A New Methodology for Dynamic Performance Simulation of a New Linear Switched Reluctance Motor based on Geometrical Dimensions

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Abstract: - The purpose of this paper is the simulation of the dynamic behavior of a Linear Switched Reluctance Machine (LSRM) taking into account the magnetic circuit saturation. The proposed methodology is based on the knowledge of the magnetization curves for one machine phase. This paper gives three possible ways to obtain the magnetization curves, and uses the fast one of them applied to a LSRM with a new topology. The other phase curves will be obtained considering symmetry and electromagnetic independency between phases.

Key-Words: - Linear Switched Reluctance Machines, performance simulation.

LIST OF SYMBOLS

b_p	Primary tooth length
b_s	Secondary tooth length
D_{cu}	Winding wire diameter
f_J	Phase J developed force
F_R	Rated force
F_{pu}	Per unit average force considering F_R as base value
f_{RES}	Load resistant force
g	Airgap length
h_b	Coil length
h_p	Primary slot depth
h_s	Secondary tooth depth
$i_J(x_J,\psi_J)$	Phase instantaneous current
I_{max}	Maximum rms value of the coil current
I_{pu}	Per unit rms current considering I_{max} value for base value
J	Numerical index of one primary phase
l_b	Coil width
m	Primary phase number
N	Number of turns per primary phase
N_2	Secondary pole number
N_{br}	Number of coils per winding parallel path
N_e	Number of turns per primary coil
N_r	Number of parallel paths of each primary phase
R	Coil resistance value
U_0	DC input voltage of power electronic converter
u_J	Applied voltage to the phase J
v	Machine linear speed
v_R	Continuous rating linear speed
w	Primary and secondary stack iron width
$W_m(x_J,\psi_J)$	Stored magnetic energy
x	Machine position considering the origin on one unaligned position of phase 1
x_J	Phase relative position
τ_P	Primary pole pitch
τ_S	Secondary pole pitch
Ψ_J	Flux linkage of a phase J
η	Expected efficiency
B	Magnetic flux density
H	Magnetic field intensity
t	Time
μ_0	Free space magnetic permeability
M	Mass of the movable part
Int	Function that returns the integer part of a decimal number

1 Introduction

Switched reluctance machine dynamic simulation is now, as in the past, a concerning subject for its control development. Many of the known techniques are greatly simplified by assuming unsaturated machines, on the other hand accurate methodologies based on finite element analysis (FEA) requiring the use of expensive software and demands for large computation times.

It is the purpose of this work to present a new methodology for the LSRM performance evaluation. This methodology is based on the application of the 4^{th} order Runge-Kutta method to the (m+2) differential equation system.

2 Machine structure

Fig. 1 shows the topology of the machine used to exemplify the presented simulation methodology, and Table I present its dimensions. As can be seen, the LSRM presents a cheap secondary and modular primary construction. On the other hand, this topology increases significantly the electric, magnetic, thermal and physical independency between phases. Thus, mutual inductances between phases may be neglected without loss of accuracy.

3 Simulation methodology

For machine simulation one considers that all phase coils are series-connected, and the DC input voltage for power electronic energy converter is given by the following equation:

$$U = N_r\, U_0 \tag{1}$$

Independently of the chosen inverter topology, by application of the 2nd Kirchhoff's law one obtains the following time dependent equation for an excited phase circuit:

$$u_J(t) = R_J\, i_J(t) + \frac{\partial \psi_J(t)}{\partial t} \tag{2}$$

where J is a natural number between 1 and m, and represents the phase index, and R_J is the equivalent phase resistance given by:

$$R_J = N_r\, N_{br}\, R \tag{3}$$

Table 1
Summary of the LSRM dimensions.

