Optimization and Simulation of Secondary Settler Models

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Abstract: - This work focus on the accomplishing of the best model to a secondary settler to use in a minimum cost optimization procedure concerning the construction and operation of a wastewater treatment plant (WWTP). Two traditional models are tested as well as a new model that results from the combination of the other two. The obtained optimal designs are then simulated in order to evaluate the model that provides the best performance.

Key-Words: - WWTP optimal design, Secondary settler modeling, Cost function minimization, Simulation design

1 Introduction

In the planning and design of a wastewater treatment plant (WWTP) the role played by the secondary settler is, most of the time, underestimated.

Due to the high costs associated with the construction and operation of WWTPs, it is convenient to conduct a careful analysis of the involved models, in particular the secondary settler model, and perform an optimization of the entire system.

The most commonly used models in literature to describe the secondary settler are the ATV [2] and the double exponential (DE) [9] models. The ATV model is usually used as a design procedure to new WWTPs. It is based on empirical equations that were obtained by experiments and does not contain any solid balances, although it contemplates peak wet weather flow (PWWF) events. The DE model is the most widely used in simulations and it produces results very close to reality. However, as it does not provide extra sedimentation area needed during PWWF events, the resulting design has to consider the use of security factors, many times inadequate.

The optimization procedures found in the literature involving secondary settlers are based on very simple models [10] or on the ATV model [1, 3, 5]. As far as we know, there have been no optimization attempts using the DE model.

We believe that the combination of the two models overcomes the limitations of each one, and this combination was used for the first time in [4].

In this work, optimization procedures were conducted, in the sense that a minimum cost design ought to be achieved, with the two traditional models (DE and ATV) separately and with a combination of both models. It is the first time that the DE model is used with an optimization goal and compared with the other models. Besides, based on the obtained optimal designs, GPS-X [11] simulations were carried out to be able to evaluate the goodness of the solutions. Some stress conditions were imposed in order to assess the most robust solution. The design that relies on the combined model (ATV+DE) to describe the secondary settler was the only one able to overcome the stress conditions.

This paper is organized as follows. In Section 2 a brief description of the mathematical models related with the activated sludge system of a WWTP is presented. Section 3 describes the cost function used in our optimization procedures. Section 4 synthesizes the obtained mathematical programming models and Section 5 reports on the obtained optimal designs as well as on the carried simulations. Finally, Section 6 contains the conclusions and the ideas for future work.

2 The Activated Sludge System

Considering the performance and associated costs, the most important treatment in a WWTP is the secondary treatment. The activated sludge system, composed by an aeration tank and a secondary settler is the most commonly found secondary treatment. These two unitary processes are intimately related, therefore, one should not be considered without the other.

To model the aeration tank the ASM1 model [8] is used. Mass balances were done to each one of the compounds considered by this model for each involved biological process.

For the secondary settler, we propose a combination of the two traditional models ATV and DE.

Besides the balances to each unit, some balances were done around the entire system.

2.1 Aeration tank

The aeration tank is where the biological reactions take place. To describe it the activated sludge model n.1, described by Henze et al. [8], is used and considers both the elimination of the carbonaceous matter and the removal of the nitrogen compounds. The tank is considered a completely stirred tank reactor (CSTR) in steady state. The balances around this unit define some of the constraints of our mathematical model. The generic equation for a mass balance around a certain system considering a CSTR is

$$\frac{Q}{V_a} = (\xi_{in} - \xi) + r_\xi = \frac{d\xi}{dt} \qquad (1)$$

where Q is the flow that enters the tank, V_a is the aeration tank volume, ξ e ξ_{in} are the concentrations of the component around which the mass balances are being made inside the reactor and on entry, respectively. In a CSTR the concentration of a compound is the same at any point inside the reactor and at the effluent of that reactor. The reaction term for the compound in question, r_ξ, is obtained by the sum of the product of the stoichiometric coefficients, $\nu_{\xi j}$, with the expression of the process reaction rate, ρ_j, of the ASM1 Peterson matrix [8]:

$$r_\xi = \sum_j \nu_{\xi j} \rho_j .$$

In steady state, the accumulation term given by $d\xi / dt$ in (1) is zero, because the concentration is constant in time. A WWTP in labor for a sufficiently long period of time without significant variations can be considered at steady state. As our purpose is to make cost predictions in a long term basis it is reasonable to do so. The ASM1 model involves 8 processes incorporating 13 different components, such as the substrate, the bacteria, dissolved oxygen, among others. We refer to [5] for details.

