COMPUTER ALGEBRA TECHNIQUE FOR ABRASIVE TREATMENT PROCESS DYNAMICS

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Abstract: The mathematical model was compiled for the studies of the dynamics of an abrasive processing in a magnetic field. An analytical solution of its nonlinear part was found synthesizing harmonic balance method and computer algebra technique. Solutions of the linear part were obtained by the traditional numerical techniques. The numeric investigation of process dynamic features was fulfilled, regimes ensuring the most effective vibratory abrasive treatment process were determined. The process optimization was fulfilled using LP search method according the maximum power of treatment process and the greatest phase difference between the settled motion of abrasive particles and of components being treated.

Keywords: computer algebra, nonlinear dynamics, dry friction, vibratory abrasive treatment

1 Introduction

When a nonlinear structure is subjected to harmonically varying forces, it first passes through a transient state and afterwards it reaches a steady state or executes chaotic motions. Some steady-states can be non-periodic but mostly the periodic steady-states are observed. Two related problems of interest are the periodic behavior of structure undergoing harmonic excitation and a free vibration of the structure.

Natural vibration of the nonlinear systems is of primary concern in studying the resonance phenomena because the backbone curves (the amplitude-frequency relations) and the modes of vibrations, i.e. dynamic characteristics of systems, are determined. Analytical expressions for the backbone curves are very complex and numerical methods are not convenient way to analyze nonlinear oscillations [1].

In some cases, such as the one in a range where the internal resonance exists, the corresponding backbone curves have a very complex shape owing to the presence of sharp peaks, looping characteristic and rapidly changing slopes. It is difficult to determine these types of backbone curves by the developed numerical methods [2,3]. Simulations by means of numerical methods are powerful tools for the investigation in mechanics however, they have serious drawbacks, e.g. finite precision, difficulties in determining transient states and steady states, and investigation of stability is errorprone and complex.

The analytical steady-state solution by hand requires a lot of routine work, is errorprone and available only for very simple systems. Here the computerized symbolic manipulation systems – so called *computer algebra* – are indispensable tools. Symbolic manipulations provided by computer algebra systems in combination with high-power number crunching abilities of traditional hardware and software really opens new way to a large-scale computations needed in steady-state solutions and stability analysis [4].

The aim of this paper is to describe a theoretical background of systematic computer algebra methods for analyzing the free and steady state periodic vibrations of nonlinear structures. There are developed many analytical steady-state solution methods but everyone has different capabilities, e.g. small parameter method gives solution in close form and harmonic balance method only converts nonlinear differential equations to algebraic. A broader understanding of mechanical phenomena can be gained by means of analytical methods. Computerized symbolic manipulation is a very

attractive means to reliably perform analytic calculations with even complex formulas and expressions. But often, a semi-analytical approach, combining the features of analytical and numerical computations, is the most desirable synthesis. This allows the analytic work to be pushed further before numerical computations start.

For numeric-symbolic computation of the nonlinear oscillation systems with several degrees of freedom computer algebra system VIBRAN [5] was used. The computer algebra system VIBRAN is a FORTRAN preprocessor for analytical computation with polynomials, rational functions and trigonometric series. Special VIBRAN's procedure can generate optimized FORTRAN code from obtained analytical expressions, which can be directly used in the programs for numerical analysis.

The vibratory abrasive treatment process [6] is one of the most popular final processing methods. The main factors of vibratory abrasive treatment components interaction are dry friction forces, which are in combination with the viscous friction, and also some other forces of non-linear character arising between the details being treated and particles of the abrasive. Therefore the dynamics of the system is usually described by nonlinear differential equations.

Usually, a solution method must correlate with the type of motion equations and at the same time the character of the initial mechanic system. The harmonic balance method was chosen for the investigation of vibratory abrasive treatment process as it is easily applied for systems where calculations are made using computer algebra methods and in this case it needs considerably less computer memory then other methods. Though in this way we obtain nonlinear algebra equation system with many expressions, but they all are very short. So it is possible and expedient to process results by numerical methods. Analytic calculation system VIBRAN is extremely effective in this case. As the adequate motion equation solution method for dynamically complicated systems has to contain a great number of analytic as well as numeric calculations a strong program connection must exist between them. Analytic calculation system VIBRAN selected in order to ensure this connection. It is foreseen in the system generating of subroutines in FORTRAN, selecting and rejecting of excessive operations while generating programs in accordance with analytic expressions. Besides, this system distinguishes itself in flexibility of input-output and the unique means for operations with sparse matrices.