F_R[N]	100	g [mm]	3	h_s [mm]	42
v_R [m/s]	0,5	b_p [mm]	18	N_e	28319
M	4	b_s[mm]	18	l_b [mm]	45
U_0 [V]	200	τ_P [mm]	132	h_b [mm]	50
N_r	2	τ_S [mm]	48	D_{cu} [mm]	0,2
N_{br}	1	w [mm]	33	I_{max} [A]	0,11
N_2	6	h_p [mm]	50	R [Ω]	68.57

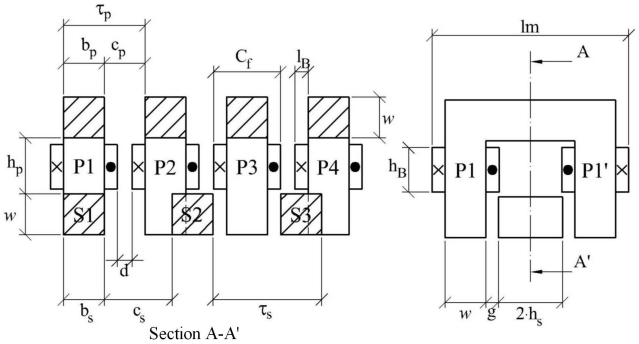

Fig.1 4-phase 8/6 Linear Switched Reluctance Machine geometry and envelope dimensions.

Alternatively, (2) can be rewritten to give the rate of change of flux linkage by phase J at instant t:

$$\frac{\partial \psi_J(t)}{\partial t} = u_J(t) - R_J\, i_J(t) \tag{4}$$

Equation (4) represents m of the (m+2) differential equations of the system. The remaining differential equations are related with the mechanical balance Thus, by Newton's law:

$$\frac{\partial v(t)}{\partial t} = \frac{\left(\sum_{J=1}^{m} f_J(x, \psi_J)\right) - f_{RES}(t)}{M} \tag{5}$$

$$\frac{\partial x(t)}{\partial t} = v(t) \tag{6}$$

where f_J is the developed force by phase J and $f_{res}(t)$ the resistant force or load force (as a result of all other forces, to be considered positive if they have the opposite direction of the speed).

Thus, knowing position, speed, and each phase flux linkage value in a time instant t_i, and applying the Runge-Kutta method to (4), (5) and (6) differential equations, considering a time step of Δt, one obtains respectively:

$$\psi_J(t_i + \Delta t) = \psi_J(t_i) \frac{K_{1\psi J} + 2K_{2\psi J} + 2K_{3\psi J} + K_{4\psi J}}{6} \tag{7}$$

$$v(t_i + \Delta t) = v(t_i) \frac{K_{1v} + 2K_{2v} + 2K_{3v} + K_{4v}}{6} \tag{8}$$

$$x(t_i + \Delta t) = x(t_i) \frac{K_{1x} + 2K_{2x} + 2K_{3x} + K_{4x}}{6} \tag{9}$$

where the K factors are obtained as follows:

$$K_{1\psi J} = \left[u_J(t_i) - R_J\, i_J(x(t_i), \psi_J(t_i))\right] \Delta t \tag{10}$$

$$K_{1v} = \frac{\left(\sum_{J=1}^{m} f_J(x(t_i), \psi_J(t_i))\right) - f_{RES}(t_i)}{M} \Delta t \tag{11}$$

$$K_{1x} = v(t_i)\, \Delta t \tag{12}$$

$$K_{2\psi J} = \left[- R_J\, i_J\left[\left(x(t_i) + \frac{K_{1x}}{2}\right), \left(\psi_J(t_i) + \frac{K_{1\psi J}}{2}\right)\right] + u_J\left(t_i + \frac{\Delta t}{2}\right)\right] \Delta t \tag{13}$$

$$K_{2v} = \frac{\left(\sum_{J=1}^{m} f_J\left[\left(x(t_i) + \frac{K_{1x}}{2}\right), \left(\psi_J(t_i) + \frac{K_{1\psi J}}{2}\right)\right]\right)}{M} - \frac{f_{RES}\left(t_i + \frac{\Delta t}{2}\right)}{M} \Delta t \tag{14}$$

$$K_{2x} = \left[v(t_i) + \frac{K_{1v}}{2}\right] \Delta t \tag{15}$$

$$K_{3\psi J}=\left[-R_J\, i_J\left[\left(x(t_i)+\frac{K_{2x}}{2}\right),\left(\psi_J(t_i)+\frac{K_{2\psi J}}{2}\right)\right]+u_J\left(t_i+\frac{\Delta t}{2}\right)\right]\Delta t \quad (16)$$