2.2 Secondary settler

Traditionally the secondary settler is underestimated when compared with the aeration tank. However, it plays a crucial role in the activated sludge system. When the wastewater leaves the aeration tank, where the biological treatment took place, the treated water should be separated from the biological sludge, otherwise, the chemical oxygen demand would be higher than it is at the entry of the system. The most common way of achieving this purpose is by sedimentation in tanks. The optimization of the sedimentation area and depth must rely on the sludge characteristics, which in turn are related with the performance of the aeration tank. So, the operation of the biological reactor influences directly the performance of the settling tank and for that reason, one should never be considered without the other.

The ATV design procedure (Fig. 1) contemplates the peak wet weather flow events, during which there is a reduction in the sludge concentration. To turn around this problem, a certain depth is allocated to support the fluctuation of solids during these events

$$h_3 = \Delta X \, V_a \frac{DVSI}{480 A_s} \qquad (2)$$

This way a reduction in the sedimentation area, A_s, is allowed. A compaction zone

$$h_4 = X_p \frac{DVSI}{1000} \qquad (3)$$

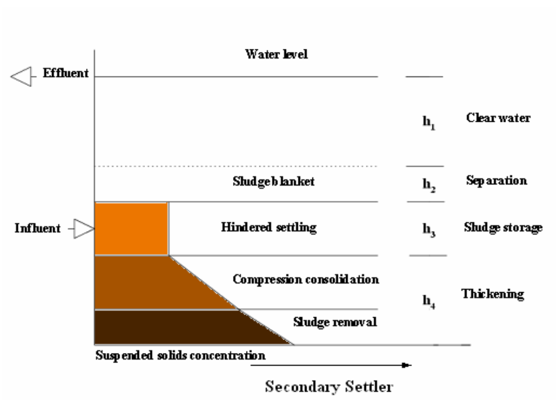

Fig. 1: Typical solids concentration-depth profile adopted by the ATV model (adapted from [2])

where the sludge is thickened in order to achieve the convenient concentration to return to the biological reactor, also has to be contemplated and depends only on the characteristics of the sludge. $DVSI$ is the diluted volumetric sludge index, ΔX is the variation of the sludge concentration inside the aeration tank in a PWWF event and X_p is the sludge concentration during a PWWF event. A clear water zone (h_1) and a separation zone (h_2) should also be considered and are set empirically ($h_1 + h_2 = 1$, say). The depth of the settling tank, h, is the sum of h_1, h_2, h_3 in (2) and h_4 in (3). The sedimentation area is still related to the peak flow, Q_p, by the expression

$$\frac{Q_p}{A_s} \leq 2400 (X_p DVSI)^{-1.34} .$$

The double exponential model assumes a one dimensional settler, in which the tank is divided into ten layers of equal thickness (Fig. 2). Some simplifications are considered. No biological reactions take place in this tank, meaning that the dissolved matter concentration is maintained across all the layers. Only vertical flux is considered and the solids are uniformly distributed across the entire cross-sectional area of the feed layer (j=7, in our case). This model is based on a traditional solids flux analysis but the flux in a particular layer is limited by what can be handled by the adjacent layer. The settling function, described by Takács et al. in [9], is given by

$$v_{s,j} = \max\left(0, \min\left(v'_0, v_0\left(e^{-r_h\left(TSS_j - f_{ns}TSS_a\right)} - e^{-r_p\left(TSS_j - f_{ns}TSS_a\right)}\right)\right)\right)$$

where $v_{s,j}$ is the settling velocity in layer j (m/day), TSS_j is the total suspended solids concentration in each of the ten considered layers of the settler and v_0, v'_0, r_h, r_p and f_{ns} are the settling parameters. Note that $TSS_7 = TSS_a$.

