Modulation Techniques for Three-Phase Four-Leg Inverters

ARMANDO BELLINI and STEFANO BIFARETTI
Department of Electronic Engineering
University of Rome “Tor Vergata”
Via del Politecnico 1, 00133 Rome
ITALY

Abstract: - Three-phase four-leg inverters are typically employed to supply unbalanced loads avoiding the use of a transformer. Usually, the inverter control is performed by modulation techniques, such as the three-dimensional one, requiring complex mathematical algorithms. The paper suggests a different approach, based on the separation of the control of the fourth leg from that of the other phases, allowing the application to the three inverter phases of traditional SVM techniques and avoiding the employment of complex procedures.

Key-Words: - Converters, Modulation, Power Generation, Uninterruptible Power Supplies.

1 Introduction

Many power electronics applications, such as Distributed Generation Systems, Uninterruptible Power Supplies or active filtering, employ an inverter feeding a star connected three-phase load with accessible neutral terminal. The currents flowing on each phase are generally not balanced so, if a transformer is not required, a connection to the neutral terminal should be provided by adding an extra wire to the inverter.

The load neutral terminal can be connected to the inverter using two different topologies:
- three-phase four-wire, in which the neutral point is connected directly to the midpoint of the supply by means of a capacitor divider;
- three-phase four-leg, employing an additional inverter leg that permits to modify the neutral point voltage.

The first solution, shown in Fig. 1, is certainly the simplest one, but the three-phase inverter turns into three independent single-phase inverters. As consequence, zero-sequence harmonics are generated; moreover, specially when the load is unbalanced or non-linear, a high voltage ripple over supply capacitors is produced by neutral currents. A further limitation is represented by the maximum voltage value that the amplitude of each phase fundamental harmonic can reach.

The second solution, shown in Fig. 2, requires two additional power switches and a more complex control strategy, but it offers different advantages, such as an increased maximum output voltage value, a reduction of neutral currents and the possibility of neutral point voltage control.

Different Pulse Width Modulation techniques for three-phase four-leg inverters were proposed in literature. The most popular techniques are denoted as three-dimensional Space Vector Modulation (SVM) and can be applied to two-level [1-2], or three-level inverters [3-5]. These techniques are based on the assumption that the fourth leg introduces additional switching combinations and, then, an increased number of space vectors, if compared with traditional three-leg inverters. To select, in each sampling interval, the space vector...
nearest to the desired voltage vector, generally complex mathematical algorithms, based on three dimensional geometrical figures, are employed.

In order to simplify the control of four-leg inverters, a previous paper [6] proposed a different approach based on the separation of the control of the fourth leg from that of the other phases, allowing the application to the three inverter phases of traditional SVM techniques and avoiding the employment of complex procedures. This paper continues the previous study taking into account uniform sampling modulation techniques and a different topology of the fourth leg.

To make more intelligible the waveforms of the voltages and currents, a quite low value of the modulation frequency has been selected; in particular, it has been assumed that the fundamental harmonic of the voltages applied to the load has a frequency $f_r$ equal to 50 Hz and that frequency $f_c$ of the carrier is equal to 750 Hz, i.e. ratio $k$ between the carrier and the reference frequencies is equal to 15. However, the results obtained by the different solutions were validated using a rather bigger value of ratio $k$. Besides, it has been supposed that supply voltage $V_{dc}$ is equal to 800 V and that the three phases of the load must be supplied with a RMS voltage equal to 220 V.

The waveforms and the connected harmonic contents were obtained by simulation, using the Simulink-SimPowerSystems tools, taking into account both the real behaviour of the inverter components and the dead times necessary for the commutations.

2 Modulation Techniques

As said in the introduction, the paper propose to employ, for the three inverter phases (a, b and c), a standard modulation technique. To this aim, the triangle intersection technique based PWM, employing the zero sequence injection principle, described in [7] is taken into consideration. As shown in the Simulink block diagram of Fig. 3, this technique determines modulated signals $v_{ma}$, $v_{mb}$ and $v_{mc}$, that produce the inverter phases commutations, by comparing a symmetric triangular carrier $v_c$ with three reference voltages, $v_{ra}$, $v_{rb}$ and $v_{rc}$, obtained by adding, to three sinusoidal reference voltages, a suitable zero sequence signal $v_0$. In this paper the technique denoted in [7] as SVPWM, applying the zero sequence signal depicted in Fig. 4, is used. The modulation produced by the comparison between each reference signal ($v_{ra}$, $v_{rb}$ and $v_{rc}$) and the carrier signal can be produced employing either a natural sampling of the reference signals or a uniform sampling.