$$K_{3v}=\frac{\left(\sum_{J=1}^{m} f_J\left[\left(x(t_i)+\frac{K_{2x}}{2}\right),\left(\psi_J(t_i)+\frac{K_{2\psi J}}{2}\right)\right]\right)}{M}-\frac{f_{RES}\left(t_i+\frac{\Delta t}{2}\right)}{M}\Delta t \quad (17)$$

$$K_{3x}=\left[v(t_i)+\frac{K_{2v}}{2}\right]\Delta t \quad (18)$$

$$K_{4\psi J}=\left[-R_J\, i_J\left[\left(x(t_i)+K_{3x}\right),\left(\psi_J(t_i)+K_{3\psi J}\right)\right]+u_J(t_i+\Delta t)\right]\Delta t \quad (19)$$

$$K_{4v}=\frac{\left(\sum_{J=1}^{m} f_J\left[\left(x(t_i)+K_{3x}\right),\left(\psi_J(t_i)+K_{3\psi J}\right)\right]\right)}{M}-\frac{f_{RES}(t_i+\Delta t)}{M}\Delta t \quad (20)$$

$$K_{4x}=\left[v(t_i)+K_{3v}\right]\Delta t \quad (21)$$

As can be seen, several values for current $I(x,\psi)$ and force $f(x,\psi)$ are necessary and are obtained by using the magnetization curves lookup table.

4 Magnetization curves

As explained, this simulation method requires the use of a lookup table representative of the magnetization curves.

For exact determination of the machine magnetization curves from dimensions there are two possible ways:

- from finite element analysis
- from static test measurements [1]

In this work the used methodology to obtain the magnetization curves is presented in [2,3] and is made through the linearization of the airgap length mean value between the unaligned and aligned positions.

According to [2] and considering all series--connected phase coils, the following equations represent a simplest and fast method to obtain ψ from *B-H* characteristic represented in Fig. 2:

$$\psi = N\, B\, w\, b_p, \quad (22)$$

where N is the number of turns per phase, given by:

$$N = N_r\, N_{br}\, N_e \quad (23)$$

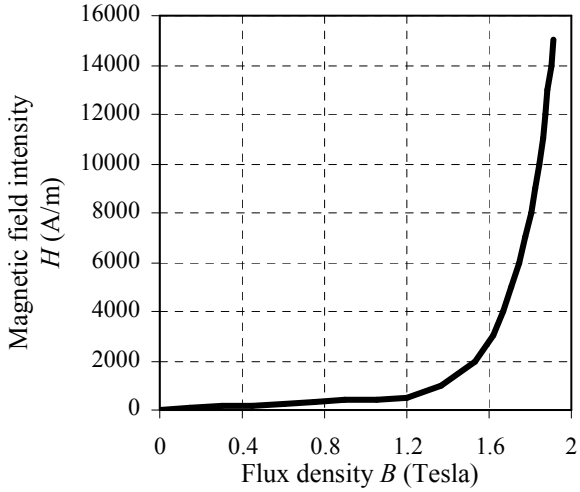

Fig.2 Magnetization curves for the magnetic circuit material.

Then, the input coil current for each relative position is obtained by using the following equation:

$$I(x,\psi)=\frac{H\, l_f(x)+\dfrac{\psi\, l_g(x)}{N\, w\, b_p\, \mu_0}}{N} \quad (24)$$

where l_g and l_f are respectively the airgap and iron average lengths of flux path, for each position.

Assuming x=0 for the unaligned position and for $0<x<0.5\tau_S$, l_g and l_f are given respectively by the following equations:

$$l_g(x)=2(g+h_s)-\frac{4h_s\, x}{\tau_s} \quad (25)$$

$$l_f(x)=2\left(h_p+2(g+h_s+w)\right)-l_g(x) \quad (26)$$

Fig. 3 illustrates the airgap average length of flux path versus relative position characteristic. Note that both $x = 0$ and $x = \tau_S$ are unaligned positions, whereas $x = 0.5\tau_S$ is the aligned position.