The solids flux due to the bulk movement of liquid may be up or down, v_{up} and v_{dn} respectively, depending on its position relative to the feed layer, thus

$$v_{up} = \frac{Q_{ef}}{A_s} \quad \text{and} \quad v_{dn} = \frac{Q_r + Q_w}{A_s}.$$

As to the subscripts, r concerns the recycled sludge, w the wasted sludge and ef the treated effluent.

The sedimentation flux, J_s, for the layers under the feed layer (j=7,…,10) is given by

$$J_{s,j} = v_{s,j} TSS_j$$

and above the feed layer (j=1,…,6) the clarification flux, J_{clar}, is given by

$$J_{clar,j} = \begin{cases} v_{s,j} TSS_j & \text{if } TSS_{j+1} \le TSS_t \\ \min\left(v_{s,j} TSS_j, v_{s,j+1} TSS_{j+1}\right) & \text{otherwise,} \end{cases}$$

where TSS_t is the threshold concentration of the sludge. The resulting solids balances around each layer, considering steady state, are the following:

- for the top layer (j=1)

$$\frac{v_{up}\left(TSS_{j+1} - TSS_j\right) - J_{clar,j}}{h/10} = 0,$$

- for the intermediate layers above the feed layer (j=2,…,6)

$$\frac{v_{up}\left(TSS_{j+1} - TSS_j\right) + J_{clar,j-1} - J_{clar,j}}{h/10} = 0,$$

- for the feed layer (j=7)

$$\frac{\frac{Q\,TSS_a}{A_s} + J_{clar,j-1} - \left(v_{up} + v_{dn}\right)TSS_j - \min\left(J_{s,j}, J_{s,j+1}\right)}{h/10} = 0,$$

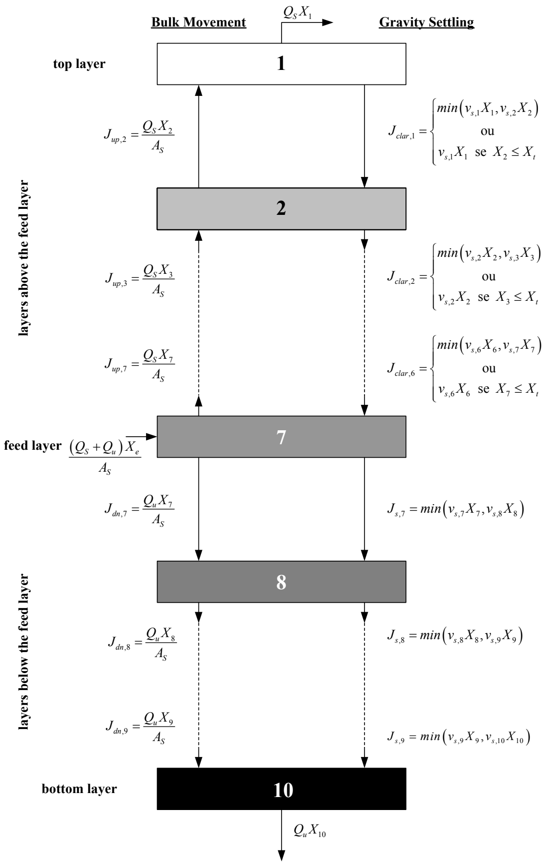

Fig. 2: Solids balance around the settler layers according to the double exponential model (adapted from [9])

- for the intermediate layers under the feed layer (j=8,9)

$$\frac{v_{dn}\left(TSS_{j-1} - TSS_j\right) + \min\left(J_{s,j}, J_{s,j-1}\right) - \min\left(J_{s,j}, J_{s,j+1}\right)}{h/10} = 0,$$

