Bifurcation Lines Calculations of Period-1 Ferroresonance

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Abstract: - The principal contribution of this paper is to develop a numerical tool to calculate the bifurcation lines of the fundamental ferroresonance (i.e., period-1). The developed process uses the same methods for construction of the bifurcation diagrams : Galerkin method and continuation by the pseudo-arclength method. The obtained Galerkin algebraic equations are nonlinear. The applied iterative method of resolution is that of Newton-Raphson. The determination of Jacobien of this problem requires computation of the matrix determinant derivatives. Except for a few simple cases, it is difficult to express this derivation analytically. To solve this difficulty, a numerical relation is developed.
Applied to a series ferroresonant circuit, describing an opening operation of a circuit-breaker supplying a voltage transformer, we could continue the bifurcation point in a plan with two parameters. With this type of curve, it is possible to estimate the associated safety margin, and thus to operate the electrical supply network with total safety. Several results obtained numerically, starting from a real case, are illustrated and discussed. In addition, to validate these results, a simplified analytical method is developed.

Key-Words: - Transformer, Nonlinear, Ferroresonance, Bifurcation, Galerkin method, Continuation method, Bifurcation line.

1 Introduction

Ferroresonance is a nonlinear resonance phenomenon that can affect power networks. The main problem with ferroresonance is that an overvoltage (as for fundamental ferroresonance), or an overcurrent (as for subharmonics) is generated. This is often dangerous for the electrical equipement [1]. These phenomena are not transient and are present during normal operation [2, 3, 4].
The engineer's practical problem is to know whether these dangerous phenomena may appear in his circuit. Simple simulation of the representative equations is not suited to the problem. Indeed, many parameters are poorly known in a real circuit : losses, saturation curves, switching-in instants, etc. To ensure that there is no risk, many variants must be simulated before any conclusion can be drawn -and transient states are long to simulate. In addition there is a risk of being at the limit of the dangerous zone without knowing it. This is why the engineer wants to have an overall view of his circuit's behavior. He wants to know whether he has a good safety margin or not. To have an overall view of the phenomenon, the latter must be placed within an appropriate mathematical framework.
Modeling this problem results in a system of time differential, nonlinear equations (known as the dynamic system) which depends on various physical parameters. The mathematical framework adapted to the study of these dynamic systems is the bifurcation theory [2, 5, 7].
We are especially interested here in the study of the dynamic system of figure 1, which describes the practical problems of series ferroresonance. Most commonly, this type of situation is achieved when a magnetic voltage transformer (the nonlinear inductance) is connected to a busbar separated by the grading capacitance of an open circuit-breaker (the series capacitance) [4, 6, 8].

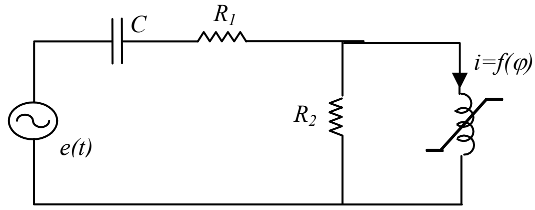


Fig.1 : Series, single-phase, nonlinear ferroresonant circuit.

The physical parameters of this circuit are:
E : amplitude of the sinusoidal voltage source
e(t) = E sin (100πt),
C : equivalent capacitance of the circuit, corresponding to the capacitance of the open circuit breaker and to all the capacitances to earth of the voltage transformer and the connexion
R_1 : series losses of the circuit, and
R_2 : parallel losses of the circuit.

The magnetic characteristic of nonlinear inductance $i(\varphi)$, where i is the inductance current and φ, its flux, is modeled by a polynomial function with an odd power:

$$i(\varphi) = k_1\varphi + k_n\varphi^n \quad ; n \in N \text{ , } (k_1, k_n) \in R^+ \qquad (1)$$

Numerical data, corresponding to the parameters of the circuit illustrated in figure 1, are:

$R_1 = 32$ kΩ ; $R_2 = 714$ MΩ ; $C = 0.4$ nF.

$i(\varphi) = 10^{-8}\varphi + 2.34 \cdot 10^{-34}\varphi^9$, corresponding to a real voltage transformer 400/20 kV.

This paper aims at a global response to the ferroresonance phenomenon in permanent state. For a specific circuit, this answer is given by the continuation of the bifurcation points when two or several parameters vary simultaneously. The obtained curve is called bifurcation line. Then it will be possible to study the limits of the existence zones of singular phenomena, to take a safety margin, between the studied case and the close dangerous solutions, and to operate the network under totally safe conditions.

