Congestion Management Incorporating Demand Elasticity Effects

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Abstract: In electric power market operation, the price of electricity may be volatile and has spikes due to transmission congestions. In conventional congestion management, demand responses are not fully considered. Using the maximum of total social welfare as objective function, a congestion management model considering demand elasticity is proposed in this paper and a two-level approach is developed for solving the congestion problem. The solutions include not only the nodal prices and generations but also the nodal demands. Case studies on a three buses network and the IEEE 30-bus system are presented to demonstrate the application of the approach.

Keywords: Congestion Management; Elastic demand; Nodal price; Power market.

I. Introduction

Electricity markets worldwide continue to be opened to suppliers and buyers of electricity. The creation of mechanisms for suppliers, and sometimes for consumers, to openly trade electricity is at the core of these changes. Most of the mechanisms being used and developed are based on an assumption that consumer demands are unchanged. However, the report of California Power Exchange incidents reveals that lack of response on consumer side may enhance the market power of generators and lead to a high power price [1]. It is necessary to study demand elasticity, the functional relationship between the price and the demand of electricity in the new competitive environment [2].

In a possible restructured energy marketplace, producers and demands of electric power are called suppliers and consumers. A system operator acts as a middleman between the suppliers and customers. Congestion management (CM) is one of the tasks of the system operator. CM is required to ensure that all the transactions are feasible without violating transmission capacity of power networks. Typically, the task of CM and pricing is vested in the hands of an Independent System Operator (ISO), who uses an optimal power flow based tool to determine the necessary reschedule generation actions to relieve congestion and to determine nodal prices. Congestion management approaches for pool and bilateral models have been reported in [3] and [4-5]. In these methods ISO will curtail pool transactions on the basis of their bids on congestion conditions. Responses of consumers, however, are not considered in conventional approaches of congestion management. Congestion may result in price volatility and leads to price spikes in competitive electricity markets because of lack of demand response.

In this paper, a demand elasticity factor at its expected demand is used to express the response of consumers on electricity. A congestion management model considering demand elasticity is proposed. A two-level approach is developed using the elasticity factor and expected demand as input parameters for solution of the model. This approach can maximize benefits of all the participants in power market. The nodal price, generation dispatches and nodal demands are all the solutions of the congestion management approach. Case studies are presented on a three-bus network and on IEEE 30-bus system. They are presented to demonstrate the application of the approach.

II. Demand Elasticity Model

Whether power systems operate in a pool model or a bilateral contract model, many consumers (especially large consumers) are likely to seek the marginal short run spot price of electricity [6]. Prices of electricity generally is set by laws of supply and demand and vary spatially (across suppliers and customers) and temporally (with time). It is usually

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related to: available generations, available transmission and distribution capacities, and demands.

Fig. 1 shows the curves of market supply and demand to price of electricity for a typical hour in June 2000 in California [7]. The demand curve is lack elastic at low consumption levels but becomes more elastic and responsive to prices when consumers' basic subsistence needs in electricity are met. All consumers require a basic amount of electricity for which they are willing to pay at relative high price. But beyond the basic needs, their demand becomes flexible and responsive to an increase in the price

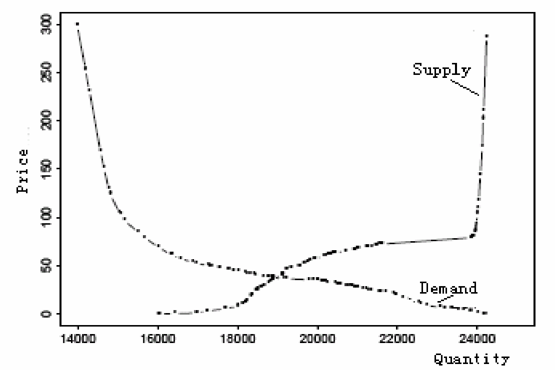

Fig. 1: A typical demand and supply bid curves for electricity in June 2000, California Electricity Market

Customers respond to electricity prices usually in two basic ways:

1. Modify usage: If the price is very high, they may reduce usage because the value of the services is less than the price. On the other hand, if the price is low, they might increase usage to receive more services than that they normally wouldn't buy.
2. Reschedule usage: If the price is high during some hours of the day and low during other hours, customers may reschedule usage to shift some their demand from high price hours to low price hours.