- and, for the bottom layer (j=10)

$$\frac{v_{dn}\left(TSS_{j-1} - TSS_j\right) + \min\left(J_{s,j-1}, J_{s,j}\right)}{h/10} = 0.$$

2.3 General balances

The system behaviour, in terms of concentration and flows, may be predicted by balances. In order to achieve a consistent system, these balances must be done around the entire system and not only around each unitary process. They were done to the suspended matter, dissolved matter and flows. The equations for particulate compounds, generically represented by $X_?$, (organic and inorganic) have the following form:

$$(1+r)Q_{inf}\ X_{?ent} = Q_{inf}\ X_{?inf} + (1+r)Q_{inf}\ X_? - \frac{V_a\ X}{SRT\ X_{?r}}\left(X_{?r} - X_{?ef}\right) - Q_{inf}\ X_{?ef}$$

where X represents the particulate COD.

For the solubles ($S_?$) we have:

$$\left(1+r\right)Q_{inf}\ S_{?ent} = Q_{inf}\ S_{?inf} + r\,Q_{inf}\ S_?$$

where r is the recycle rate, SRT is the sludge retention time and $Q_?$ represents the volumetric flows. As to the subscripts, *inf* concerns the influent wastewater, *ent* the entry of the aeration tank, r the recycled sludge and *ef* the treated effluent.

For the flows, the resulting balances are:

$$Q = Q_{inf} + Q_r \text{ and } Q = Q_{ef} + Q_r + Q_w .$$

2.4 Other important definitions

The other important group of constraints in the mathematical model is a set of linear equalities that define composite variables. In a real system, some state variables are, most of the time, not available for evaluation. Thus, readily measured composite variables are used instead. They are the chemical oxygen demand (COD), volatile suspended solids (VSS), total suspended solids (TSS), biochemical oxygen demand (BOD), total nitrogen of Kjeldahl (TKN) and total nitrogen (N).

It is also necessary to add some system variables definitions, in order to define the system correctly. In this group we include the sludge retention time (SRT), the recycle rate (r), hydraulic retention time (HRT), recycle rate in a PWWF event (r_p), recycle flow rate in a PWWF event (Q_{r_p}) and maximum overflow rate (Q_p/A_s).

All the variables in the model are considered nonnegative, although more restricted bounds are imposed to some of them due to operational consistencies. For example, the dissolved oxygen has to be always greater or equal to 2 mg/L. These conditions define a set of simple bounds on the variables.

Finally, the quality of the effluent has to be imposed. The quality constraints are usually derived from law restrictions. The most used are related with limits in the COD, N and TSS at the effluent. In mathematical terms, these constraints are defined by portuguese laws as $COD_{ef} \leq 125$, $N_{ef} \leq 15$ and $TSS_{ef} \leq 35$.

3 The Cost Function

The cost function is used to describe the installation and operation costs of a WWTP, in a way that reflects the behaviour of each unitary process. In the present study, only the aeration tank and the secondary settler are considered.

The basic structure of the cost function, based on the work done by Tyteca [10], is $C = aZ^b$, where C represents the cost and Z the variable that most influences the design of the unitary process under study. The parameters a and b are estimated according to the costs associated with the unit under study and depend on the local conditions where the WWTP is being built.

Although the model is nonlinear it can be easily linearized yielding

$$\ln C = \ln a + b \ln Z .$$

The parameters a and b were estimated by a least squares technique considering real data collected from a portuguese WWTP building company. At the present, the collected data come from a set of WWTPs in design, therefore no operation data are available. However, from the experience of the company, it is known that the maintenance expenses for the civil construction are around 1% during the first 10 years and around 2% in the next 10. To the electromechanical components, the maintenance expenses are negligible, but all the material is usually replaced after 10 years. The energy cost is directly related with the air flow. The power cost (P_c) in Portugal is 0.08 €/KW.h. For the sake of simplicity, no pumps were considered, which means that all the flows in the system move by the effect of gravity. Also, all the fixed costs are neglected as they do not influence the optimization procedure.