In this study, our effort is mainly devoted to the development of a numerical tool, based on the Galerkin method and the pseudo-arclength continuation method. It is a tool for computation of bifurcation lines of the fundamental series ferroresonance case (i.e., period-1), in plans with two parameters. Our objective is to determine the critical borders between the ferroresonant and normal zones in the two plans (losses, applied voltage) and (capacitance, applied voltage).

2 Construction of the bifurcation lines

The fundamental ferroresonance is a periodic phenomenon of the same frequency as the source. That is why a model equation is adopted by the Galerkin method [2]. To validate the results of this method, a simplified analytical approach is developed.

2.1 Analytical method

Fundamental ferroresonance is essentially characterized by discontinuous variations of the flux amplitude when the source voltage is constantly modified. These variations - commonly called “ferroresonnant jumps” - occur at different source voltages according to the variation direction (figure 2).

On this figure, one sees that the values E_1 and E_2 are limiting values or critical values of the supply voltage parameter. The two corresponding solutions PL_1 and PL_2 are called bifurcation points of the limit or turning point type. These points divide the diagram into three branches : a normal branch from the origin (0,0) to point PL_1, an unstable branch from PL_1 to PL_2, and a ferroresonant branch beyond PL_2 [2, 4, 5].

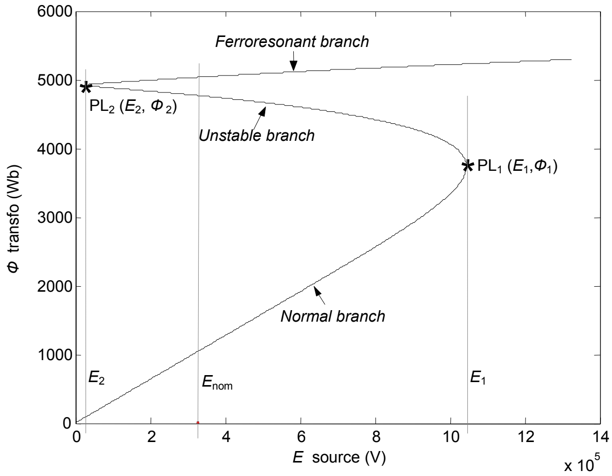


Fig.2 : S-curve bifurcation diagram of flux amplitude versus applied voltage for the fundamental ferroresonant phenomenon.

Computation of voltages E_1 and E_2 is easily performed if the harmonics of flux are neglected and only the fundamental component is considered. These voltages correspond to a null derivative of the source voltage in relation to the flux, i.e.,

$$\frac{dE}{d\varphi} = 0 \qquad (2)$$

We propose to express these jump voltages by considering the sinusoidal flux of the form :

$$\varphi(t) = \phi\cos(\omega t - \theta) \qquad (3)$$

The differential equation governing the study circuit (figure 1) as follows :

$$R_1 i_1 + \frac{1}{C}\int i_1 dt + \frac{d\varphi}{dt} = E\sin(\omega t) \qquad (4)$$

After derivation of equation (4), with suppression of the current, the following equation (5) is obtained :

$$\left(\frac{R_1}{R_2}+1\right)\frac{d^2\varphi}{d^2t} + \left[\frac{1}{R_2C} + R_1\left(k_1 + nk_n\varphi^{n-1}\right)\right]\frac{d\varphi}{dt} + \frac{1}{C}\left(k_1\varphi + k_n\varphi^n\right) = E\omega\cos(\omega t) \qquad (5)$$

which gives the following equation (6), after replacement φ by its expression (3) and keeping only the fundamental terms (which suppresses time t) :

$$E^2 = \left(A\phi\right)^2 + \left(\frac{B\phi}{\omega}\right)^2 \qquad (6)$$

where A and B are functions of ϕ^{n-1} :

$$A = \frac{R_2}{C} + R_1\left(k_1 + \alpha\phi^{n-1}\right)$$

$$B = \frac{1}{C}\left(k_1 + \alpha\phi^{n-1}\right) - \omega^2\left(1 + \frac{R_1}{R_2}\right)$$

and α is a constant of value:

$$\alpha = \frac{2n}{n+1} k_n \frac{1}{2^{n-1}} C_{n-1}^{(n-1)/2} .$$

Bifurcations occur when the relation (2) is verified, i.e.

