# Evaluation of the Proximity Effect upon the Impedance Characteristics of Subsea Power Transmission Cables

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*Abstract:* This paper presents an evaluation of the proximity effect upon the impedance characteristics of single-core and three-core subsea cables. The proximity effect is often ignored in subsea cable models as used by the computer based simulation packages that are widely used for establishing the performance of offshore electrical power systems. Two common types of subsea cables have been investigated in this paper; single-core and three-core cables. Models have been developed and results presented which clearly show that the proximity effect has almost no effect on single-core cables but has a significant effect upon three-core cables. The paper concludes that when calculating the harmonic impedance of single-core subsea cables then only skin effect needs to be accounted for but for three-core cables then both skin effect and proximity effects must be considered.

*Key-Words:* Proximity effect, Offshore, Subsea cables, Impedance, Harmonics

## 1 Introduction

The offshore power generation industry has developed very rapidly in recent years. To model offshore electrical power systems successfully, it is necessary to have accurate impedance models of the subsea cables otherwise, it is impossible to reliably predict system resonances and the effects of any generated harmonics such as by power converters. To develop accurate impedance models, a good understanding of the physical phenomena that goes into the makeup of the cable impedance is necessary. Many studies have been conducted to determine the appropriate methods for calculating impedance of underground cables [1]-[5]. This is not the case for subsea cables which are different in that they have a layer of heavy armour on the outside to give added strength both for laying and for protecting against mechanical damage e.g. fishing. Because of the heavy armour, the electromagnetic effects between the layers within the subsea cable need to be considered carefully when developing impedance models. Also, subsea cable arrangements and structures are diverse, which suggests that each cable type will generate a distinct impedance characteristic.

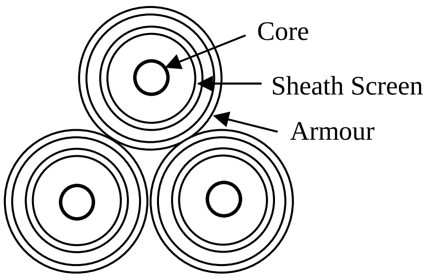
The proximity effect is a phenomenon that is seen when two conductors carrying alternating currents run parallel and close to each other. The current densities in the conductor layers on the near sides i.e. facing each other are decreased and those on the rear sides are increased because of differences in the magnetic flux densities. As a consequence there is an increase in the conductor ac resistance [6].

For single core cables having a core, a sheath and an armour layer, the impedance calculation should only need to consider the skin effect because of the concentred arrangement of cable layers. However, when other cables, such as three-phase, single-core cables or three-phase, three-core cables are being modelled then the proximity effect should be included in the analysis since there is a strong possibility that the impedance of the conductors will be affected by it. The intriguing point is how influential is the proximity effect in contributing to the overall cable impedance. It may be that for three-phase single-core cables, due to both sheath and armour, the proximity effect will be considerably small. On the other hand, for three-phase, three-core cables, the cable conductors are located within the armour so the proximity effect would certainly be expected to have an impact on impedance.

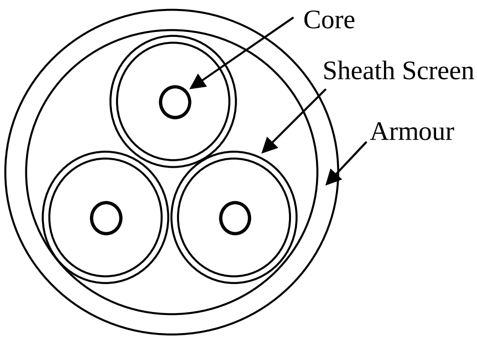
In this paper an evaluation of the proximity effect on subsea cables is presented. The evaluation has been undertaken using the theory of superposition for both determining resistance and inductance of the conductor for single-core and three-core subsea cables.

## 2 Subsea Cable Arrangements

Subsea cables are usually divided into single or multiple core types. For high power, three-phase AC transmission, cable systems are usually designed as either a three single-core cables or as a single three-core cable. Because of recent developments in cross-linked polyethylene (XLPE) insulation for high power transmission [7], then XLPE-insulated cables have been chosen for analysis in this paper. For the purpose of determining the significance of the proximity effect in subsea cables, the single-core trefoil touching formation and the three-core cable have been chosen for investigation. The single-core trefoil type is expected to be more influenced by the proximity effect as compared to the flat touching, trefoil and flat formations [8]. The two cable configurations are shown in Figure 1. The cables are considered to be lying on the sea bed at a depth of 50m and the conductor temperature is 90 °C when operational. For safety reasons, solid-bonding of both the sheath and armour have been adopted. The parameters and size of typical 150kV rating single-core and three-core subsea cable are given in the Appendix.




a. Single-Core Trefoil Formation Cable Structure.




b. Three-Core Trefoil Formation Cable Structure.

