Measuring Information Propagation and Retention in Boolean Networks and its Implications to a Model of Human Organizations

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Abstract: - A system structure, i.e., how elements of a system are connected, is a key factor for information retention and transmission through its elements. From the system dynamics, i.e., the states of the elements over time, we measure the system's ability to propagate information through its elements as the pairwise mutual information (pMI) between the elements at moments t and t + L, where L is the minimum path length between the two elements. Information retention is measured with Lempel-Ziv (LZ), a measure of the complexity of transmitted information, from the same time series of states. We propose a combined measure of information propagation and ability to retain information efficiently, to determine optimal structures for information propagation and retention. We present the results on information propagation and retention, as a function of topology (random and small world structures), connectivity, noise and clustering coefficient. The conclusions are applicable in any context where these networks are used to model the system. Here, we apply our findings to a model of human organizations and than propose a generalization of the model to capture more realistic features, such as more complex internal states for elements and simulating information exchange with the environment outside of the system. As more features are incorporated, this model will capture many important features of human organizations, and other complex systems.

Key-Words: - Organization, System, Information, Complexity, Lempel-Ziv, Information propagation and retention, Knowledge Management, Organizational Development.

1 Introduction

Structures where knowledge creation and flow of information are present are inherently dynamic and adaptive to external and internal changes [19][21] becoming resistant to environmental uncertainty [13][16]. Human organizations have these capacities, with variable degrees of effectiveness, depending on their goals and ability to innovate.

Following the Systems Theory approach [9] we focus on the organization as a whole, at the structural level, and consider the interrelationships between the different sub-systems and the importance of environmental influences [8]. Thus, we focus on the topology of the interactions between elements [3][9] and, using a general model of quantifying information transfer, show which structures optimize information propagation and retention through its elements. On the context of human organizations, this applies to organizations focused on research, but certainly not to a organization where the goal is not a free exchange of information through all members.

This optimization problem is also crucial in other systems, such as gene regulatory networks [32], where genes exchange information between them and with their environment.

Here, we study the influence of structural organization by computing mutual correlations (here measured by the pMI between time series of the elements) and knowledge acquirement (here computed with the LZ complexity [18] of each element, averaged over all elements).

Distinction between information and knowledge is not easy [19]. Information is understood as flow of signals [15], while knowledge is not just the storing of such information, but depends on the receiver's interpretation of the information [14].

Here, we do not deal with the problematic of converting information into knowledge, which as been extensively studied (see [10] e.g.). Rather, we focus in information propagation and retain, from which knowledge arises.

This article is organized as follows: after we introduce the measures of correlation of the system as a whole and the information retained by each elemen, we present and justify our modeling strategy in the context of human organizations. We then present the results for information propagation and retention for the tested topologies, as the network dynamics go from ordered to critical to chaotic [20]. We do this by varying connectivity, the influence of noise in communication and retention and the dependence on the network's local properties.

We then show that, for constant connectivity, the cause is the variation of the average clustering coefficient as the size of the network grows [28].

Next, we introduce a more complex model of organizations where the internal state of the system elements are no longer Boolean variables, thus allowing much more complex dynamics. We propose the functions by which they interact and the method for measuring correlations on their temporal patterns and information retained. Finally, we present our conclusions and future developments.

2 Problem Formulation

Although the model here proposed is general in the sense that it can be applied in any system where elements exchange information and have the ability to retain information, we shall use the human organizations model as a specific example of such systems.

Optimizing human organizations information propagation and retention is very complex. The knowledge an organization retains in its elements is a fundamental resource [11][12], expressed by how efficiently the organization functions. One problem is choosing what parameters to optimize. Many have to be optimized simultaneously, and some might "compete" with one another.

Most previous works on optimizing knowledge sharing in organizations focuses on the individuals' characteristics and how information sharing can be improved given human behavior ([19] e.g.). Here we focus at the structural level, disregarding elements' internal properties.

Several mechanisms allow communication among people within an organization.. "Regular" communication mechanisms, which provide the opportunity to share knowledge and data, creation of empathy and trust, facilitating exchange of information, and, face-to-face meetings that provide opportunities to share experience and focus explicitly on common problems and their solutions have been broadly studied [5]. Variables such as the building structure, and orientation within a building, are features to consider for an optimized work environment [1][2].