Let $B_i(q_{di})$ denote the total benefit of consumer at node i in consuming q_{di} quantity electricity. In buying electricity, if the power price is ρ_i, a clever consumer may analyze how much q_{di} could maximize his net benefits, i.e. maximum of $(B_i(q_{di})-\rho_i q_{di})$. On an assumption that function $B_i(q_{di})$ is continue and differentiable, it is easy to see that the consumer net benefits, $B_i(q_{di})-\rho_i q_{di}$, will reach its maximum at condition of

$$\partial B_i(q_{di})/\partial q_{di}-\rho_i=0 \tag{1}$$

Thus the consumer's demand function can be expressed by:

$$f_{di}(q_{di})=\partial B_i(q_{di})/\partial q_{di} \tag{2}$$

A curve of function $\rho_i = f_{di}(q_{di})$ is illustrated in Fig. 2. The demand elastic coefficient at point of $(\tilde{q}_{di},\tilde{\rho}_i)$ can be expressed by $e_i=-(q_{di}-\tilde{q}_{di})/(\rho_i-\tilde{\rho}_i)$. In the consideration of demand elasticity, demand characteristic function near $\tilde{q}_{di}$ can be approximated by a linear function shown in (3), and in this case the benefit function can be denoted by (4):

$$f_{di}(q_{di})\approx-\frac{1}{e_i}(q_{di}-\tilde{q}_{di})+\tilde{\rho}_i,\ (i=1,2,\cdots,N) \tag{3}$$

$$\mathrm{B}_i(q_{di})=\mathrm{B}_i(\tilde{q}_{di})-\frac{1}{2e_i}(q_{di}-\tilde{q}_{di})^2+\tilde{\rho}_i(q_{di}-\tilde{q}_{di}) \tag{4}$$

Where $\mathrm{B}_i(\tilde{q}_{di})$ denotes the benefit in usage of $\tilde{q}_{di}$ amount of electricity and $\tilde{q}_{di}$ represents the expected demand of consumer at node *i*. Since it is independent on the system operation conditions, $\mathrm{B}_i(\tilde{q}_{di})$ could be arbitrarily defined as zero. Generally, the electricity price is affected by exogenous factors such as weather, time and season. Therefore the elastic coefficient is usually not changeless.

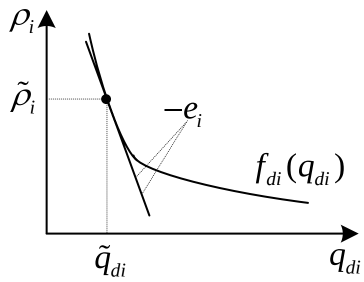

Fig. 2: Demand Price Curve

III. CONGESTION MANAGEMENT MODEL CONSIDERING DEMAND ELASTICITY

A. Objective Function

Under considering network congestions, node electricity prices would be calculated from the market clearing price. The congestion management process is performed solely by the ISO, therefore the demand elasticity offered by consumers gives ISO the necessary information for congestion management.

Assume that in each node there are at most one generator and/or one demand. Bid price function of a supplier at node i, $\rho_i = f_{si}(q_{si})$, is expressed by a quadratic function as follows:

$$f_{si}(q_{si})=c_{2i}q_{si}^2+c_{1i}q_{si}+c_{0i},\ (i=1,2,\cdots,N) \tag{5}$$

where q_{si} stands for the electricity supplied by the supplier. With the function, the generators' supply cost function can be derived as follows.

$$\mathrm{C}_i(q_{si})=\frac{1}{3}c_{2i}q_{si}^3+\frac{1}{2}c_{1i}q_{si}^2+c_{0i}q_{si} \tag{6}$$

Thus the total social welfare (*TSW*) considered in this paper is defined by the following equation.