2 Computer algebra realization

Consider the system of s degrees of freedom with Lagrangian function:

$$L = L(q_i, \dot{q}_i, t) \qquad (i = 1,2,...s) \quad (1)$$

where L - Lagrangian function, $q_i, \dot{q}_{i,} t$ - generalized coordinates and velocities of the system and time; s - number of degrees of freedom;

Equations of motion of such a system are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = F_{qi} \qquad (2)$$

These equations can be separated into the linear and nonlinear parts by formal replacements $L = L_L + L_N$ and $F_{qi} = F_{Li} + F_{Ni}$. Now equations of motion may be expressed in the form:

$$\left(\frac{d}{dt}\left(\frac{\partial L_L}{\partial \dot{q}_i}\right) - \frac{\partial L_L}{\partial q_i} - F_{Li}\right) + \left(\frac{d}{dt}\left(\frac{\partial L_N}{\partial \dot{q}_i}\right) - \frac{\partial L_N}{\partial q_i} - F_{Ni}\right) = 0 \quad (3)$$

where F_{qi}, F_{Ni}, F_{Li} - generalized force, nonlinear (polynomial with respect to generalized coordinates and periodical or Fourier expansion of time) and linear parts; L_N, L_L - nonlinear and linear parts of Lagrangian function respectively.

The linear part can be formalized for numerical analysis without difficulties and we used special VIBRAN programs to analyze the nonlinear part of the system. These steps are presented in Fig. 1 as block 1 and block 3 respectively

The proposed method provides smaller expressions for analytical computation and allows to analyze systems with greater order. After good known perturbations the equations of motion can be rewritten in the matrix form:

$$[M]\{\ddot{q}\} + [B]\{\dot{q}\} + [C]\{q\} = \{H(q,\dot{q},t)\} + \{f(t)\} \quad (4)$$

where $f(t)$ - periodic function;

$$\{f\} = \{f_0\} + \{f_1\}\cos(\omega t) + \{f_2\}\sin(\omega t) + ... \qquad (5)$$

$H(q,\dot{q},t)$ - nonlinear part of a system, calculated by special VIBRAN's program;

Solution of the above mentioned system can be expressed using harmonic balance method in the form [1]:

$$\{q\} = \{A_0\} + \{A_1\}\cos(\omega t) + \{A_2\}\sin(\omega t) + ... \qquad (6)$$

where $\{A_i\}$ are the unknown vectors that can be found from the nonlinear algebraic equations. Following harmonic balance method these equations for the first three vectors' coefficients in matrix form are:

$$[U]\{A\} - \{f\} - \{H(A)\} = \{0\} \qquad (7)$$

where f_i are coefficients of function *f(t)* Fourier expansion. Analogously equations for other harmonics could be found by VIBRAN's program shown in block 4 of Fig.1. The expressions of H_i and theirs required derivatives are expressed in closed form using computer algebra techniques with FORTRAN code generation procedure. Special modifications were done for terms with dry friction, integration procedure was developed according Malkin's method [1]. The programs' structure is shown in Fig. 1.

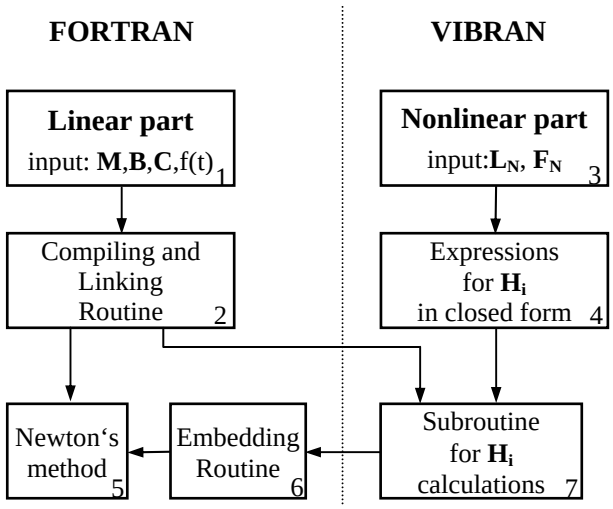

Fig.1. Programs' structure for nonlinear oscillation systems analysis.