Through the process of transfer of learning, both among its members and with outside information sources, organizations develop their competitive advantages [4]. Several studies measure the effects of organizational structure on the learning process, suggesting that organizational structure is fundamental for enhancing learning, and affecting organization performance (e.g., [5][6][7]).

To define the structure of the system we focus on direct work relations, where proximity is high and interactions more stable. We consider such relations as those of direct dependency ("boss"–employee) and (college-college), and disregard less stable and indirect interactions. We model this structure as a directed graph between elements.

The system dynamics are represented by a synchronous Boolean network. The element states (which define the system state) are Boolean variables and, at time t+1 are defined by a random Boolean function, assigned to each element, given the state of its inputs at time t.

The relation between this model and the system we wish to model is the following: each message, sent from one element to another, has an information content (in this model, either 0 or 1), and, regardless of the content, is perceived,. This content will act on the receiving element, according to internal properties (such as mimicking individual levels of perception for each element), and is here modeled by assigning to each element a random Boolean function which, given the combination of inputs values, determines the output value.

Each element stores all the states experienced over time. This constitutes the messages received, mimicking the information received and from which knowledge acquisition arises (when the element provides meaning to such information [10][19], here not modeled). We focus only on the quantity of distinct messages.

Information loss as a message, transmitted and/or stored, is modeled introducing a probability that the element will, at each time step, do the opposite of what the inputs states and Boolean function determine.

Given the model, we now focus on how to measure the system's ability to propagate information through the elements and store information in each element from the time series of the elements' states.

When a person receives information, his "internal state" changes, provided that there is something new in such message. Namely, a message can be "irrelevant" given what the person previously knew. Here, an "irrelevant message" is one already received.

Also, the elements that send the information (inputs) and the receiver (output) should be more correlated if the information is relevant. pMI captures this correlation, since it is zero between two elements if one of the elements time series is "frozen" (no relevant information being propagated, thus, no state change), and for random or uncorrelated time series (no information being propagated in such a way that the elements states become correlated via information transfer). Yet, it is high when inputs and output states change over time in a correlated manner.

Notice that "changing state", due to receiving a message, in all time steps in not necessarily good. That is, if one element has a periodic behavior with a small period, no new information is being transmitted, although states vary in time. The diversity of messages sent should be maximized in a system that has as its primary purpose effective information transmission between its elements with relevant meaning (messages from which the receiver can extract new knowledge).

To measure the diversity of messages sent, we compute the normalized (also called relative) Lempel Ziv complexity [17] (LZ) of each element time series. This quantity increases with the diversity of messages sent, independent of the size of the time series. To measure the influence of structural local properties, we use the average clustering coefficient [24] (Cp) of the structure.

In the next section, we describe in detail how these quantities are computed from the model here proposed.

2.1 Measure of information transmission

Mutual information [15] (MI) measures correlation between variables. Applied to Boolean networks (BNs), mutual information can be used to measure the correlation between nodes time series of states. Here, we restrict to pMI, i.e., between pairs of nodes.

This measure can be used to solve the inverse problem [22], i.e., given the elements time series, find the structure and logic of the network. Yet, in our framework, the structure is known and one is interested in the average global correlation among all elements, whether they are directly connected or not. We can compute such correlation as a function of the minimum distance, i.e., the minimum path length (L) between the two elements (see, e.g., [24]), attained using Dijkstra's algorithm [23]. If two elements are L steps apart, the correlation should be measured as a function of such distance, since that is the time that a message takes to travel within the network between the two elements.

We use this quantity as a measure of correlation between pairs of elements' time series. Given the distance L_{ij} between elements i and j, we compute their pMI with (1):

$$MI[i(t),j(t+L_{ij})] = H[i(t)]+H[j(t+L_{ij})]-H[i(t),j(t+L_{ij})] \quad (1)$$

From this we compute the average pMI, as a function of path length, using formula (2), in a network of n elements:

$$< MI\left[i(t), j(t+L_{i,j})\right] >= \frac{\sum_i \sum_j MI(i(t), j(t+L_{i,j}))}{n^2} \quad (2)$$

We measured the average pMI of the time series of elements states, obtained from the model simulator, using the algorithm developed in [22].

2.2 Measure of information retention and complexity of transmitted messages

Lempel-Ziv (LZ) measures [17] an individual sequence complexity over a finite alphabet (here {0,1}) counting the number of new sub-strings (words) found as the sequence is read from left to right. The algorithm separates the sequence into shortest words that haven't occurred previously and the complexity equals the number of such unique words, except the last word, which may not be unique [18].