$$TSW=\sum_{i=1}^{N}B_i(q_{di})-\sum_{i=1}^{N}C_i(q_{si}) \tag{7}$$

where N denotes the buses number of power systems. The basic economic information includes demand benefit of the consumers and cost of the suppliers. When the consumers' demands are inelastic (i.e. $e_i = \infty$, $q_{di} = \tilde{q}_{di}$, and $\mathrm{B}_i(q_{di}) = \mathrm{B}_i(\tilde{q}_{di})$), the total social welfare equals to the negative total generation cost (*TGC*), i.e.:

$$TGC = \sum_{i=1}^{N} C_i(q_{si}) \qquad (8)$$

To compare different congestions' impacts, an economic index of Transmission Surplus (*TS*) is introduced as:

$$TS = \sum_{i=1}^{N} \rho_i\, q_{di} - \sum_{i=1}^{N} \rho_i\, q_{si} \qquad (9)$$

As shown in Fig.3, in power market operation the sum curve of the supply cost function will intersect the sum curve of consumer demand at point C (corresponding to the market clearing price ρ_{C}) if network constraints are not considered. In this case, $q_{d\Sigma} = q_{s\Sigma}$, i.e. *TS* equal to zero. In Fig.3 $q_{d\Sigma} = \sum_{i=1}^{N} q_{di}$ and $q_{s\Sigma} = \sum_{i=1}^{N} q_{si}$.

Under considering network constraints, node electricity prices would deviate from the market clearing price ρ_{C}. In this case *TS* becomes unequal to zero.

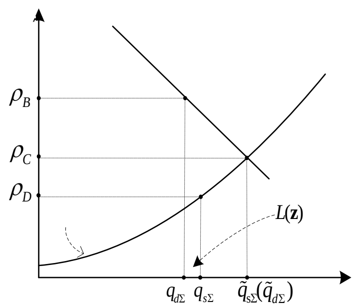

Fig. 3: An Illustration of Congestion Management in considering demand elasticity

B. Network Constraints

In this paper, the DC power flow approach is used to constructing the network constraints. Consider a power network with N nodes and L lines. The injection power at node i is defined by: $q_i = q_{si} - q_{di}$

1) Energy balance constraint

In the assumption that the time unit used in electricity trade is one, e.g. one hour, thus the energy balance constraint can be expressed by:

$$\sum_{i=1}^{N} q_{si} - \sum_{i=1}^{N} q_{di} - \tilde{L} = 0 \qquad (10)$$

where $\tilde{L}$ represents the network transmission losses. Let $\boldsymbol{z}$ stands for the vector of line active power. The vector $\boldsymbol{z}$ can be calculated by [6]:

$$\boldsymbol{z} = \boldsymbol{H} \cdot \boldsymbol{q}_{-1} \qquad (11)$$

where $\boldsymbol{H} \in R^{L} \times R^{N-1}$ is a sensitivity transfer matrix and $\boldsymbol{q}_{-1}$ denoted the vector of node injection power after eliminating the relax node. Thus $\tilde{L}$ can be evaluated by:

$$\tilde{L} = \boldsymbol{q}_{-1}^{T} \cdot \boldsymbol{H}^{T} \cdot \boldsymbol{R} \cdot \boldsymbol{H} \cdot \boldsymbol{q}_{-1} \qquad (12)$$

where $\boldsymbol{R}$ is for the network ($L \times L$) branch resistance matrix.

2) Transmission capacity constraint

Let K_l represent the transmission capacity of line l. Thus the transmission capacity constraints, i.e. the congestion inequality constraints, can be expressed by:

$$\left| \sum_{i=1}^{N-1} h_{li} q_i \right| \le K_l,\ (l = 1, \cdots, L) \qquad (13)$$

where h_{li} is for the element of matrix $\boldsymbol{H}$ on row l and the column i.

C. Models of congestion management

The node price analysis of this paper includes two steps. With customers' expected demand, $\tilde{q}_{di}$, the first step is to evaluate consumers' nodal spot prices $\tilde{\rho}_i$ without considering the network constraints (i.e. the market clearing price) using the following optimization model.

$$\begin{aligned} &\mathit{Min}\ T\mathrm{GC} \\ &\text{s.t.}\ \ \sum_{i=1}^{N} q_{si} = \sum_{i=1}^{N} \tilde{q}_{di} \end{aligned} \qquad (14)$$

The solution of (14) corresponds to the price $\tilde{\rho}_i$.