Obviously matrix of coefficients [*U*] consists of independent submatrix blocks located at the main diagonal. Therefore the linear part of the matrix equation decomposes into m separate systems (*m* - number of harmonics in the solution vector) of size *2s* and one system (corresponding to zero harmonic) of size *s*. The obtained solution of these systems serves as initial approximation for further computation.

The Newton's iteration formula claims

$$\left[U - H'(A^i)\right]\left\{A^{i+1} - A^i\right\} - \left[U\right]\left\{A^i\right\} - \left\{f\right\} - \left\{H(A^i)\right\} = \left\{0\right\} \quad (8)$$

where [*U*-*H'*(*A*)] is the Jacobian matrix of the whole system (including linear part as well). Substituting these two formulas Newton iteration becomes

$$\left(\left[U\right] - \left[H'\left(A^i\right)\right]\right)\left\{A^{i+1}\right\} = \left[U - H'\left(A^i\right)\right]\left\{A^i\right\} - \left[U\right]\left\{A^i\right\} + \left\{H\left(A^i\right)\right\} + \left\{f\right\} \quad (9)$$

Starting the iterations:

$$\left[U\right]\left\{A^0\right\} = \left\{f\right\} \quad (10)$$

This linear matrix equation must be solved on every iteration step. This equation can be rewritten in simple form

$$\left(\left[U\right] + \left[H'\right]\right)\left\{A\right\} = \left\{F\right\} \quad (11)$$

It's not expedient to sum up matrices [*U*] and [*H'*] in advance because they have quite a different structure. Matrix [*U*] as it was already mentioned is block-banded, i.e. consists of separate submatrix blocks located at the main diagonal. The structure of matrix [*H'*] depends on the type and location nonlinear terms in the initial differential equation system but it's always sparse and can possess nonzero elements far from main diagonal. Thus matrix [*U*] is not stored in the computer memory at all and its every element is computed by the reference to the special subroutine. Matrix [*H'*] is stored in the main memory as a sparse matrix. Of course, such storage demands corresponding modifications in the solution algorithm itself.

In many applications the solution of differential equation system must be obtained in a specified domain of some varied parameters (frequency, stiffness, mass, etc.). Therefore the program is designed in such a way that any parameter of initial system can be varied with a regular or logarithmic step. Note that analytical computation is performed only once while numerical calculations are repeated every time when the value of any parameter of the system is being changed.

3 Abrasive treatment process

In most of the cases connected with vibratory abrasive treatment problems the work with miniature and micro-miniature brittle details is not taken into proper consideration. During the intensive treatment these parts acquire cracks and splits or even break up. For this reason when improving the existing and developing new, reliable and more effective methods and devices one must try to make them suitable for the treatment of brittle and miniature details.

The new method for the treatment of miniature ring shape details was proposed [9] where the internal surface of the detail is treated as well as an external one. During the vibratory treatment process the working medium particles are constantly striking each other. As a result, slight scratches and crater shaped crevasses occur on the surface of treated details forming the surface micro- relief. In this way, abrasive friction and impacts to the treated details perform the treatment process.

The application of an electromagnetic field is one of the possible methods to increase the treatment efficiency and avoid the direct interaction between treated details when they may be damage or even break up.