For example (described in [18]), consider the sequence 01100101101100100110. The first digit, 0, is a new word since it hasn't been seen before. So is the second digit, 1. The third digit, also a 1, has been seen before, so one must increase the length of the word by one, resulting in a new word "10", and so on. Repeating this process, the sequence gets parsed as follows: 0•1•10•010•1101•100100•110, where the dots delimit new words. Thus, the LZ complexity of this word is 7. All words, except the last one, are unique and, using this definition of LZ complexity, the search for previous occurrences of a word can span across previously seen word boundaries [18]. Repetition results in lower LZ; e.g., the complexity of the sequence 01010101010101010101 is 3. In general, time series with repetitive or simple patterns have a low LZ, whereas series with a complex pattern structure exhibit high LZ.

Previous work [18] showed that "networks in the ordered or critical regimes exhibit lower LZ complexities of the sequences generated by each node due to their pattern-like behavior over time, as compared to networks in the chaotic regime, which give rise to more random gene behavior". We measured the average LZ complexity (<(LZ)> of the time series of elements states, using the algorithm suggested in [25].

As shown here, the <pMI> decreases significantly in the chaotic regime, since nodes become uncorrelated, thus a compromise must be reached to maximize both quantities. To measure the efficiency of both information propagation and retention, one can combine the <pMI> and <LZ> in the following relation:

$$X = n. < pMI > + m. < (LZ) > \quad (3)$$

The constants n and m depend on the relevance of information transmission between elements and information storage in each element for a particular system. Notice that, the way to combine these quantities depends on the system in question and what one intends to maximize. Here we analyze these measures independently, as a function of several network parameters.

As we shall see, these quantities are very sensitive to "how" the nodes are connected at a "local level", that is, to their nearest neighbors. Therefore, we need a measure that captures such characteristics.

Here, we propose and show that the network clustering coefficient captures this important feature.

2.3 Clustering Coefficient

From its definition [24], with E_i representing the number of connections between the k_i elements connected to a certain element i, the network clustering coefficient, C(p), is:

$$\langle Ci \rangle_i = \left\langle \frac{2.E_i}{k_i.(k_i - 1)} \right\rangle_i \quad (4)$$

This quantity is smaller for random than for small world topologies. A perfectly random structure should have a null clustering coefficient.

Therefore, we use this quantity to explain why the average connectivity k for which pMI is maximized varies as the network number of nodes grows (for small number of nodes).

In the following section we present the results.

3 Results

We generate networks initialized at random states, and the average pMI as a function of path length, average LZ complexity, and average C_p are computed for 1000 runs of 1000 time steps each so that the results represent an average behavior, following the ensemble approach procedure [20].

Notice that, given such time series length, in the cases where there is no noise, for most of the time the system is in an attractor.

We study the variation of <LZ> and <pMI> for two topologies: [i] random, as connectivity varies (Fig. 1); and [ii] small world [24][26][27], as the probability p of rewiring regular connections randomly varies (Fig.s 2a and 2b). The first one is chosen since it allows the generation of a wide variety of networks, and the second is a topology present in many known natural networks [28][29][30].

We then study the influence of noise (Fig.s 3a and 3b) for random networks with connectivities of 1, 2 and 3. Finally, we analyze the influence of the size of the network, i.e., total number of nodes (Fig. 4), maintaining all other parameters constant and the effects of local structural connectivity by analyzing the connectivity for which pMI is maximum, as the Cp varies with the network number of elements increase, maintaining average connectivity (k).

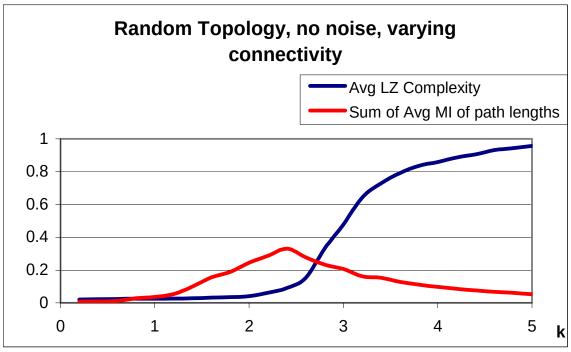

Fig. 1: <pMI> and <LZ> for random topology, variable connectivity and no noise. Maximum pMI occurs for k = 2.4, while LZ "phase transition" begins at k = 2, when the network dynamics goes from ordered to critical, and then chaotic [20].