Fig. 4 shows the magnetization characteristics of the machine phases. Note that for each position it is necessary to calculate the relative position of each phase considering the following relationship:

$$x_J(t_i)=\left(x(t_i)-\frac{J-1}{m}\tau_S\right)-\tau_S\cdot Int\left\{\frac{x(t_i)}{\tau_S}-\frac{J-1}{m}\right\} \quad (27)$$

where x_J is the relative position of the phase J and *Int* function returns the integer part of a decimal number.

As can be seen, the lookup table values of the magnetization characteristics are stored in the form of necessary current in a phase to achieve a specific value of linkage flux in a given relative phase position.

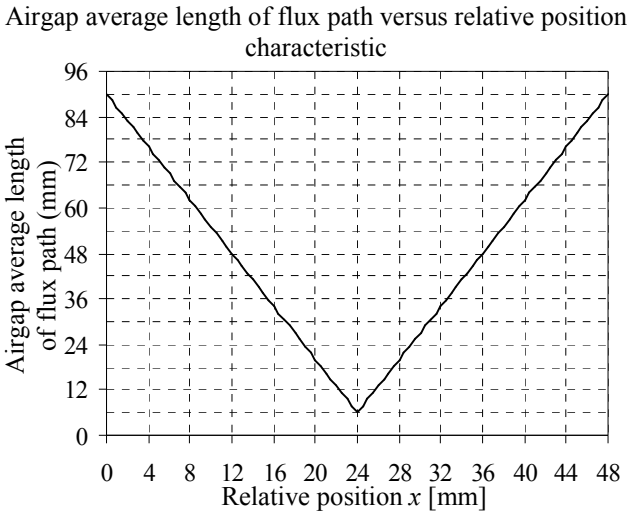

Fig.3 Airgap average length versus relative position.

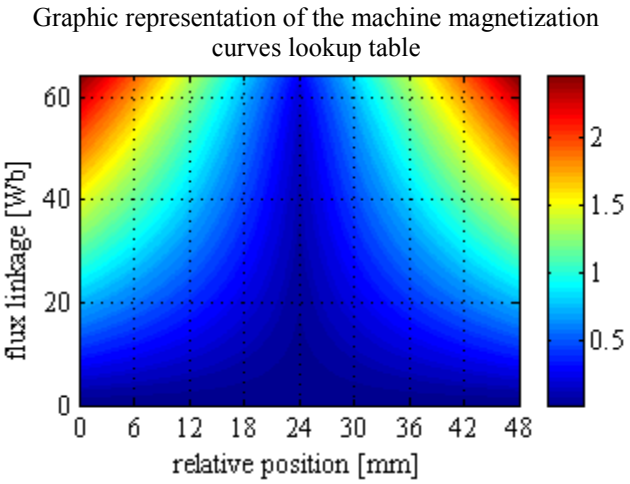

Fig.4 Phase current [A] versus relative position and flux linkage (magnetization curves).

5 Lookup table based calculations

There are three machine quantities stored in and extracted from lookup tables, phase current, stored magnetic energy and developed force. Thus, the necessary values obtained directly from a lookup table, are obtained from numerical interpolation using Cubic Spline Interpolation.

As well know, the computation time required for Cubic Spline Interpolation is directly related with the dimension of the data table. Thus, two different dimensions of the lookup tables are considered to analyze their dimension influence in the results.