![Fig. 3 - Triangle intersection technique block diagram.](image)

![Fig. 4 Waveform of voltage $v_0$.](image)

When the control of the inverter is performed by a microprocessor based board, the sampling is certainly of uniform type. Therefore in this paper a uniform sampling has been preferred; besides sampling period $T_s$ has been selected equal to an half period of the carrier: $T_s=1/2f_c$, so that the sampling instants coincide with those in which the carrier assumes the maximum and minimum values.

Taking into consideration an interval characterized by a negative slope of the carrier, the comparisons of the reference signals with the carrier subdivide the sampling interval in four subintervals, whose lengths are denoted as $t_0$, $t_1$, $t_2$ and $t_3$, as shown in Fig 5.

![Fig. 5 - Subdivision of the sampling interval.](image)

Employing the zero sequence signal depicted in Fig. 4, length $t_0$ is equal to $t_1$; so in each sampling interval only lengths $t_1$ and $t_2$ and the sequence of
the commutations must be determined. Similar shapes occur in the sampling intervals in which the carrier has a positive slope.

In three-leg inverters with balanced load, voltage $v_n$, applied to the load neutral point and referred to the midpoint of the supply voltage, is, at every instant, equal to the mean value of the three output voltages $v_a$, $v_b$, and $v_c$. Therefore, as shown in Fig 6, its waveform depends only on lengths $t_1$ and $t_2$. It is possible to observe that the mean value of voltage $v_n$ during the sampling interval is equal to $V_{dc} \frac{t_2 - t_1}{6T}$, and it is proportional to the value assumed by zero sequence signal $v_0$ sampled at the beginning of the sampling period.

The waveform of voltage $v_n$, illustrated in Fig. 7, is characterized by a period equal to the third part of that of the output voltages; so only harmonics at frequencies multiple of $3f_r$ are present.

When the load is balanced, the waveform of each phase voltage has a very good harmonic content because all the harmonics of order multiple of three, which appear in the phase output terminal voltages, appear also in the load neutral point voltage and practically disappear in phase voltages; so the waveforms of the load neutral point voltage, illustrated in Figs. 6 and 7, can be considered as the ideal neutral point voltage shapes. Table 1 shows the amplitudes of the first harmonics of a phase voltage, referred to the fundamental one. Taking into account all the harmonics up to the frequency of 1 kHz, the Total Harmonic Distortion (THD%) is equal to 30.10.

When the load is unbalanced, the sum of the instantaneous values of the three phase currents assumes values different from zero; thus, it is necessary to connect the load neutral point to the inverter.

![Fig. 6 - Shape of voltage $v_n$ in a sampling period.](image)

![Fig. 7 - Waveform of the load neutral point voltage.](image)

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Amplitude %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>1.59</td>
</tr>
<tr>
<td>7</td>
<td>4.27</td>
</tr>
<tr>
<td>9</td>
<td>0.12</td>
</tr>
<tr>
<td>11</td>
<td>11.55</td>
</tr>
<tr>
<td>13</td>
<td>19.58</td>
</tr>
<tr>
<td>15</td>
<td>0.06</td>
</tr>
<tr>
<td>17</td>
<td>14.36</td>
</tr>
<tr>
<td>19</td>
<td>7.35</td>
</tr>
</tbody>
</table>

As shown by Table 3, this technique reduces the amplitudes of the first harmonics (specially that of the third harmonic) but those of the thirteenth,
fifteenth and seventeenth and the value of THD% (equal to 114.31%) slightly increase; besides, the amplitude of the fundamental harmonic of each phase voltage cannot become greater than $V_{dc} / 2$, while the SVPWM allows a maximum value equal to $V_{dc} / \sqrt{3}$.

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Amplitude %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.38</td>
</tr>
<tr>
<td>5</td>
<td>0.11</td>
</tr>
<tr>
<td>7</td>
<td>0.09</td>
</tr>
<tr>
<td>9</td>
<td>0.10</td>
</tr>
<tr>
<td>11</td>
<td>0.39</td>
</tr>
<tr>
<td>13</td>
<td>24.09</td>
</tr>
<tr>
<td>15</td>
<td>107.79</td>
</tr>
<tr>
<td>17</td>
<td>29.42</td>
</tr>
<tr>
<td>19</td>
<td>1.72</td>
</tr>
</tbody>
</table>

Table 3 - Harmonics of a SPWM phase voltage.

To increase the maximum voltage value, it is necessary to employ the four-leg inverter structure shown in Fig. 2, which requires two additional power switches ($S_{n1}$ and $S_{n2}$); by this way output voltages $v_a$, $v_b$ and $v_c$ can be obtained by the SVPWM technique, while the shape of voltage $v_m$ applied to the load neutral point, must be selected so to reduce the phase voltage harmonics.

3 Choice of the Neutral Point Voltage

The model of the three-phase four-leg inverter and its modulation system was implemented in Matlab-Simulink environment. Fig. 8 illustrates the structure of the conversion circuit.