From Fig. 1, for a system where no noise exists, both pMI between elements and the diversity of messages received by each element (LZ) are maximum for nearly chaotic dynamics (the phase transition from ordered to chaotic occurs for an average connectivity of 2.0 [20]). We show (in Fig 5) that this maximum occurs each time "closer" to the phase transition as the size of the network grows, and Cp diminishes, making loops of connections among small numbers of elements less common.

From Fig.1 it is clear that a network with the purpose of propagating and retaining information should be at the "edge of chaos" [32].

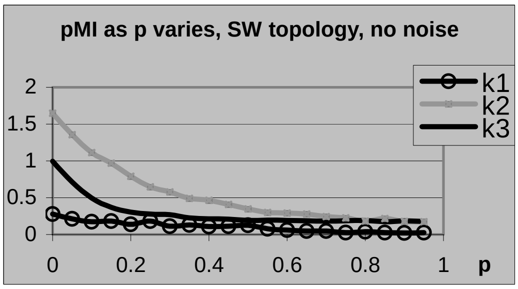

Fig. 2a: <pMI> as the ratio p of random connections varies in a Small World topology, where p is the probability of rewiring a regular connection randomly, for k=1,2 and 3.

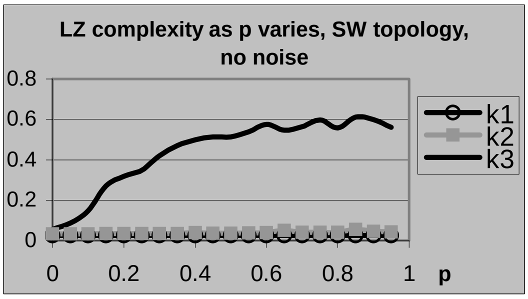

Fig. 2b: <LZ> as the ratio of random connections varies in a small world topology, for k=1, 2 and 3.

Using a small world topology constructed using the algorithm proposed in [24], we began our tests, starting with ring lattice of k = 1, 2 and 3 (corresponding approximately to the three dynamical regimes, order, critical and chaotic) and rewiring probability varying from 0 to 0.95 (Fig.s 2a and 2b).

For an average k of 1, the network is always in the ordered regime, and both pMI and LZ are low, as expected from the previous results on random networks. Randomization of connections (increase of p) is not enough to increase LZ and pMI due to several nodes with no inputs ("frozen").

For an average k of 3, the network has, for small values of p, a behavior near the critical regime, and as p grows, the network goes deep into the chaotic regime and pMI drops to near null.

Using a average connectivity of 2, one observes that the network goes from ordered to chaotic, having the highest pMI between all nodes. It is between k = 2 and 3 that the both LZ and pMI are maximized, as in random networks.

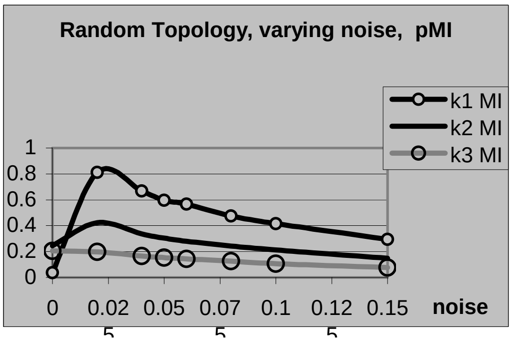

Fig. 3a: pMI values for random topology, connectivity 1, 2 and 3, varying noise.

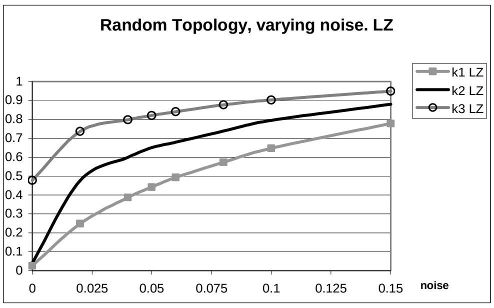

Fig. 3b: LZ complexity for random topology, k = 1, 2 and 3, varying noise.