The operation cost is usually in anual basis, so it has to be updated to a present value with the parameter Γ:

$$\Gamma = \sum_{j=1}^{n} \frac{1}{(1+i)^j} = \frac{1-(1+i)^{-n}}{i}$$

with i the discount rate and n the life span of the WWTP. We used i=0.05 and n=20 years. For each unit, the total cost is given by the sum of the investment (IC) and operation costs (OC):

$$TC = IC + OC . \quad (4)$$

For the aeration tank, the influent variables are the tank volume (V_a) and the air flow (G_S). The investment cost is given by

$$IC_a = 148.6 V_a^{1.07} + 7737 G_S^{0.62} , \quad (5)$$

and the operation cost by

$$OC_a = \left(0.01\Gamma + 0.02\Gamma(1+i)^{-10}\right)\left(148.7 V_a^{1.07}\right) + (1+i)^{-10} 7737 G_S^{0.62} + 115.1 \Gamma P_c G_S . \quad (6)$$

In the secondary settler, the sedimentation area (A_s) and the depth (h) are the influent variables. For the investment and operation costs we obtained

$$IC_s = 955.5 A_s^{0.97} \quad (7)$$

and

$$OC_s = \left(0.01\Gamma + 0.02\Gamma(1+i)^{-10}\right)\left(148.6 (A_s h)^{1.07}\right) \quad (8)$$

respectively. According to (4), the objective function is then the sum of the cost terms (5) – (8).

4 The Optimization Problems

A mathematical programming problem results from the set of constraints that were described in Section 2 with the objective function presented in Section 3.

The mathematical model that relies on the ATV model to describe the settling tank has 57 parameters, 82 variables and 64 constraints, where 28 are nonlinear equalities, 35 are linear equalities and there is only one nonlinear inequality. 71 variables are bounded below and 11 are bounded below and above. We refer to [3] for details.

When the DE model is used to describe the settling tank, the mathematical model has 64 parameters, 113 variables and 97 constraints, from which 62 are nonlinear equalities, 34 are linear equalities and one is a nonlinear inequality. 102 variables are bounded below and 11 are bounded below and above.

The mathematical model that combines both ATV and DE equations (see [4]) has 64 parameters, 115 variables and 105 constraints, where 67 are nonlinear equalities, 37 are linear equalities and there is only one nonlinear inequality. 104 variables are bounded below and 11 are bounded below and above.

The chosen values for the stoichiometric, kinetic and operational parameters that appear in the mathematical formulation of the problems are the default values presented in the GPS-X simulator, and they are usually found in real activated sludge based plants for domestic effluents.

The three problems have been coded in the AMPL mathematical programming language [6] and were solved with the software package SNOPT [7], available in the NEOS Server (http://www-neos.mcs.anl.gov/).

5 Comparative Results

The main purpose of the paper is to carry out a comparative analysis of the resulting designs and use the GPS-X simulator to assess their robustness under stress conditions. First, all the problems were solved with identical conditions in the influent to the system.

Table 1 reports the optimal values of the aeration tank volume, sedimentation area, depth of the secondary settler, aeration air flow and the design total cost (TC) in millions of euros, for the three models. The most immediate result is that the ATV model is the most expensive. This is expected as the model only uses empirical equations that contemplate large security factors to account for PWWF events.

Table 1: Results for the three mathematical models

model	V_a	A_s	h	G_S	TC
ATV	884	97	6.1	10863	5.6
DE	1503	48	1	2825	2.5
ATV+DE	1744	97	10.5	5163	3.8

The DE model turns out to be the cheapest one and this is due to the fact that the PWWF events are not taken into account. Only average conditions based on balances are contemplated and the extra settler depths (h_3 and h_4) are not incorporated in the model. The most equilibrated solution is obtained when a combination of the two models is considered. The total cost of this design is between the other two. The resulting model is prepared to turn around the PWWF events without over dimensioning because it also incorporates balances. We point out that with this model the achieved quality of the effluent, in terms of *COD* and *TSS,* is under the demanded by law in contrast with the other two models where the limit values were attained.