**Fig. 1 Subsea Cable Configurations**

## 3 Cable Impedance Calculations

The key to determining the proximity effect in subsea cables is to develop and use equations for impedances of multi-layer cylinders. These equations are well established but they need to be modified according to the subsea cable structure and the physical arrangement under consideration in order to evaluate accurately the impact of the proximity effect.

The general equation for magnetic potential from Faraday's law with flux linkage can be expressed by Maxwell's equations as follows [5][9]:

$$\nabla^2 \cdot \mathbf{A} = \mu \cdot \mathbf{J} \tag{1}$$

$$\mathbf{J} = \frac{j\omega}{\rho} \cdot \mathbf{A} \tag{2}$$

where, $\mathbf{A}$ is magnetic vector potential; $\mathbf{J}$ is the current density; $\mu$ is the corresponding permeability; $\rho$ is the corresponding resistivity and $\omega$ is the angular velocity.

Using cylindrical coordinates:

$$\frac{\partial^2 \mathbf{A}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{A}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \mathbf{A}}{\partial \theta^2} - j\frac{\mu\omega}{\rho}\mathbf{A} = 0 \tag{3}$$

The general solution using Bessel's equations is given as:

$$\mathbf{A}(r,\theta) = \sum_{n=0}^{\infty}[A_n I_n(mr) + B_n K_n(mr)]\cos(n\theta) \tag{4}$$

where the $I_n(x)$ is first kind modified Bessel's function, order n; $K_n(x)$ is second kind modified Bessel's function, order n; $A_n$ and $B_n$ are constants need to be determined using the boundary condition; $m = \sqrt{\frac{j\mu\omega}{\rho}}$ .

While considering the vector potential within the material conducting layer such as conductor, sheath and armour. Use the Bessel's equation (4) to derive the impedance [10]:

$$E(r,\theta) = Z \cdot I = \rho \cdot J(r,\theta) \tag{5}$$

$$Z = \rho[A_0 I_0(mr) + B_0 K_0(mr)] + \rho\sum_{n=1}^{\infty}(\frac{s}{r})^n[A_n I_n(mr) + B_n K_n(mr)] \tag{6}$$

In this formula, the first term of the right hand side is due to the skin effect while the second term is due to the proximity effect. Where, $s$ is the distance between conductors and the constants $A_0$, $B_0$, $A_n$ and $B_n$ can be referred to in [10].

### 3.1 Impedance of Single-Core Cables

For single-core subsea cable, the impedance is described in [11].

$$Z_{cable} = [Z_i] + [Z_{earth}] \tag{7}$$

For a three-phase system, the matrix can be express as:

$$Z_{cable} = \begin{bmatrix} Z_{i1} & 0 & 0 \\ 0 & Z_{i2} & 0 \\ 0 & 0 & Z_{i3} \end{bmatrix} + \begin{bmatrix} Z_{earth11} & Z_{earth12} & Z_{earth13} \\ Z_{earth21} & Z_{earth22} & Z_{earth23} \\ Z_{earth31} & Z_{earth32} & Z_{earth33} \end{bmatrix} \quad (8)$$

Where, the first term of the right hand side is the internal impedance matrix of one single-core cable of a three-phase transmission system with the conductor, sheath and armour layers considered and derived in [11] and [12] as:

$$\begin{bmatrix} Z_{cc} & Z_{cs} & Z_{ca} \\ Z_{sc} & Z_{ss} & Z_{sa} \\ Z_{ac} & Z_{as} & Z_{aa} \end{bmatrix} \quad (9)$$

The second term is the earth return impedance matrix which includes self and mutual impedances using Wedepohl approach [13].