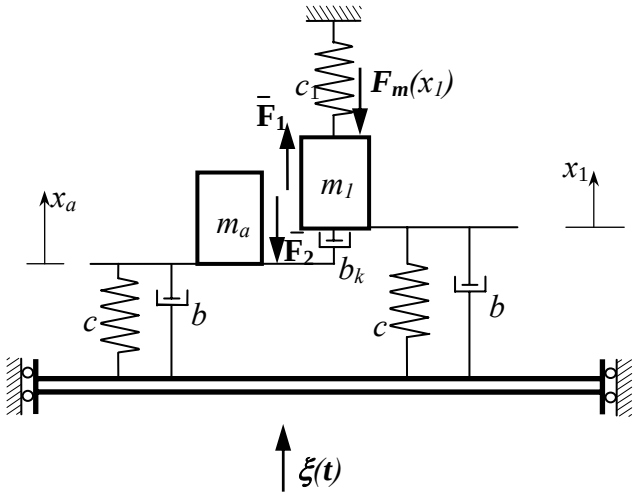

Fig. 2. The dynamics model of vibratory abrasive treatment process.

Fig. 2 shows a nonlinear dynamic model with concentrated parameters of the vibratory treatment process in which the load and elasto-dissipative properties of the supporting magnetic field are taken into consideration. This model is made for the analysis of the motions of both treated details and an abrasive particle as well as the efficiency of the vibratory abrasive treatment [4,9].

Equations of motion describe the dynamics of vibratory abrasive treatment process:

$$\begin{cases} m_1\ddot{x}_1 + b\dot{x}_1 + b_k(\dot{x}_1 - \dot{x}_a) + (c + c_1)x_1 = \\ \quad = c\xi(t) - F_m(x_1) - F_1 sign(\dot{x}_1 - \dot{x}_a) + b\dot{\xi}(t), \\ m_a\ddot{x}_a + b\dot{x}_a + b_k(\dot{x}_a - \dot{x}_1) + cx_a = \\ \quad = c\xi(t) + F_2 sign(\dot{x}_1 - \dot{x}_a) + b\dot{\xi}(t). \end{cases} \quad (12)$$

where m_1 – mass of components being treated; m_a – mass of abrasive particles; the load mechanical properties are evaluated by elasticity c and working medium resistance b; F_1, F_2 – forces of dry friction between component and abrasive; b_k –viscosity resistance; the elasticity of magnetic field is evaluated by a stiffness coefficient c_1, additional excitement to detail was given generating a variable component of magnetic field - $F_m(x_1)$. Its stiffness properties were obtained experimentally by the least squares method:

$$F_m(x_1) = \tilde{u}_0 + \tilde{u}_1 x_1 + \tilde{u}_2 x_1^2 + \tilde{u}_3 x_1^3 \quad (13)$$

The kinematics excitation of a vessel with load is $\xi(t) = A\sin\omega t$.

Having substituted the variable ($x_1 - x_a = x_2$) into equations of motion (12), transferring system to a "slow time" ($\tau = \omega t$), and introducing the new coefficients, finally equations (12) could be expressed:

$$\begin{cases} \ddot{x}_1 + k_1\dot{x}_1 + k_2\dot{x}_2 + k_3x_1 + u_0 + u_1x_1 + u_2x_1^2 + u_3x_1^3 + \\ \quad + S_1 sign\,\dot{x}_2 = f_1(\tau) \\ \ddot{x}_2 + k_6\dot{x}_1 + k_7\dot{x}_2 + k_8x_1 + k_9x_2 + u_0 + u_1x_1 + u_2x_1^2 + \\ \quad + u_3x_1^3 + S_2 sign\,\dot{x}_2 = f_2(\tau) \end{cases} \quad (14)$$

here

$$f_1(\tau) = k_4\sin\tau + k_5\cos\tau,\ f_2(\tau) = k_{10}\sin\tau + k_{11}\cos\tau \quad (15)$$

The computer algebra system VIBRAN is used for solution of system (15). Steady state solutions are generated in the form of trigonometric series:

$$\begin{cases} x_1(\tau) = a_{10} + \sum_{n=1}^{\infty}(a_{1n}\cos n\tau + b_{1n}\sin n\tau) \\ x_2(\tau) = a_{20} + \sum_{n=1}^{\infty}(a_{2n}\cos n\tau + b_{2n}\sin n\tau) \end{cases} \quad (16)$$

here A in expression (6) substituted by letters a, b.