$$\frac{dE}{d\phi} = 0 \qquad (7)$$

which gives for equation (8):

$$a\phi^{2n-2} + b\phi^{n-1} + d = 0 \qquad (8)$$

where *a, b* and d are constants depending on the circuit's parameters and on the coefficients of the non-linear element :

$$a = \alpha^2 n\left(R_1^2\omega^2 + \frac{1}{C^2}\right)$$

$$b = \alpha\left(n+1\right)\left[R_1\omega^2\left(R_1k_1 + \frac{1}{R_2C}\right) - \frac{\omega^2}{C}\left(1 + \frac{R_1}{R_2}\right) + \frac{k_1}{C}\right]$$

$$d = \frac{k_1^2}{C^2} + \omega^2\left(R_1k_1 + \frac{1}{R_2C}\right)^2 + \omega^4\left(1 + \frac{R_1}{R_2}\right)^2 - \frac{2k_1\omega^2}{C}\left(1 + \frac{R_1}{R_2}\right)$$

Equation (8) is a quadratic equation in ϕ^{n-1}. Both solutions, if available, are the values ϕ_1 and ϕ_2 of the flux at the turning points. The corresponding voltages E_1 and E_2 are given by relation (6).

To specify the critical borders between the two zones in which the network behavior is either ferroresonnant or normal, it is necessary to trace the bifurcation lines in spaces at two parameters. That enables us to specify the values domains of different parameters controlling the physical model, where the solution will have the desired behavior.

We apply this method to the study of the circuit of the figure1; it has enabled us to draw certain bifurcation lines (figures 3).

2.2 Galerkin method

2.2.1 Model and equations

This method consists in finding an approximate periodic solution of the nonlinear differential equation (4) by minimizing the error associated with this solution . The idea is to seek this solution in the form of Fourier series [2]. Here the method is exposed only for the case of the first harmonic, but it can be generalized for an unspecified harmonic rate.

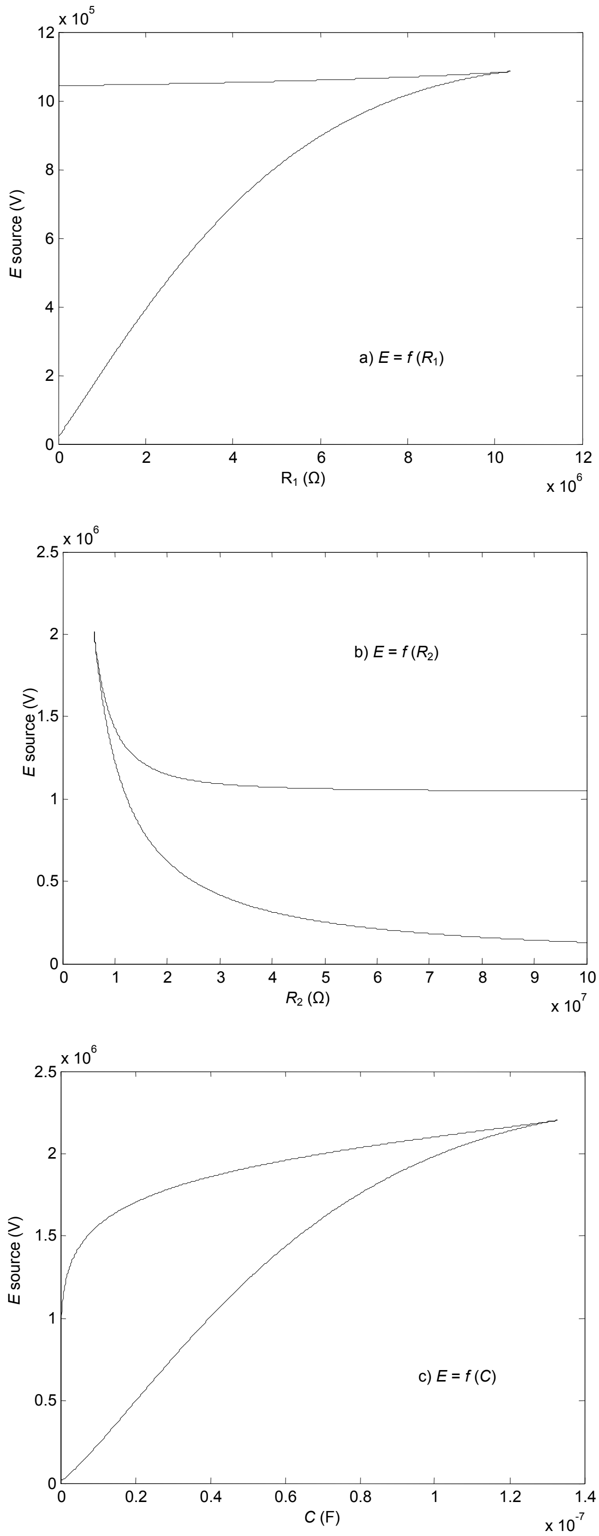


Fig.3 : Bifurcation lines – Analytical method.