Due to the symmetrical arrangement within a single-core cable, the proximity effect will be zero when considering the internal impedance alone. However, outside the cable core there are eddy currents flowing in the conducting layers due to the presence of other phases and the proximity effect will influence these, especially the outer most circulating current loops, thereby affecting the armour and sea return impedance matrix. The cable impedance matrix should therefore include the proximity effect:

$$Z_{cable} = [Z_i] + [Z_{pe}] + [Z_{earth}] \quad (10)$$

where,

$$[Z_{pe}] = \begin{bmatrix} Z_{pe-self11} & Z_{pe-mutu12} & Z_{pe-mutu13} \\ Z_{pe-mutu21} & Z_{pe-self22} & Z_{pe-mutu23} \\ Z_{pe-mutu31} & Z_{pe-mutu32} & Z_{pe-self33} \end{bmatrix}$$

To obtain the self-impedance $Z_{pe-self}$ and mutual impedance $Z_{pe-mutu}$ when including the proximity effect of a single-core subsea cable, equation (6) can be modified [10]:

$$Z_{pe-self} = 2\sum_{n=1}^{\infty} \frac{j\omega\mu_0 r_{ao}{}^{n-1}}{\pi} \left(\frac{r_{ao}}{s}\right)^n \left(\frac{1}{s}\right)^n \cdot \frac{I_n(r_{ao}m_a)}{\frac{n}{r_{ao}} I_n(r_{ao}m_a) + \frac{m_a}{\mu_a} I'(r_{ao}m_a)} \quad (11)$$

$$Z_{pe-mutu} = \sum_{n=1}^{\infty} \frac{j\omega\mu_0 r_{ao}{}^{n-1}}{\pi} \left(\frac{r_{ao}}{s}\right)^n \left(\frac{1}{s}\right)^n \cdot \frac{I_n(r_{ao}m_a)}{\frac{n}{r_{ao}} I_n(r_{ao}m_a) + \frac{m_a}{\mu_a} I'(r_{ao}m_a)} \quad (12)$$

where, $m_a = \sqrt{\frac{j\omega\mu_a}{\rho_a}}$ ; $\mu_a$ is the relative permeability of armour; $r_{ao}$ is the inner radius of armour.

## 3.2 Impedance of Three-Core Cables

When considering the impedance of three-core cables, the equations for Pipe-Type (PT) cable can be utilised. The impedance matrix is expressed as [11]:

$$Z_{cable} = [Z_i] + [Z_{pi}] + [Z_{conn}] + [Z_{earth}] \quad (11)$$

where $Z_i$ is the pipe internal impedance; $Z_{conn}$ is the connection impedance of inner and outer surface of armour; and $Z_{earth}$ is the earth return impedance. The proximity effect, $Z_{pi}$ is subject of further analysis. The remaining impedances $Z_i$ , $Z_{conn}$ , and $Z_{earth}$ can be found in [11].

According to [14], because of non-concentric of cables the proximity effect needs to be taken into account. The matrix equation for $Z_{pi}$ is as:

$$Z_{pt} = \begin{bmatrix} Z_{pi-self11} & Z_{pi-mutu12} & Z_{pi-mutu13} \\ Z_{pi-mutu21} & Z_{pi-self22} & Z_{pi-mutu23} \\ Z_{pi-mutu31} & Z_{pi-mutu32} & Z_{pi-self33} \end{bmatrix} \quad (12)$$

Where, $Z_{pi-self}$ is the self impedance and $Z_{pi-mutu}$ is the mutual impedance between inner conductors with respect to the armour inner surface and is given by [11][14]:

$$Z_{pi-self} = \frac{j\omega\mu_0\mu_a}{2\pi}\left[\frac{K_0(m_a r_a)}{m_a r_a \cdot K_1(m_a r_a)}\right] + \frac{j\omega\mu_0}{2\pi} Q_{self} + \frac{j\omega\mu_0\mu_a}{\pi}\sum_{n=1}^{\infty}\frac{C_n}{n(1+\mu_a) + m_a r_a K_{n-1}(m_a r_a)/K_n(m_a r_a)} \quad (13)$$

$$Z_{pi-mutu} = \frac{j\omega\mu_0\mu_a}{2\pi}\left[\frac{K_0(m_a r_a)}{m_a r_a \cdot K_1(m_a r_a)}\right] + \frac{j\omega\mu_0}{2\pi} Q_{mutu} + \frac{j\omega\mu_0\mu_a}{\pi}\sum_{n=1}^{\infty}\frac{C_n}{n(1+\mu_a) + m_a r_a K_{n-1}(m_a r_a)/K_n(m_a r_a)} \quad (14)$$

where, $\rho_a$ is resistivity of armour; $r_a$ is the inner radius of armour.

$$Q_{self} = \ln[\frac{r_a}{r_c}(1-(\frac{d_c}{r_a})^2)] \quad (15)$$

$$Q_{mutu} = \ln[\frac{r_a}{s}] - \sum_{n=1}^{\infty}\frac{C_n}{n} \quad (16)$$

$$C_n = (\frac{d_c}{r_a})^{2n} \cdot \cos(n\theta) \quad (17)$$

where, $r_c$ is the radius of the conducting layer right inside the armour; $d_c$ is the distance between individual cable to the amour centre; $\theta$ is the angle between cables in respect to armour centre.