The second step computes the generation schedule, the node demand and the nodal prices ρ_i in considering the network constrains (10) and (12) using model (15). In the solution of (15), all customers' demand will respond to price volatility due to system congestions. Therefore, model (15) is used in congestion management considering demand elasticity.

$$\begin{aligned} &\mathit{Min}\ \ \text{-}TSW \\ &\text{s.t.}\ \ \sum_{i=1}^{N} q_{si} - \sum_{i=1}^{N} q_{di} - \tilde{L} = 0 \\ &\qquad \left| \sum_{i=1}^{N-1} h_{li} q_i \right| \le K_l,\ (l = 1, \cdots, L) \end{aligned} \qquad (15)$$

Problem of (15) can be solved using the Lagrange multiplier method [8-11]. This paper does not discuss the method in detail.

IV. CASE STUDIES

The proposed method has been programmed in a MATLAB platform using its optimization tools to solve the congestion management problem and tested in a three-bus network and the IEEE 30-bus system [12].

■ *Case 1: Three- bus network*

A three-bus system with a congested line as shown in Fig. 4 is used to illustrate and examine the proposed method.

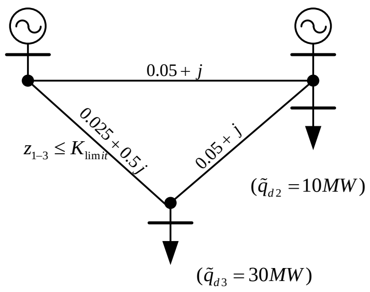

Fig. 4. A Simple 3-bus Power System

The suppliers' cost function and consumer demand data are shown in Table I without considering generation limit constraints.

TABLE I

SUPPLIER AND CONSUMER DATA OF THREE BUS SYSTEM

Suppliers	Cost function (*$*)	
Gen1	$0.0003q_{s1}^2 + 0.1q_{s1} + 2$	
Gen2	$0.0003q_{s2}^2 + 0.2q_{s2} + 1$	
Consumers	$\tilde{q}_{di}$	*e*
Con2	10	1.0
Con3	30	1.0

In the congestion management method, the first step, based on optimization model (14), obtained the imaginary price $\tilde{\rho}_i = 4.25$ *$/MWh*. Then the consumers' benefit functions (refer to (4)) are shown in Table Ⅱ.

TABLE Ⅱ

CONSUMERS' BENEFIT FUNCTION

Consumer	Benefit function
Con2	$-0.5q_{d2}^2 + 14.25q_{d2} - 92.5$
Con3	$-0.5q_{d3}^2 + 34.25q_{d3} - 577.5$

To show the impacts of different demand elasticity on the behavior of the nodal prices, the transmission capacity K_l of line 1-3 are set as: 8.0, 10.0, 15.0 and 17.0 *MW* and the rest lines do not consider the transmission capacity. The generation management results using (15) are shown in Table III.

TABLE Ⅲ

RESULTS OF CM WITH DIFFERENT CONGESTIONS

Parameter	Transmission capacity of line 1-3 *(MW)*			
s	8	10	15	17
q_{s1}	1.19	5.72	17	21.57
q_{s2}	29.12	26.57	20.2	17.65
q_{d2}	7.50	8.01	9.29	9.8
q_{d3}	22.80	24.27	27.95	29.42
ρ_{s1}	2.12	2.57	3.7	4.15
ρ_{s2}	6.82	6.31	5.04	4.53
ρ_{d2}	6.82	6.31	5.04	4.53
ρ_{d3}	11．55	10.05	6.374	4.9
-TSW	187.33	161.99	123.9	118.77

■ *Case 2: IEEE 30-bus System*

The proposed method is also studied with standard IEEE 30-bus test system. The test system, system parameters and initial buses data could be found in [12]. The expected demands $\tilde{q}_{di}$ are shown in Fig. 5.