Analytic expressions obtained according VIBRAN program concludes the part of analytic calculations. The analytical result for H_i in equation (4) is (direct program printout):

H0=k3*a10+u1*a10+.5*u2*a11**2+.5*u2*b11**2+u2*a10**2+1.5*u3*a10*a11**2+1.5*u3*a10*b11**2+u3*a10**3+u0

H1=a20*k9+k8*a10+u1*a10+.5*u2*a11**2+.5*u2*b11**2+u2*a10**2+1.5*u3*a10*a11**2+1.5*u3*a10*b11**2+u3*a10**3+u0

H2=b21*w*k2+w*b11*k1+k3*a11+a11*u1+2*u2*a11*a10+.75*u3*a11**3+.75*u3*a11*b11**2+3*u3*a11*a10**2+1.27324*ks1

H3=-a21*w*k2-w*a11*k1+k3*b11+b11*u1+2*u2*b11*a10+.75*u3*a11**2*b11+.75*u3*b11**3+3*u3*b11*a10**2

H4=b21*w*k7+a21*k9+w*b11*k6+k8*a11+a11*u1+2*u2*a11*a10+.75*u3*a11**3+.75*u3*a11*b11**2+3*u3*a11*a10**2+1.27324*ks2

H5=-a21*w*k7+b21*k9-w*a11*k6+k8*b11+b11*u1+2*u2*b11*a10+.75*u3*a11**2*b11+.75*u3*b11**3+3*u3*b11*a10**2,

where ks1 and ks2 are calculated by special VIBRAN procedure.

The corresponding derivatives are very simple and there are only 25 nonzero terms (direct printout):

H01 = k3+u1+2*u2*a10+1.5*u3*a11**2
+1.5*b11**2*u3+3*u3*a10**2

H02 = a11*u2+3*a11*u3*a10

H03 = b11*u2+3*b11*u3*a10

H11 = k8+u1+2*u2*a10+1.5*u3*a11**2
+1.5*b11**2*u3+3*u3*a10**2

H12 = a11*u2+3*a11*u3*a10

H13 = b11*u2+3*b11*u3*a10

H14 = k9

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H51 = 2*u2*b11+6*u3*a10*b11
H52 = -k6*w+1.5*u3*b11*a11
H53 = k8+u1+2*u2*a10+.75*u3*a11**2+2.25*u3
    *b11**2+3*u3*a10**2
H55 = -k7*w
H56 = k9
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All properties complying with the dynamic pattern of the process are investigated in numerical way and the program itself is composed in FORTRAN language. For this reason for calculation of factors analytic expressions and their partial derivatives two FORTRAN subroutines are generated where one is for a dictionary compiling and another is for calculation of expressions themselves. Besides, the program created applying a harmonic balance method for differential equation systems in addition to equations where amplitudes and constant components are found also presents equation derivatives from unknown quantities. In this case one or some criteria of further numeric parameter optimization may be calculated.

4 Numerical results

Numerical investigation of process dynamic properties was carried out in order to investigate in a qualitative way the dynamics of vibratory abrasive treatment process and to discover optimal process parameters sets ensuring the greatest efficiency of the treatment process. Fig. 3 shows amplitude frequency characteristics' dependence upon the working medium elasticity *c*.

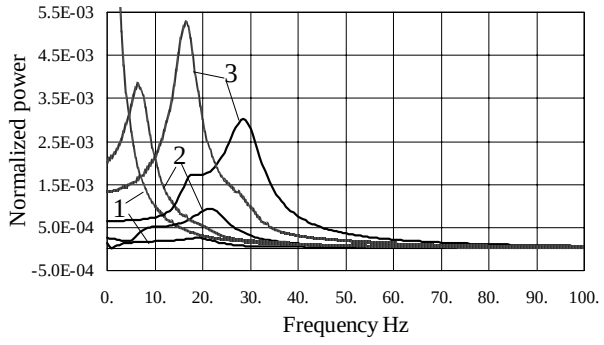

Fig.3. Amplitude frequency characteristics' dependence upon the working medium elasticity *c*: 1 - *c* = 1 N/m, 2 - *c* = 20 N/m, 3 - *c* = 100 N/m (black curves - for treated details, grey - for abrasive).