The proximity effect is the third term on the right hand sides of equations (13) and (14). This term will be included when the proximity effect needs to be considered. If proximity effect is not to be taken into account then this term can be ignored.

## 4 Results and Discussions

The harmonic resistance and inductance of the single-core and the three-core subsea cables are plotted against frequency from the fundamental frequency, 50Hz, up to 30 orders in Figures 2 and 3. For simplicity of comparison, the harmonic resistance and inductance magnitudes are expressed as a ratio to the magnitudes found at the fundamental frequency when considering the proximity effect. The significance of the proximity effect in both types of cables is demonstrated in these results. In general, the resistance increases as the frequency increases due to the skin effect. However, due to losses due to the circulating currents between the layers, the curve is non-linear. Also, the inductance of the conductor due to cancellation of flux leakages between layers tends to decrease as frequency increases.

The results in Figure 2 compare the harmonic resistance and inductance of a steel armour single-core subsea cable. The harmonic resistance curves with and without inclusion of the proximity effect term can be seen to be very close, overlapping each other. This is also seen for the inductance curves. The proximity effect only affects the outer most current loop i.e. the circulating current that flows between the armour and earth. The proximity effect has significantly reduced the effect on the impedance of the conductor, located inside the sheath screen.

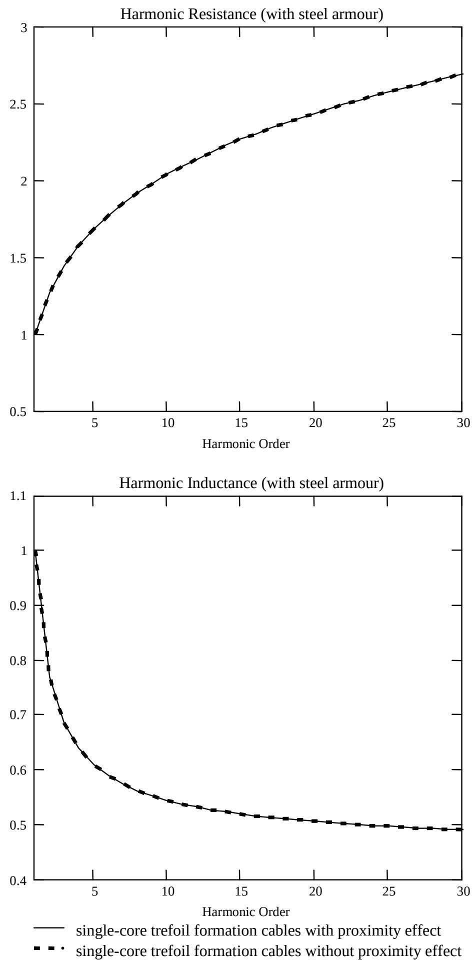



**Fig. 2 Harmonic Resistance and Inductance of a Steel Armour Single-Core Subsea Cable**

However, in Figure 3 which compares the harmonic resistance and inductance of a steel armour three-core cable with and without the proximity effect term, than it is seen that there are differences between the curves. This is because three-core cables are located within a common armour shield. The phases are much closer to each other with only a sheath screen surrounding individual conductors. To observe in more detailed, Figure 4 shows the differences in the results obtained with and without the proximity effect. The differences in the harmonic resistances and inductances are expressed as a percentage of the value obtained with proximity effect included. Again, for the single-core cable, there is nearly no difference between the resistance and inductance. On the other hand, for three-core cable, there are differences of up to 19% at fundamental frequency. Also, it is higher up to 48% difference at the third order (150Hz) for the two methods. This clearly shows that for harmonic impedance evaluations then for three-core cables the consideration of proximity effect is necessary.

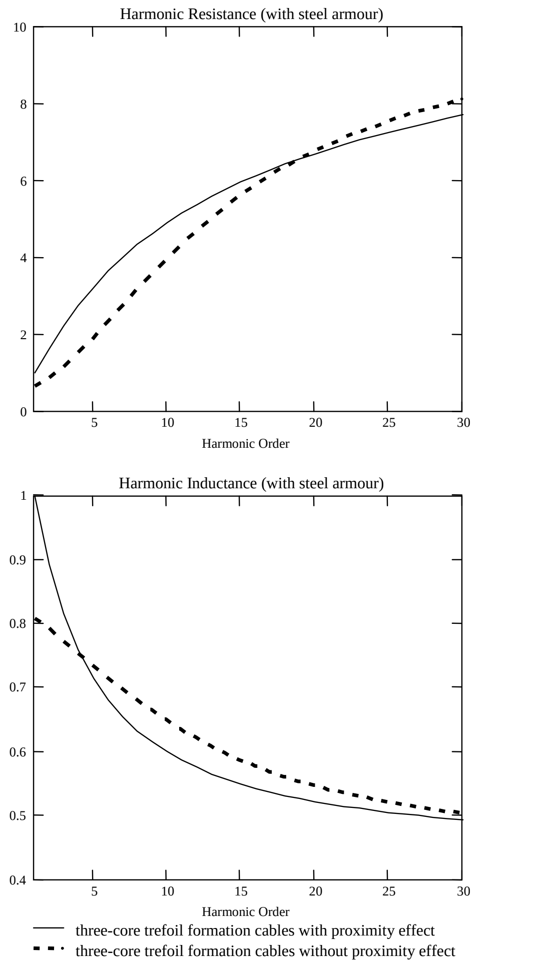



**Fig. 3 Harmonic Resistance and Inductance of a Steel Armour Three-Core Subsea Cable**

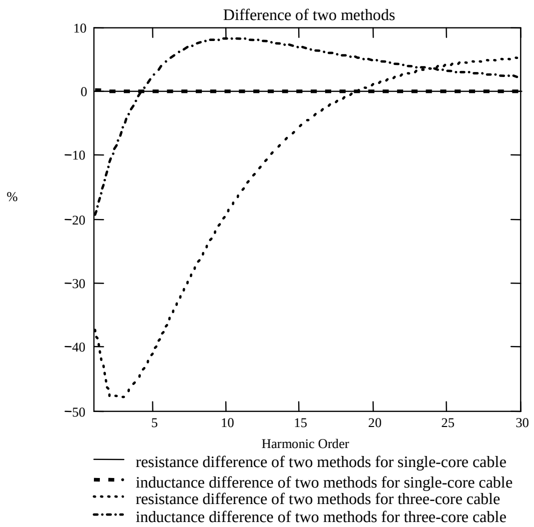



**Fig. 4 The difference of two methods (with and without proximity effect)**

# 5 Conclusions

This paper has demonstrated that the impedance for single-core cable is unaffected by the proximity effect and can therefore be neglected. However, the proximity effect is a significant factor in three-core cables and it should therefore be taken into account when calculating harmonic impedance. The results also imply that knowledge of the geometric arrangements and structures of cable are essential for determining the influence of the proximity effect. How conductors are shielded, the layer arrangements in respect to each cable and relative to each phase are critical features for deciding whether or not the proximity effect is influential. For the purposes of harmonic analysis of the subsea cable transmission system, then these issues need to be carefully considered at the very early stages to avoid significant errors.


# 6 Acknowledgements

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# 7 Appendix

**For Single-Core AC cables:**

| | |
|---|---|
| Copper Conductor Diameter: | 37.9 mm |
| XLPE Insulation Thickness: | 17 mm |
| Relative Permittivity of XLPE: | 2.5 |
| Diameter over Insulation: | 78.5 mm |
| Lead Sheath Thickness: | 2.5 mm |
| Outer Diameter of Cable: | 102.2 mm |
| Steel wire Number: | 54 |
| Steel wire Diameter: | 5mm |

**For Three-Core AC cables:**

| | |
|---|---|
| Copper Conductor Diameter: | 29.8 mm |
| XLPE Insulation Thickness: | 17 mm |
| Relative Permittivity of XLPE: | 2.5 |
| Diameter over Insulation: | 69 mm |
| Metallic Cable Sheath Thickness: | 2.4 mm |
| Outer Diameter of Cable: | 187 mm |
| Steel wire Number: | 106 |
| Steel wire Diameter: | 5mm |

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