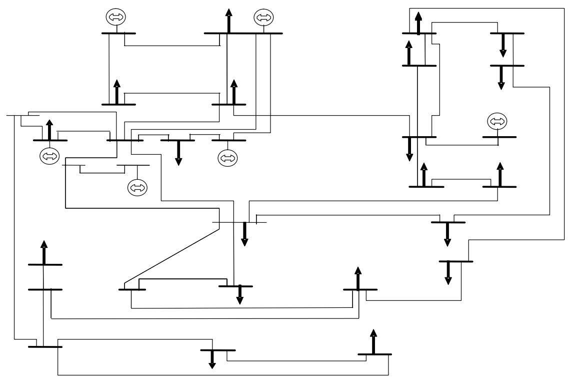

Fig. 5. The IEEE 30-bus Test System

Each generator and customer has its own cost and benefit function. Generators Costs functions used are shown in Table Ⅵ.

TABLEⅥ

COST FUNCTION COEFFICIENTS OF GENERATORS

Bus	c_2	c_1	c_0	q_s^{min}	q_s^{max}
1	0.005	0.32	2	0	200
2	0.0053	0.51	2	0	200
5	0.0074	0.83	2	0	200
8	0.01	0.03	2	0	200
11	0.01	0.83	2	0	200
13	0.0025	0.53	2	0	200

To show the impacts of demand elasticity in congestion management, the entire customer's demand-price curves have been introduced with the uniform elasticity coefficient. Here five elasticity coefficients, e_i , of 0.0, 0.2, 1.0 5.0 and ∞ are used in this study. The case $e_i = 0$ implies customer demands are fixed and $e_i = \infty$ indicates that demand prices at each bus are constant.

In the studies, the following binding system constraints are utilized in the congestion management:

- *Cog. 1*: Capacity of line 1-2 is 20.0*MW*.
- *Cog. 2*: Capacities of lines 1-2 and 25-27 are 20.0*MW* and 10.0*MW*, respectively.

Fig. 6 shows the *TS* after the congestions managements with different demand elasticity factors. It is easy to see that the value of *TS* decreases with an increase in demand elasticity; the *TS* of *Cog.2* is larger than that of *Cog.1*, which implies that the congestions of *Cog.2* are more severe; the biggest *TS* happed at $e_i = 0$, while for $e_i = \infty$, the *TS* is zero.

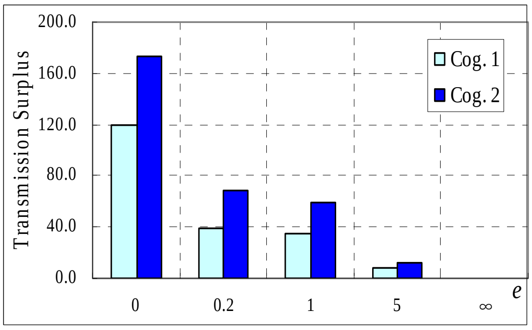

Fig. 6. *TS* with different demand elasticity levels

Fig. 7 depicts the ratios of nodal demand prices to system the clearing price with different demand elasticity levels in case of Cog. 1. The average level of prices during congestion is lower for higher values of elasticity and the standard deviation of the demand prices is much lower if compared to the case without load responsiveness. In the uniform zero elasticity case, the resulting demand prices are very high; while in the uniform infinite elasticity case all demand prices equal to the market clearing price.

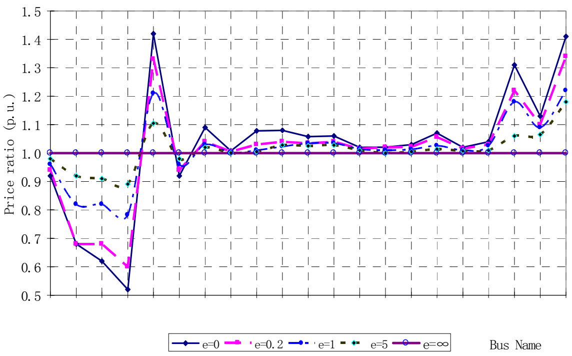

Fig. 7. Price ratios to different demand elasticity levels

V. Conclusions

The demand responsiveness on electricity price is expressed with demand elasticity coefficient in this paper. Method of congestion management in considering demand elasticity is studied. In this method, electricity demands are used as additional decision variables in the congestion management. Case studies presented show that with the demand elasticity increases the demand price decrease and the impacts of congestion are alleviated.

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