As it has been mentioned the results of analytic calculations are formed in the shape of completed programs, which may be included into the common process of numerical investigation. It must be emphasized that analytic arrangements by the VIBRAN system are made only once while numerical calculations are repeated every time when the value of any parameter is changed.

The investigation of dynamic system is represented by frequency characteristics of amplitude, phase and power. The power of vibratory abrasive process in one case was calculated evaluating only friction between components being treated and abrasive particles. Here friction force work at the time of steady vibration period was calculated:

$$N_1 = \frac{1}{T}\int_0^T F(\dot{x}_2 - \dot{x}_1)\ dt\,. \qquad (17)$$

The greatest influence for the treatment power in this case has the motion phase difference between abrasive particles and details being treated. As the treatment power in this case is the greatest when details and abrasive particles move in opposite directions it has been established that such kind of motion is possible in actual parameters alteration limits.

In the second case the power was calculated when the treatment was performed by friction and impact interaction between abrasive particles and details being treated. In this case the total treatment power was calculated using the formula:

$$N = N_1 + N_2 \qquad (18)$$

and the treatment power achieved by impact interaction using the formula:

$$N_2 = \frac{1}{T}\int_0^T b_y(\dot{x}_2 - \dot{x}_1)^2\ dt\,, \qquad (19)$$

where impact interaction efficiency coefficient b_y isn't the same viscosity resistance b_k; it shows which part of impact interaction energy is used for treatment. The values of parameters b_y and F were varied coordinating results of experimental and theoretical investigations.

In the power frequency characteristics total power and components features are shown separately. Power values on the secondary axis are normalized, i.e., $\tilde{N}_i = N_i / N_{max}$.

While analyzing frequency characteristics we may see that the most treatment efficiency is achieved in the field of resonance frequencies. However the before phase motion of components being treated and abrasive particles at certain parameters may remain during the whole range of frequencies. When working medium resistance coefficient increases the frequency interval of effective treatment by friction and impact interaction extends very much (Fig. 4).

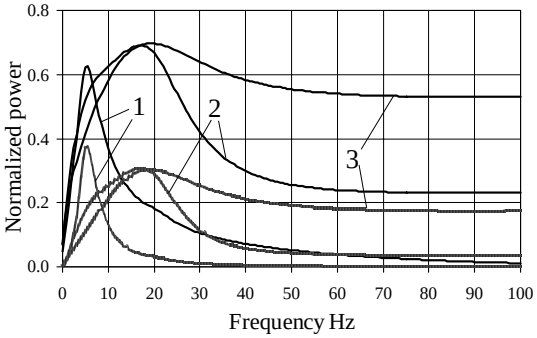

Fig. 4. Power components (black - by friction, grey - by impacts) frequency characteristics' dependence upon the working medium resistance *b:* 1 - *b* = 0,1 kg/s, 2 - *b* = 0,5 kg/s, 1 - *b* = 1,0 kg/s,

The optimization of vibratory abrasive treatment process was fulfilled using LP search method (according Sobol points) [10].

As the criteria for optimization the treatment process power and the maximum phase difference between abrasive particles and details being treated were selected. 600 experiments were performed and optimal parameter sets were selected which maximize these criteria.

5 Conclusions

On the basis of non-linear differential equations solving by a harmonic balance method and the synthesis of analytic calculation system VIBRAN the investigation method of non-linear systems with dry friction effect was created. This method combines the advantages of analytic calculation methods and computer algebra. They are compound on the principle of parallel analytic – numerical calculation where analytic rearrangements are applied only to the non-linear part of the system, and at the same time the linear part of the system could be easily solved in numerical way. The proposed method provides smaller expressions for analytical computation and allows analyzing systems with greater order.

This method is applied for the investigation of vibratory abrasive treatment process dynamics. The new method for the treatment of miniature ring shape details under the action of the magnetic field was proposed. On the basis of vibratory abrasive treatment process survey a two - mass dynamic model with elastic dissipation interaction was prepared and the investigation of this dynamics system with dry friction was made for the treatment process of friable miniature products. Using the LP search method the process optimization was executed according the maximal treatment process power and the largest phase difference between abrasive particles and the processed details settled movements.

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