5.1 Force calculation

The force calculation requires a more careful approach. Thus, taking into account that the exposed method uses both the flux linkage and relative position as state variables, the developed force is derived from D'Lambert principle as follows [4]:

$$f_J(x_J,\psi_J) = -\left.\frac{\partial W_m(x,\psi)}{\partial x}\right|_{\psi=\psi_J} \quad (28)$$

$$f_J(x_J,\psi_J) = \frac{W_m[(x_J-\Delta x),\psi_J] - W_m[(x_J+\Delta x),\psi_J]}{2\cdot\Delta x} \quad (29)$$

where Δx should tend to zero and must be lower than x_J, and W_m is the stored magnetic energy for one phase, and is represented by the following relationship:

$$W_m(x,\psi_J) = \left.\int_{\psi=0}^{\psi=\psi_J} i(x,\psi)\partial\psi\right|_{x=Const.} \quad (30)$$

At this point two possible ways can be adopted as follows:

Way No.1) To make a lookup table of stored magnetic energy, whose values are graphically presented in Fig. 5, and to use (29) during the simulation process. Concerning Fig. 5 one obtains the stored magnetic energy as a function of both relative position and flux linkage.

Way No.2) To use the stored magnetic energy lookup table to construct a lookup table of forces, like the one whose values are represented by Fig. 6, to be applied during the simulation process. In this lookup table, one obtains the developed force as a function of both relative position and flux linkage.

At first time the second way is more suitable to be used, however the force function presents a strong discontinuity for x_J equal to 0 and $0.5\tau_S$, where the force is equal to zero. That force discontinuity is directly related with the discontinuity of the derivative of l_g in order to x. Thus, some errors may be introduced by using the force lookup table around those positions.

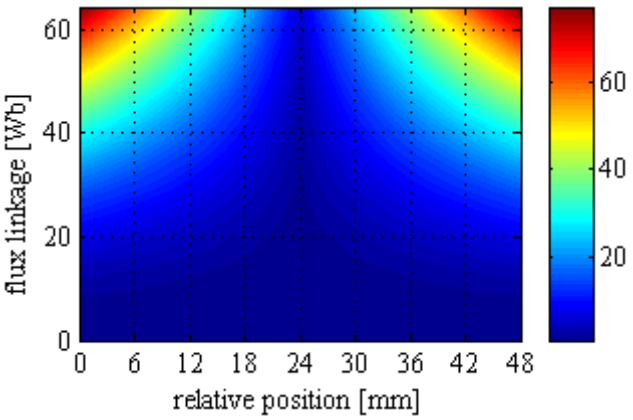

Fig.5 Stored magnetic energy [Joule] versus relative position and flux linkage, for one machine phase.

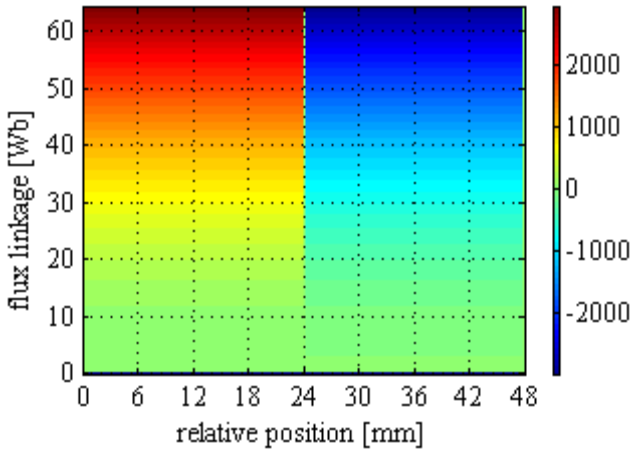

Fig.6 Phase developed force [N] versus relative position and flux linkage.

Note that for $0<x_J<0.5\tau_S$, due to the linearization of the airgap length mean value, the force value is only dependent from the linkage flux, remaining constant for all relative positions if the linkage flux remains constant. Note that constant flux in different positions implies different current values.

6 Results

In this work a 100 kg machine start up situation was considered. The simulation also considers a constant resistive force of 100N and the following control strategy:

- If $0<x_J<0.4\tau_S$ and $i_J\leq 0.24$A then $u_J=U$.
- If $0<x_J<0.4\tau_S$ and $i_J\geq 0.25$A then $u_J=-U$.
- If $0<x_J<0.4\tau_S$ and $0.24<i_J<0.25$A then u_J remain constant.
- If $x_J\geq 0.4\tau_S$ and $i_J>0$ then $u_J=-U$
- If $x_J\geq 0.4\tau_S$ and $i_J\leq 0$ then phase J is open
- If $x_J\leq 0$ then phase J is open.