For that, a network modeling is adpted, without the nonlinear element, based on the equivalent Thévenin model (figure 4).

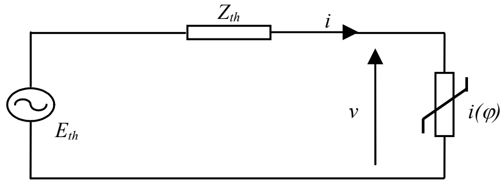


Fig.4 : Equivalent study circuit.

The complex equation of circuit (figure 4) for the fundamental one is as follows :

$$j\omega\phi_1 + Z_1 I_1 - E_1 = 0 \tag{9}$$

where ω is the pulsation at 50 Hz of the excitation, ϕ_1, E_1, Z_1 and I_1 represent respectively the complex components at this pulsation of the flux in the nonlinear element, of the supply voltage, the equivalent impedance of Thevenin and the current traversing the circuit .

Flux $\varphi(t)$ is supposed sinusoidal (3) and by adopting the following complex notations :

$$\begin{aligned} \phi_1 &= \phi_{1c} - j\phi_{1s} \\ I_1 &= I_{1c} - jI_{1s} \\ E_1 &= E_{1c} - jE_{1s} \\ Z_1 &= R_{1c} + j\phi X_{1s} \end{aligned} \tag{10}$$

equation (9) is converted into a nonlinear algebraic system of 2 equations, as follows :

$$\begin{aligned} \omega\phi_{1s} + R_1 I_{1c} + X_1 I_{1s} - E_{1c} \equiv \xi_{1c} = 0 \\ \omega\phi_{1c} + X_1 I_{1c} + R_1 I_{1s} + E_{1s} \equiv \xi_{1s} = 0 \end{aligned} \tag{11}$$

Since we know the nonlinear characteristic $i(\varphi)$, it is possible to compute the harmonic components of the current in terms of flux components. The system of equations to be solved is thus as follows :

$$\xi(\phi, E) = 0 \tag{12}$$

where ϕ is the unknown vector formed by the fundamental component of flux and E is the amplitude of the supply voltage.

To solve this system (12), the Newton-Raphson method is used which requires the computation of the Jacobien J of the system, i.e.,

$$J = \begin{bmatrix} \dfrac{\partial \xi_{1c}}{\partial \phi_{1c}} & \dfrac{\partial \xi_{1c}}{\partial \phi_{1s}} \\ \dfrac{\partial \xi_{1s}}{\partial \phi_{1c}} & \dfrac{\partial \xi_{1s}}{\partial \phi_{1c}} \end{bmatrix} \tag{13}$$

of which the elements are expressed in a general way by :

$$\frac{\partial \xi(\phi, I)}{\partial \phi} = \frac{\partial \xi}{\partial \phi} + \frac{\partial \xi}{\partial I} \cdot \frac{\partial I}{\partial \phi} \tag{14}$$

2.2.2 Bifurcation lines – Galerkin method

To determine a fundamental ferroresonnant state, system (12) has to be solved; this can be written as follows :

$$\xi(\phi, E, P) = 0 \tag{15}$$

where :

ϕ , state variable : vector formed by the two Fourier components of the flux ;

E, first parameter : scalar representing the supply voltage ;

P, second parameter : scalar representing for example the capacitance of the circuit or the losses, etc.

Using the pseudo-arclength continuation method [9], curve $\phi = f(E)$ will be followed, where E is variable and P is constant.

If we assume that the conditions of derivability are satisfied, by derivation of system (15), we get :

$$\frac{\partial \xi}{\partial \phi} \cdot d\phi + \frac{\partial \xi}{\partial E} \cdot dE = 0 \tag{16}$$

Like at the limit point (figure 2), there are $dE = 0$, whereas $d\phi \neq 0$, the Jacobien J is singular when its determinant is null, i.e.:

$$\det\left[\frac{\partial \xi}{\partial \phi_1}(\phi, E, P)\right] = 0 \tag{17}$$

Thus, when this condition is met, the local solution is not unique; and this marks the emergence of a bifurcation.