Then, the three obtained designs were introduced in the GPS-X simulator and some stress conditions were imposed for a period of 30 days in order to assess the goodness of each solution. At first an average flow at the entry of the system was applied. At day 6, a stress flow value was imposed, and the resulting impact on the soluble *COD* (dark line) and on the *TSS* (light line) was registered, as shown in Fig. 3 to 5.

Only the combined ATV+DE model is able to support these conditions. When the design relies on the ATV model, the system gives a positive response for 18 simulation days. From that point on the system no longer gives an appropriate answer and is not able to support such an increasing flow (Fig. 3). This is the case where the demanded air flow is greater (Table 1), meaning that the sludge concentration within the reactor is bigger. In a situation of PWWF the system cannot maintain the correct concentration inside the tank, and an excess of solids is released with the treated effluent, compromising its quality.

The DE model is the most sensitive to flow variations. From the moment that the flow raises, the *TSS* stays very close to its limit (35) and from a certain point it starts to grow progressively until the 22^{nd} day, where there is saturation and the system cannot give a correct answer (Fig. 4).

When the combined ATV+DE model is used, the most robust design is achieved, and the model can support the adverse conditions for at least the period of 30 simulation days (Fig. 5).

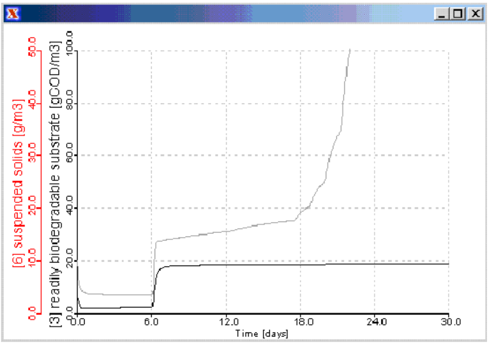

Fig. 3: Simulation using the ATV model

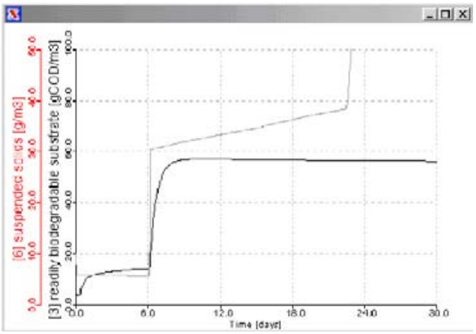

Fig. 4: Simulation using the DE model

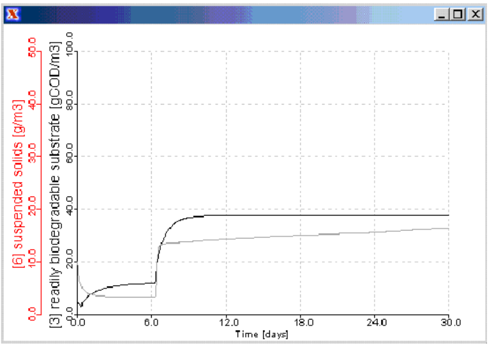

Fig. 5: Simulation using the combined ATV+DE model

6 Conclusions

The main conclusion from this comparative study concerning the optimization procedures is that the most suitable model to describe the process inside the secondary settler is the one that combines the ATV design procedure with the double exponential model. The limitations of the ATV and DE models when considered separately are overlapped by each other in the combined model and the advantages are powered. The result is a more equilibrated model in terms of cost and performance. The considered simulation procedures also show that the combined model is the most robust even under adverse conditions on the influent. The confirmed success in the secondary settler modeling is giving us some confidence to consider in the near future the optimization of the whole treatment plant with the incorporation of the sludge digestion and the final disposal processes.

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