As well known the maximum error of the Runge-Kutta method is equal to $(\Delta t)^4$, but in this case some errors can be introduced in the interpolation processes. Thus, the lookup table dimension, in terms of expressed values for each (x,ψ) pair, have a significant influence in the simulation accuracy.

In this work one set of lookup tables with 15 ψ values and 500 x values, and one set of lookup tables with 15 ψ values and 2000 x values was considered.

In Fig. 7 one can observe the influence of the lookup tables dimensions and force calculation methodology respectively on speed and developed force estimation.

As can be seen, there are no significant differences between the characteristics. A more careful and zoomed observation detects that:

- Both characteristics A) and C) have equal values.
- All characteristics are equal while the machine is stopped, because the only supplied phase is far away from both aligned and unaligned positions.
- Characteristic D) presents a smaller difference to A) and C) than B).

Considering this facts, and taking into account that A) and B) have a significantly lower computation time one will use configuration A) to observe the driver voltages and currents, as shown in Fig. 8.

7 Conclusions and future work

In authors´ opinion, it can be concluded that the proposed and developed procedure, concerning the LSRM simulation, is a good method to be applied in the development way of the control algorithm.

As future work, the authors intend to:

- Search a more accurate function for the airgap average length characteristic.
- Extend this methodology to other LSRM topologies.

Acknowledgment

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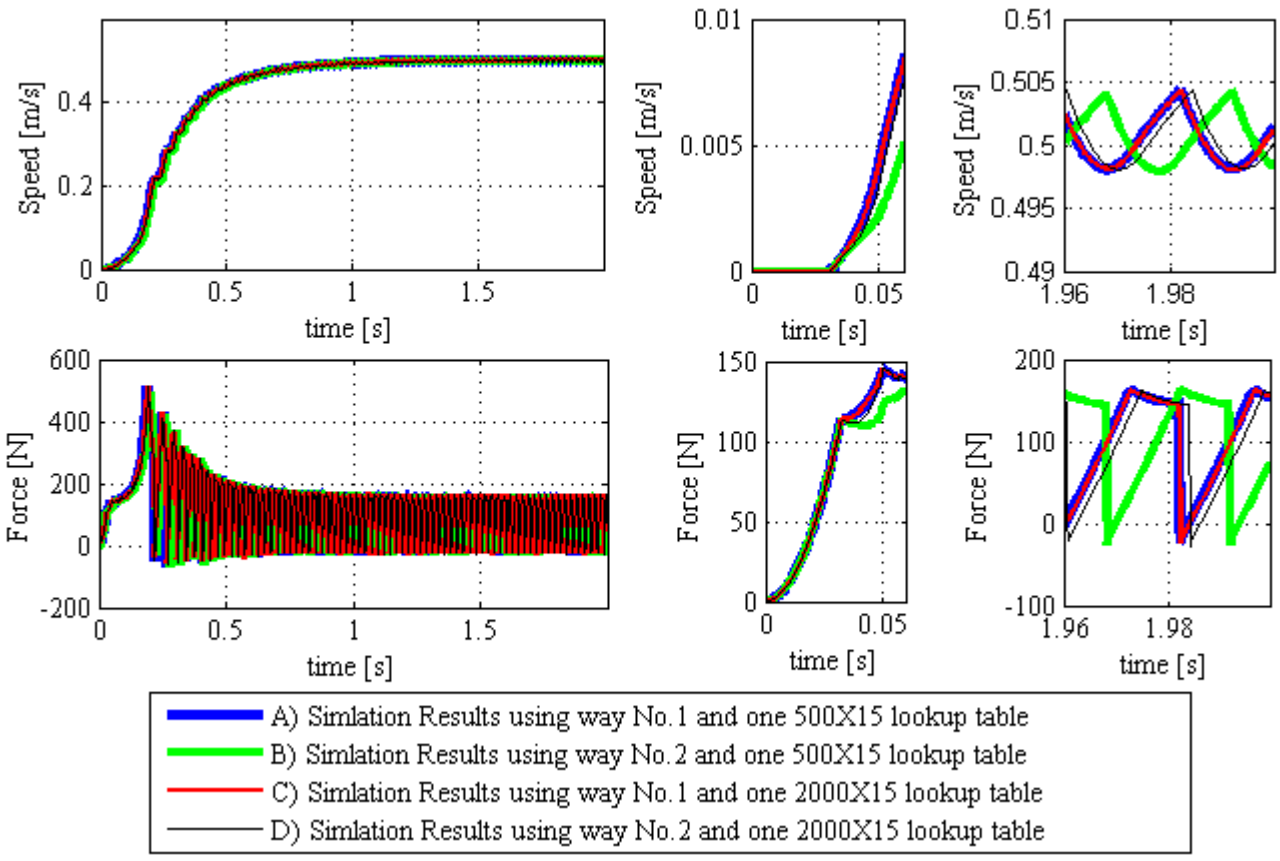