In Figs. 3a and 3b we study the effect of noise in random topologies of average k of 1, 2 and 3. As noise increases, the elements of the network become more and more uncorrelated (shown with the decrease of pMI). Yet, for low connectivity values, noise (smaller then 0.05) allows "unfreezing" of elements, increasing both LZ and pMI. Above that threshold, noise level pMI decreases since elements become more and more uncorrelated.

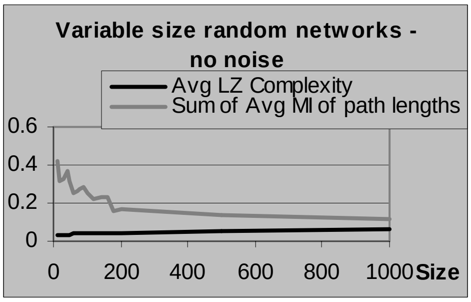

Fig. 4: Random topology, varying the size of the network (number of nodes), with average k of 2 and no noise.

For networks with more than 200 nodes, the two quantities do not vary significantly (Fig 4) for a large variation in the number of nodes, while for smaller networks, the small size allows high fluctuations, even though these results are averages of 1000 independent runs. We computed the standard deviation of pMI of Fig. 4, and, for small size networks, it has the same magnitude as those of the pMI itself, confirming the high fluctuations.

These fluctuations in smaller networks are explained by the highest probability, for nodes connected to another node, to share a connection between themselves, thus creating small "cycles" that are responsible for "un-correlating" more nodes than if such "local loop structures" did not exist.

Also, higher Cp for small networks causes the maximum pMI to occur for higher connectivities than 2, since for k =2 the network is still ordered, and the phase transition between order and chaos occurs for higher k values (between 2.4 and 2, as we show below) than 2.0 (the predicted value for random networks).

To prove these statements, we now study the variation of Cp with the size of random networks whose connection distribution follows a Poisson distribution (Fig. 5a).

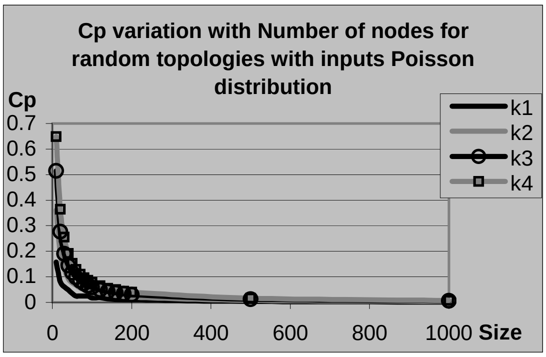

Fig. 5a: Cp average values for random topologies, k =1,2, 3 and 4, where connections are placed following a Poisson distribution due to a uniform probability of connectivity for all nodes.

From Fig. 5a, one observes that, for small sized networks, the Cp value is relevant and its relevance is higher for networks with higher average connectivity. This, as seen is Fig. 5b, explains why the maximum of the <pMI> shifts to higher connectivity values for smaller sized networks, since higher Cp allows a higher chance of "closed circuits" which tend to "freeze" in a steady state.

It is, therefore, important to show for which connectivity values pMI is maximized, as the size of the network grows (Fig 5b).

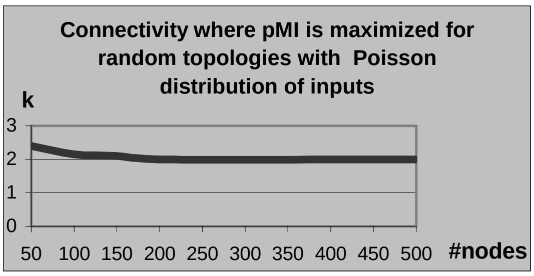

Fig. 5b: Values of connectivity for random topologies of networks as size varies, for which the pMI is maximum.

As expected, but only for large networks (N > 200), the k for which pMI is maximized corresponds to the critical regime.

To show if the reason pMI is maximized for different k values is Cp variation we kept Cp constant for variable size networks. The pMI maximum occurred for k = 2.0 in all cases. Also, using regular ring lattices where Cp is constant, pMI is constant as size varies (Fig 6).