To determine all bifurcation points for each value of P, it is possible to solve the new system (18) with three variables (ϕ , E) :

$$\begin{aligned} &\xi(\phi, E, P) = 0 \\ &\det\left[\frac{\partial \xi}{\partial \phi_1}(\phi, E, P)\right] \equiv \xi_d = 0 \end{aligned} \tag{18}$$

by the Newton-Rapson method which requires the computation of the new Jacobien J_1 :

$$J_1 = \begin{bmatrix} \begin{bmatrix} & & \\ & J & \\ & & \end{bmatrix} & \dfrac{\partial \xi}{\partial E} \\ \dfrac{\partial \xi_d}{\partial \phi} & \dfrac{\partial \xi_d}{\partial E} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} & & \\ & J & \\ & & \end{bmatrix} & \begin{matrix} \dfrac{\partial \xi_{1c}}{\partial E} \\ \dfrac{\partial \xi_{1s}}{\partial E} \end{matrix} \\ \dfrac{\partial \xi_d}{\partial \phi_{1c}} \quad \dfrac{\partial \xi_d}{\partial \phi_{1s}} & \dfrac{\partial \xi_d}{\partial E} \end{bmatrix} \tag{19}$$

All elements of the last column, representing derivatives in relation to unknown E, are simple to calculate. To express J_1 in its entirety, the elements of the last line must be computed.

2.2.3 Computation of the last line of matrix J_1

The elements of the last line of the matrix J_1, representing the derivatives of the Jacobien J determinant in relation to the unknown parameters of the problem, are expressed numerically by the following theorem:

Theorem: *Let us suppose that the elements of matrix* $J(\phi)=\left[J_{ij}(\phi)\right]$ *(square of order m) are derivable functions of the unknown variable* ϕ *, then the derivative in relation to* ϕ *of* $\det\left[J(\phi)\right]$*, that is to say* $\frac{d}{d\phi}\left(\det\left[J(\phi)\right]\right)$*, is the sum of m determinants obtained by replacing in all possible ways the elements of the one of the lines (columns) of* $\det\left[J(\phi)\right]$ *by their derivatives in relation to* ϕ *.*

Based this theorem, the elements of the last line are expressed in a general way by:

$$\frac{d}{d\phi}\left(\det\left[J(\phi)\right]\right)=\sum_{i=1}^{m}\det\left[J_i^{'}(\phi)\right] \quad (20)$$

where $J_i^{'}(\phi)$ is the matrix obtained starting from J by replacing each element $J_{ij}(\phi)$ of line i by its derivative $\frac{d}{d\phi}\left(J_{ij}(\phi)\right)$.

By using the development of Laplace according to line i, the expression of $\det\left[J_i^{'}(\phi)\right]$ is given by the scalar product of line i of $J_i^{'}(\phi)$ by the vector of the corresponding cofactors:

$$\det\left[J_i^{'}(\phi)\right]=\sum_{j=1}^{m}\left[\frac{d}{d\phi}\left(J_{ij}(\phi)\right)\cdot cofactor\left(J_{ij}(\phi)\right)\right] \quad (21)$$

with $cofactor\left(J_{ij}(\phi)\right)=(-1)^{i+j}\cdot M_{ij}\left(J_i^{'}(\phi)\right)$

where $M_{ij}\left(J_i^{'}(\phi)\right)$ represents the minor of the coefficient $J_{ij}(\phi)$.

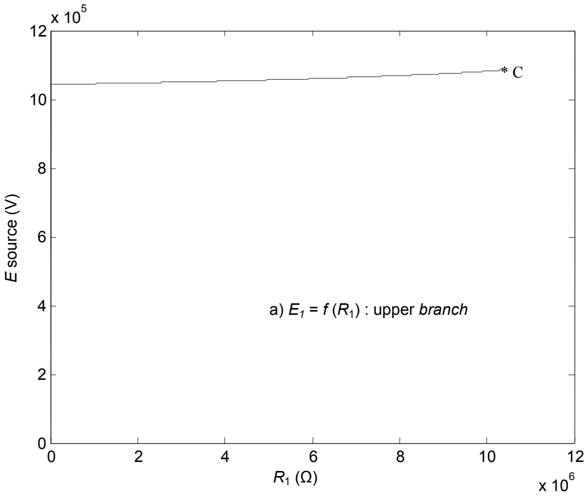
That makes it possible to conclude, by using the relation (21), that the elements of the last line of the matrix J_2 are given by the following relation:

$$\frac{d}{d\phi}\left(\det\left[J(\phi)\right]\right)=\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{d}{d\phi}\left(J_{ij}(\phi)\right)\cdot cofactor\left(J_{ij}(\phi)\right) \quad (22)$$

To draw a bifurcation line, we need simply the apply a continuation method to system (18). Given solution (ϕ_0, E_0) with value P_0 in the 2nd parameter, it is possible to seek the solution for $P_0+\Delta P$ by initializing with (ϕ_0, E_0). The parameter P is used as a continuation parameter.