Fig.7 Machine expected output performance.

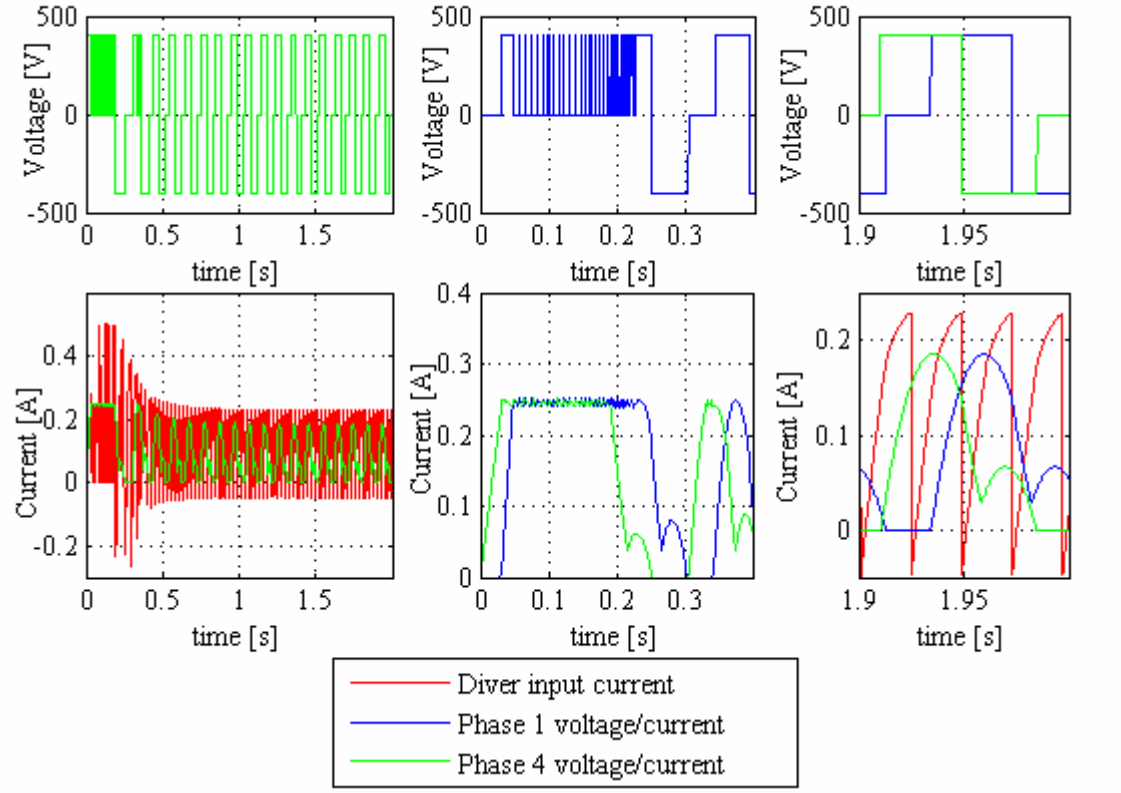

Fig.8 Machine expected input characteristics using Way No.1 and the 15X500 lookup tables.