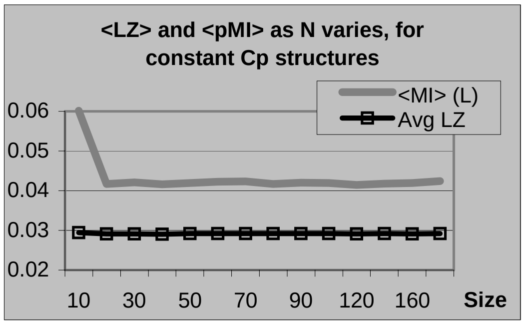

Fig. 6: <pMI> and <LZ> for ring topology networks of varying size but equal Cp.

The results (Fig 6) show that ring lattices where Cp is conserved, do not vary <pMI> or <LZ> with size, except for 50 nodes, where the network is so small that large deviations occur for small network topology differences.

We investigated the reason for this. The Cp is a measure of small loop structures occurrence in the network. Only "small" loops of a few nodes will affect the pMI average. The probability of those loops to occur diminishes as size grows. Yet, keeping Cp constant only takes in consideration 3 nodes loops.

Larger loops, of 4 or 5 nodes still affect pMI average. That's the reason why, using random topologies where we forced Cp to be null, although Cp is constant, for the k = 3, pMI still varies for sizes smaller then 100 nodes, significantly.

Finally, the <LZ> quantity had no significant variation with size, was expected. Its mainly constant with very small variations In fact, from all cases here analyzed we observed that, unlike the pMI, this quantity is independent of Cp, varying only with average k (see Figs 4 and 6).

4 Conclusion

We proposed a model of information transmission and retention in Boolean networks, and its application on human organizations, by representing humans as the elements of the network, and communication among people as connections in the network.

Using pMI as a function of the minimum path length from one node to the other, we studied how information propagates through the network. The LZ complexity of the resulting time series of states for each element, quantifies how the structure affects information retention.

Testing different topologies (random and small world), we have shown that pMI is highest in a network with rich dynamics, where k is high enough to allow complex interactions (corresponding to the critical regime in Boolean networks), but not too high so that the other nodes don't drown a node out.

The maximum pMI occurs, in all cases, in the network critical regime, between order and chaos, while LZ grows continuously from order to chaotic.

Varying the level of noise in the dynamics of random topologies of k = 1, 2 and 3 we observed that as noise increases, pMI diminishes. Yet, small noise levels ($p_{noise} < 0.05$) allows "unfreezing" elements, increasing both LZ and pMI

By computing the standard deviation, we showed that small random networks have high fluctuations in pMI and LZ, and proved that the property that causes not only these variations, but also differences in systems of equal connectivities (but different sizes) of dynamical regime, is the clustering coefficient.

We showed how the average Cp varies with the size of the random networks, of equal connectivity where the connections are placed following a Poisson distribution. By generating ring topology networks of different size but maintaining the Cp constant, the maximum of the pMI occurred always for k = 2, given that no noise existed in the dynamics. The effect of noise is to shift the pMI maximum for smaller k values, and, invariably, increase of LZ.

When we tested the same using random topologies of constant Cp value, we observed that larger loops of 4 and 5 nodes, are still capable of varying pMI as the network size varies.

The results and conclusions are general in the sense of being a property of Boolean networks and applicable in any context where these networks are used to model systems.

Under the context of gene regulatory networks [20], the results attained here show under which conditions these networks become more capable of transmitting information, while retention, in this framework, takes the meaning of complex patterns of genes activity, being maximized according to the LZ maximization conditions.

A more advanced model would be able to give results that are better correlated with the real world, depending on the system being modeled. In the future, we intend to incorporate more realistic features into the model of human organizations and stochastic gene regulatory networks [33].

5 Final Remarks and Future Work

Any message can be coded in binary code. Yet humans respond to messages in an extremely wide variety of ways, not fully captured using only binary variables. Also, they combine messages in more complex way then Boolean logic. Allowing more complex states of activity is a necessity for realistic models of information transmission and retention.

It is also necessary to consider each element with a different rate of "misinterpretation" and degree of "error" (how different from the initial message will the perceived message be) depending on each person's characteristics. More specifically, the interpretation of messages by humans should be improved such that it includes the person's beliefs system (e.g. [31]). Using the "belief networks" framework will allow modeling the "noise" due to personal interpretation more realistically, while here this was modeled by assigning a random Boolean function to each node.

These generalizations are also relevant to deal with other information propagation networks, such as gene regulatory networks, or, protein interaction networks and other metabolic networks.

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