2.2.4 Results obtained

This method is applied to draw the branches of the bifurcation lines of the circuit, illustrated in figure 1, in the plan (E, R_1). Figures 5 represent the obtained results. Classically these branches meet and they coincide perfectly with those obtained by the analytical method (figure 3).



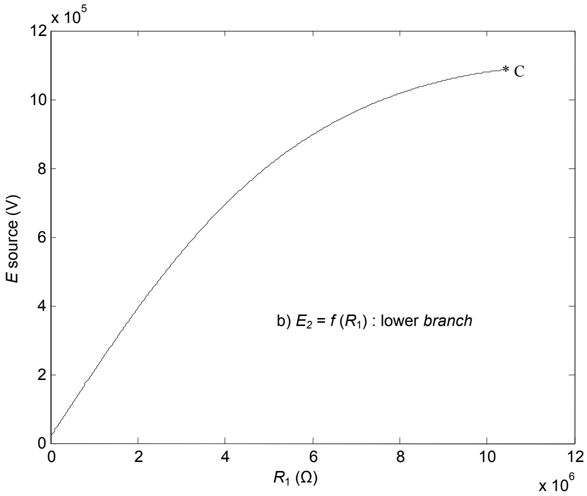


Fig.5 : Bifurcation Lines – Galerkin method.

When the determinant of the Jacobien J_1 becomes null (corresponding to the limiting value of the parameter R_1), the Galerkin method does not converge. Bifurcation point "C" corresponding to this value is called " Cusp ".

It is possible to overcome the difficulties due to the non-inversibility of J_1 and, consequently, wholly draw the bifurcation lines. We need simply apply Galerkin and pseudo-arclength methods [2] simultaneously. The problem can thus be solved.

2.3 Pseudo-arclength method

A continuation principle is still applied. However, instead of going from a point M to M+1 moving length ΔP along the parameter axis, we move by length S on the tangent to point M.

The principle of this method is to add an equation to system (18) so that the Jacobien of the new system becomes inversible. In this new system, ϕ_0 and E are no longer the only unknowns, since parameter P is also unknown. Additional parameter S is used as a continuation parameter.

The principle used to determine the additional equation is similar to that used in [2]. We need simply express the tangent to the curve in (ϕ_0, E_0, P_0) which is :

$$\left.\frac{\partial \xi}{\partial \phi}\right|_{(\phi_0,E_0,P_0)} \cdot U + \left.\frac{\partial \xi}{\partial E}\right|_{(\phi_0,E_0,P_0)} \cdot V + \left.\frac{\partial \xi}{\partial P}\right|_{(\phi_0,E_0,P_0)} \cdot W = 0 \quad (23)$$

where (U, V, W) is a tangent vector.

The additional equation is given by :

$$U \cdot (\phi - \phi_0) + V \cdot (E - E_0) + W \cdot (P - P_0) - S = 0 \quad (24)$$

The new system to be solved is composed of equations (18) and (24), i.e.,

$$\begin{aligned} &\omega\phi_{1s} + R_1 I_{1c} + X_1 I_{1s} - E_{1c} \equiv \xi_{1c} = 0 \\ &\omega\phi_{1c} + X_1 I_{1c} + R_1 I_{1s} + E_{1s} \equiv \xi_{1s} = 0 \\ &\det\left[\frac{\partial \xi}{\partial \phi_1}(\phi, E, P)\right] \equiv \xi_d = 0 \\ &U \cdot (\phi - \phi_0) + V \cdot (E - E_0) + W \cdot (P - P_0) - S \equiv \xi_p = 0 \end{aligned} \quad (25)$$

for which the Newton-Raphson method will be used, thus requiring computation of the Jacobien J_2 of system (25), i.e.,

$$J_2 = \begin{bmatrix} \begin{bmatrix} & J_1 & \end{bmatrix} & \begin{matrix} \dfrac{\partial \xi}{\partial P} \\ \dfrac{\partial \xi_d}{\partial P} \end{matrix} \\ \dfrac{\partial \xi_p}{\partial \phi} \quad \dfrac{\partial \xi_p}{\partial E} & \dfrac{\partial \xi_p}{\partial P} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} & J_1 & \end{bmatrix} & \begin{matrix} \dfrac{\partial \xi_{1c}}{\partial P} \\ \dfrac{\partial \xi_{1s}}{\partial P} \\ \dfrac{\partial \xi_d}{\partial P} \end{matrix} \\ \dfrac{\partial \xi_p}{\partial \phi_{1c}} \quad \dfrac{\partial \xi_p}{\partial \phi_{1s}} \quad \dfrac{\partial \xi_p}{\partial E} & \dfrac{\partial \xi_p}{\partial P} \end{bmatrix} \quad (26)$$

J_1 is already determined. The last line of J_2 represents the components of the tangent vector, i.e. (U, V, W). Finally, computation of the elements of the last column of J_2 does not involve any difficulty.

$$\frac{\partial \xi}{\partial P} = \frac{\partial \xi}{\partial Z} \cdot \frac{\partial Z}{\partial P} \quad (27)$$

Matrix J_2 is then fully determined.

Applied to the study circuit of figure 1, the proposed method gives good results. Indeed, the bifurcation lines obtained (figure 6) perfectly coincide with those of the analytical method.

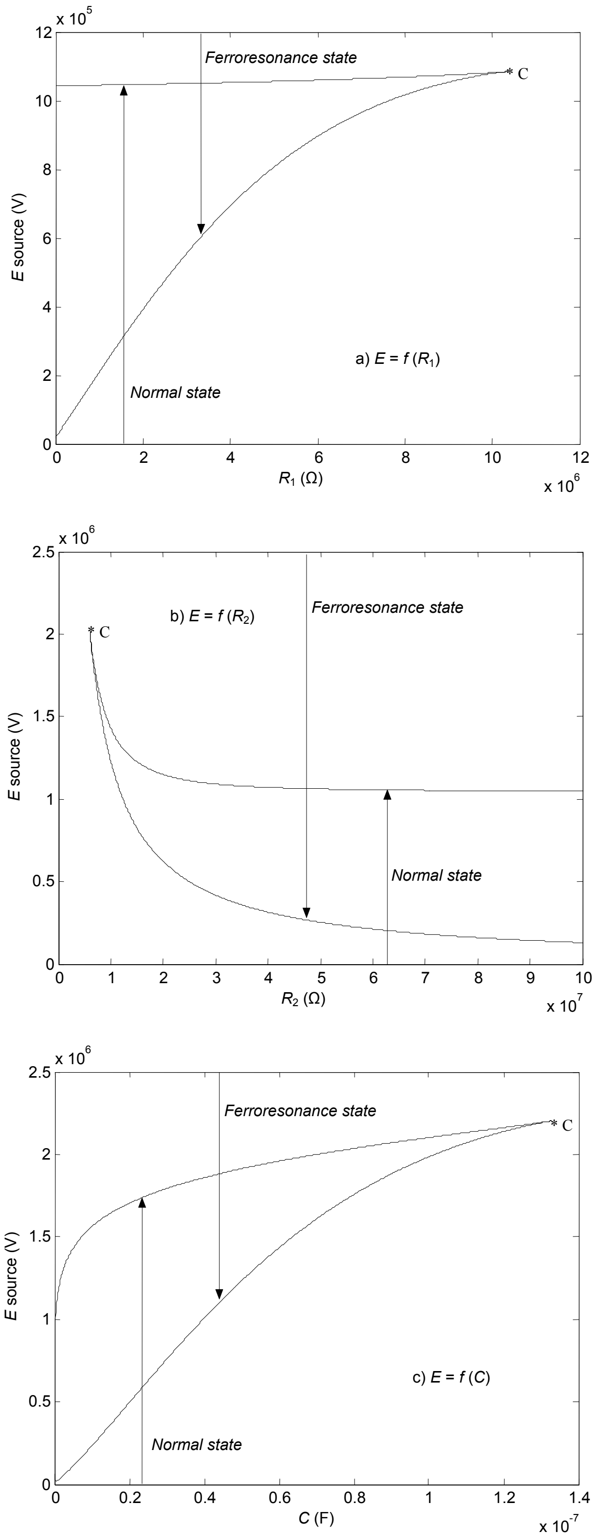


Fig.6 : Bifurcation lines – Pseudo-arclength method.

2.4 Interpretation of results

The results obtained by these various methods (analytical, Galerkin and pseudo-arclength continuation) perfectly coincide with each other.

With the pseudo-arclength method, we have a tool well suited to the study of ferroresonance in the electrical networks, in particular, the plotting of bifurcation lines. These lines make it possible to obtain a more global view of the system's behavior. They correspond to state stability limits. They also provide the existence of various zones in the parameters' plan where diverse states can occur. In the case of fundamental ferroresonance, these lines actually show the zones corresponding to a normal state and a ferroresonnant one. We can observe (figure 6) that, beyond certain values of the applied voltage, the state shows fundamental ferroresonance and that, below certain values, the state is normal. There is also an intermediate zone where the state is either normal or fundamental ferroresonance. The occurrence of one or the other depends on the initial conditions.

Taking into account the great sensitivity of the phenomenon of the circuit parameters, it is interesting to release a third parameter to see how bifurcation lines evolve in a plan, so as to anticipate ferroresonance risks with a wider safety margin. The result of this parametric study is shown by figures 7.

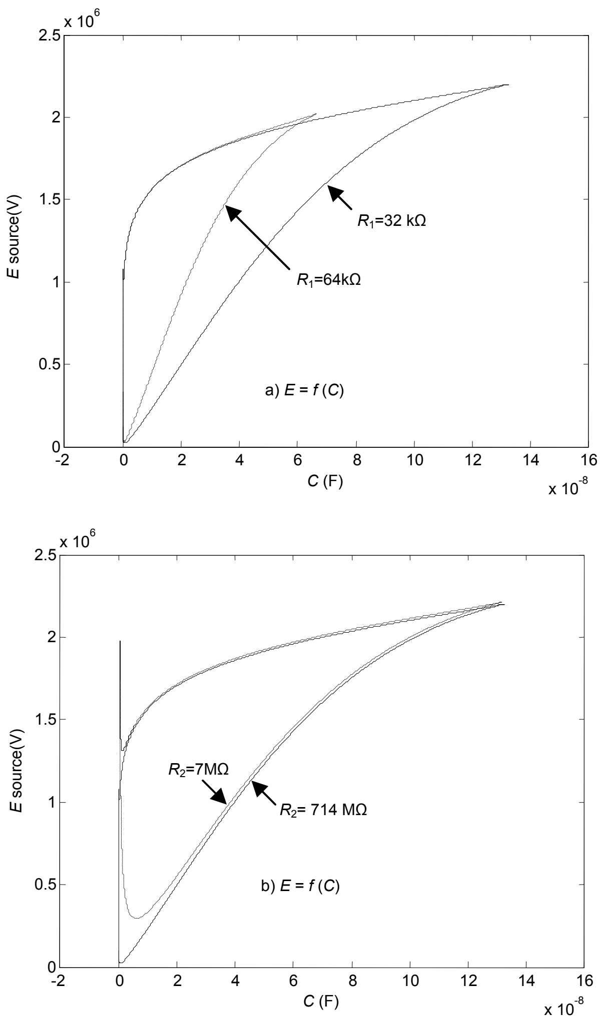


Fig.7 : Evolution of the bifurcation lines in relation to circuit parameters.

We notice that the bifurcation lines in plan (E, C) are isolates and have no intersection with banal solutions.

Figure 7a shows that as the lower threshold values of the voltage for the occurrence of fundamental ferroresonance get smaller the circuit's series losses get weaker. We also observe that, for a given series resistance, the ferroresonance phenomenon disappears altogether when the circuit capacitance exceeds a certain value (C>66.3 nF for R_1=64 kΩ and C>132.6 nF for R_1=32 kΩ).

Iron losses of the nonlinear element have little influence on the existence limits of ferroresonance. Figure 7b shows that, for a given capacitance value, the lower voltage thresholds are driven up to values which become higher as R_2 is smaller (i.e. for larger losses).

3 Conclusion

To study a ferroresonant circuit, simple temporal simulation is not enough to understand the general behavior of the circuit. The phenomena of jumps, the multiplicity of solutions for a given set of parameters, the sensitivity to initial conditions, etc. make it difficult to apply such a method or , at least, lead to excessive computation.

The mathematical framework which must be applied to understand ferroresonance is the bifurcation theory.

The answers to concrete problems faced by the system operator can be obtained with diagrams and, above all, with bifurcation lines. The numerical methods described (Galerkin's method and the pseudo-arclength continuation method) here permit efficient construction of these curves. For the determination of the Jacobien of this problem, a numerical relation is developed allowing the calculation of derived from a determinant of a matrix.

Using these lines, it is possible to learn the values of parameters which guarantee the non-occurrence of dangerous phenomena. A safety margin is chosen in relation to these values to operate the network with total safety.

The application of the methods presented in this paper covers fundamental ferroresonance with satisfactory results. The extension of this study to more complex cases of ferroresonance (subharmonic, harmonic ferroresonance) is currently being studied.

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