A first way to select the neutral point voltage waveform is to use, for the fourth leg modulation, the same carrier employed for the other legs and voltage $v_0$ as reference. The structure of the whole modulation system is depicted in Fig. 9. Fig. 10 represents the voltage $v_n$ waveform, referred to the midpoint of the supply voltage, during the sampling period examined in Fig 5. Voltage $v_n$ assumes only two values (-$V_{dc} / 2$ and +$V_{dc} / 2$) whose durations are respectively equal to:

$$t_a = t_0 + (2t_1 + t_2) / 3$$
$$t_p = t_0 + (t_1 + 2t_2) / 3 = T_s - t_a.$$ 

The obtained neutral point voltage has the same mean value as the ideal neutral point voltage illustrated in Fig. 6, but its shape is rather different; therefore, the harmonics of order multiple of three are only partially reduced.

Applying the described voltage waveform, a THD% equal to 63.12 and the harmonic content shown in Table 4 is produced by a phase voltage. Table 4 highlights a significant reduction of the voltage distortion versus the four-wire solution.

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Amplitude %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.21</td>
</tr>
<tr>
<td>5</td>
<td>0.81</td>
</tr>
<tr>
<td>7</td>
<td>0.52</td>
</tr>
<tr>
<td>9</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>8.78</td>
</tr>
<tr>
<td>13</td>
<td>14.67</td>
</tr>
<tr>
<td>15</td>
<td>56.52</td>
</tr>
<tr>
<td>17</td>
<td>17.82</td>
</tr>
<tr>
<td>19</td>
<td>13.34</td>
</tr>
</tbody>
</table>

Table 4 - Harmonics of the four-leg phase voltage.

![Fig.8. - Simulink model of the three-phase four-leg inverter circuit.](image-url)
When a natural sampling modulation is used, additional reductions of the harmonic amplitudes can be obtained by choosing, for the modulation of the fourth leg, a different reference signal and a carrier characterized by a frequency equal to $2f_c$. These modifications, described in [6], do not produce improvements when a uniform sampling is used. A different modification is suggested by the observation that the waveform of the ideal load neutral point voltage can be better approximated by a three-level waveform than by a two-level one.

### 4 NPC Leg Modulation

A NPC leg is constituted by four switches, controlled in pairs, and can provide a three-level output voltage. Therefore, in the sampling interval shown in Fig. 6, the NPC leg can supply the neutral point voltage illustrated in Fig. 11.

The mean value of the voltage supplied by the NPC leg is equal to that of the ideal neutral point voltage, if lengths $t_x$ and $t_z$ are selected so that $t_z - t_x = (t_2 - t_1)/3$.

Moreover, to minimize the RMS value of the difference between the two shapes, it is convenient to select the interval lengths as:

$$
t_x = t_0 + t_1/3
$$

$$
t_z = t_0 + t_2/3
$$

$$
t_y = T_s - t_x - t_z.
$$

With this choice, the difference between the shape of the voltage supplied by the NPC leg and the ideal one becomes that depicted in Fig. 12.

To obtain the described waveform of the output voltage, the NPC leg can be controlled by means of two modulators which compare, as shown in Fig. 13, the same carrier used for the other inverter legs with two suitable reference signals, $v_{r1}$ and $v_{r2}$ selected as:

$$
v_{r1} = 1 - 2t_x/T_s
$$

$$
v_{r2} = -1 + 2t_z/T_s.
$$

The proposed conversion structure provides a phase voltage waveform characterized by a THD% equal to 34.97 and by the harmonic content illustrated in Table 5.
Comparing Table 5 with Tables 1 and 4, it can be observed that, adopting the proposed conversion structure, the phase voltages imposed by the four-leg inverter are characterized by a fully acceptable harmonic content.

### 5 Conclusion

The paper set its focus on three-phase four-leg inverters used to supply unbalanced loads without using a transformer.

Three different converter structures were considered, analyzing their performance by using suitable models implemented in Matlab-Simulink environment:

- three-phase four-wire, in which the load neutral point is connected directly to the midpoint of the supply voltage by means of a capacitor divider;
- three-phase four-leg, employing an additional two-level inverter leg;
- three-phase four-leg, employing an additional three-level NPC inverter leg.

The first conversion structure supplies an output voltage characterized by an unacceptable harmonic content; so a three-phase four-leg inverter must be used. The second conversion structure, suitably controlled, gives rise to an acceptable harmonic content, whilst a significant improvement is obtained using the third solution, even if the conversion structure, using a NPC fourth leg, is more complex.

The proposed approach can be applied also to four-leg Neutral Point Clamped (NPC) inverters, which employ four switches on each leg. For the modulation of NPC inverters, particularly employed in high-power applications, efficient SVM techniques, investigated also by authors in several papers [8-9], should be applied.

### References:


