Contact problem in shape modelling of Multi-Bellows Air Springs

L.PRASIL, V.KRACIK, D.FRYDRYCH
Department of Design of Machine Elements and Mechanisms, Department of Applied Mathematics,
Department of Electromechanical Systems
Technical University of Liberec
Halkova 6, 461 17 LIBEREC
CZECH REPUBLIC

Abstract: - In this article is described an advanced model for solving of contact problem during calculation of geometrical characteristics of multi-bellows air springs. Geometrical characteristic (Volume, Effective area and Index of effective area) are the most significant static parameters of Bellows Air Spring (BAS). Knowledge of them is very important for designer of BAS. Using this model, designer can change the design of retainers and spring to BAS their properties, before manufacturing.

Key-Words: - bellows air spring, static characteristic, Newton's method, Runge-Kutta's method

1 Introduction

In this article is assumed that the fabric reinforced elastomer bellows are ideal (by ideal is meant, absolute rigid in tensioning and absolute elastic in bending). This assumption reduces a spatial problem to a planar problem [3]. The design of a bellow is sketched on the figure 1. Force generating axial shift (vertical moving of the upper retainer, denoted *z*), is given only by air's work. For practical purposes is used negative direction for extension and positive direction for compression.

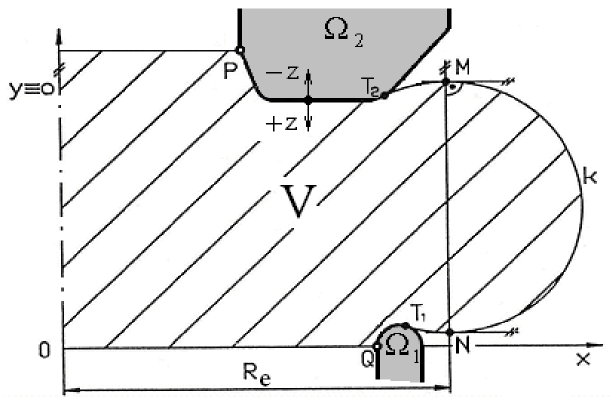

Figure 1 Design of the bellows – partial axial section; Ω_1-lower retainer; Ω_2-upper retainer; k–meridian curve of fabric-reinforced elastomer bellows; R_e–radius of the circular effective area; P, Q-terminal points of bellows; T_1, T_2-contact points; M, N-fictive points of release

The entire axial force of spring is caused by air pressure on the circle area with radius *Re*. Part of the *k* curve laying on the right side from line *M*, *N* can be taken off, as forces in these points are only in radial direction. This circle area with radius Re is known as effective area and is defined by equation

$$S(z) = \pi . R_e^2(z) \quad , \tag{1}$$

where $R_e(z)$ is radius of release points *M* or *N*.

The utilization principle of virtual work gives

$$S(z) = -\frac{dV(z)}{dz} \quad , \tag{2}$$

where *V(z)* is volume of air spring.

Last of major parameter of the BAS is index of effective area

$$U(z) = \frac{dS(z)}{dz} \quad . \tag{3}$$

1.1 Theory of multi bellows air spring

Demand of enhancement of the axial shift is solved by serial assembly of bellows for the same value of effective area (loading).

The separate bellows are isolated by detach rings. Detach rings have usually circle crosscuttoday. In big depression, neighbor bellows can get in contact between each other [2]. In the specific conditions (high temperature of elastomer) bellows can get "glued" together. When this bond is broken in next opening of the bellows, pieces of rubber can be torn out. This cause

faster wear and tear and has negative impact of their lifetime.
In new trends, designer try do develop detach rings, which inhibit the contact of neighbor bellows.

1.2 Equations of multi bellows air spring

There is a common bellows air spring assembly by n bellows see figure 2.
All bellows are part of one spring - they share inside pressure, then

$$p = p_i \qquad i = 1,2,...,n \quad , \tag{4}$$

where p is pressure inside BAS, p_i is pressure inside i^{th} bellow and n is number of BAS bellows.

For serial type of bellows then govern action/reaction law is

$$F = F_i = pS_i \qquad i = 1,2,...,n \quad , \tag{5}$$

where F is loading capacity of BAS, F_i is loading capacity of i^{th} bellow and S_i is effective area of i^{th} bellow.
From equations above follows, that every bellows in multi-BAS must have the same effective area

$$S_i = S(z) \qquad i = 1,2,...,n \quad , \tag{6}$$

Figure 2 Design of the multi-bellows air spring; H-height of whole multi-BAS; H_i-height of one bellow of multi-BAS

where $S(z)$ is effective area of multi-BAS for given shift z.
As concerns height H_i of bellows in multi-BAS similar equation applies

$$H = \sum_{i=1}^{n} H_i \quad , \tag{7}$$

where H is sum height of whole multi-BAS and H_i is height of i^{th} bellow.

2 One Bellow Problem Formulation

The principal idea of model is based on substitution of the free part of the meridian k by a part of circle [1].

2.1 Free part

This is described by three equations

$$f_1 \equiv v_1 + v_2 + R\left(\varphi_2(v_2) - \varphi_1(v_1)\right) - L = 0 \quad , \tag{8}$$

$$\mathbf{f}_{2,3} = \begin{bmatrix} f_2 \\ f_3 \end{bmatrix} \equiv \left(\mathbf{x}_1(v_1) + R\,\mathbf{n}_1(v_1)\right) - \left(\mathbf{x}_2(v_2) + R\,\mathbf{n}_2(v_2)\right) = \mathbf{0} \quad , \tag{9}$$

where v_1 is length of meridian part witch is laying on boundary domain Ω_1 between points Q and T_1, v_2 is length of meridian part, which is laying on boundary

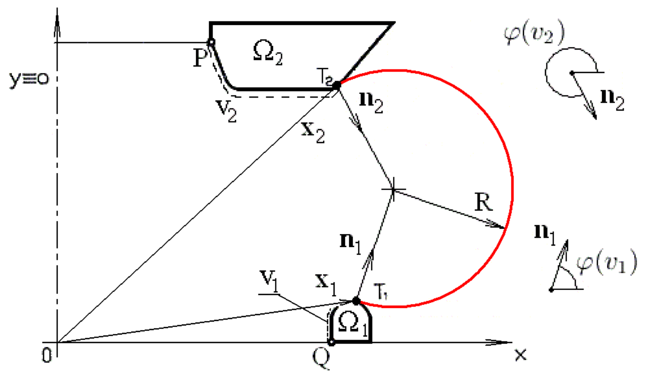

Figure 3 Model of the free part of bellow; Ω_1-lower retainer; Ω_2-upper retainer; P, Q-terminal points of bellows; T_1, T_2-contact points; $\mathbf{x}_1$, $\mathbf{x}_2$-vectors of coordinates of contact points T; $\mathbf{n}_1$, $\mathbf{n}_2$-unit vectors of exterior normal of boundary of domain Ω in contact points T; R-radius of the circular arc of bellow

domain Ω_2 between points P and T_2, R is radius of the circular arc of bellows, φ_1, φ_2 are arguments of vectors n_1, n_2 see figure 3.

The system of three equations (4) and (5) with three unknown variables v_1, v_2 and R is solved numerically by the Newton's method.

2.2 Lower limit

For this case is assumed, that inner and outer radius of the meridian circle is the same.

Set of equations (8) and (9) is extended by variable g_D see figure 4

$$f_1 \equiv v_1 + v_2 + R\,(\varphi_2(v_2) - \varphi_1(v_1)) + g_D - L = 0 \ , \quad (10)$$

$$\mathbf{f}_{2,3} = \begin{bmatrix} f_2 \\ f_3 \end{bmatrix} \equiv \left(\mathbf{x}_1(v_1) + R\,\mathbf{n}_1(v_1) + \begin{bmatrix} g_D \\ 0 \end{bmatrix} \right) - (\mathbf{x}_2(v_2) + R\,\mathbf{n}_2(v_2)) = \mathbf{0} \ . \quad (11)$$

Set of three equations contains four unknowns – v_1, v_2, R and g_D – so, this set has infinite of solutions. But, for this configuration is value R uniquely defined by variable v_1.

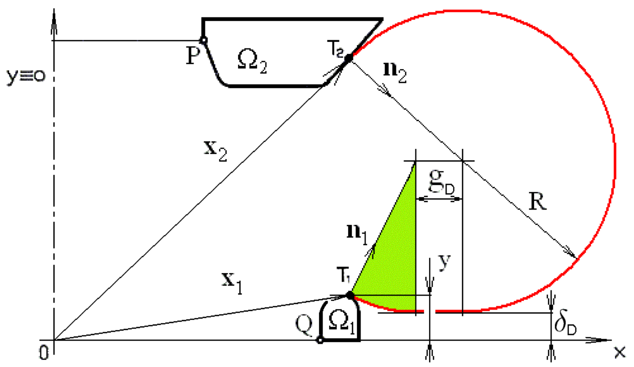

Figure 4 Model of the lower limitation of bellow; Ω_1-lower retainer; Ω_2-upper retainer; P, Q-terminal points of bellows; T_1, T_2-contact points; $\mathbf{x}_1$, $\mathbf{x}_2$-vectors of coordinates of contact points T; $\mathbf{n}_1$, $\mathbf{n}_2$-unit vectors of exterior normal of boundary of domain Ω in contact points T; R-radius of the circular arc of bellow; g_D-lower contact line

Radius R is function of variable $v_1 - R(v_1)$, see figure 4

$$R(v_1) = \frac{(y_{T_1} - \delta_D)}{1 - \sin \varphi_1(v_1)} \quad . \quad (12)$$

Equation (10) can be transformed in the form where variable g_D is function of variables v_1 and v_2

$$g_D(v_1, v_2) = L - [v_1 + v_2 + R(v_1).(\varphi_2(v_2) - \varphi_1(v_1))] \quad . \quad (13)$$

Rank of the set of equations decreases by one degree. Two unknowns' v_1 and v_2 have to be found only. For numerical solution the Newton's method is used.

2.3 Upper limit

For this case can be used the same approach as for the lower limit problem.

Set of equations (8) and (9) is extended by variable g_H see figure 5

$$f_1 \equiv v_1 + v_2 + R\,(\varphi_2(v_2) - \varphi_1(v_1)) + g_H - L = 0 \ , \quad (14)$$

$$\mathbf{f}_{2,3} = \begin{bmatrix} f_2 \\ f_3 \end{bmatrix} \equiv (\mathbf{x}_1(v_1) + R\,\mathbf{n}_1(v_1)) - \left(\mathbf{x}_2(v_2) + R\,\mathbf{n}_2(v_2) + \begin{bmatrix} g_D \\ 0 \end{bmatrix} \right) = \mathbf{0} \ . \quad (15)$$

Set of three equations contains four unknown – v_1, v_2, R and g_H – so, this set has infinite of solutions. But, for this

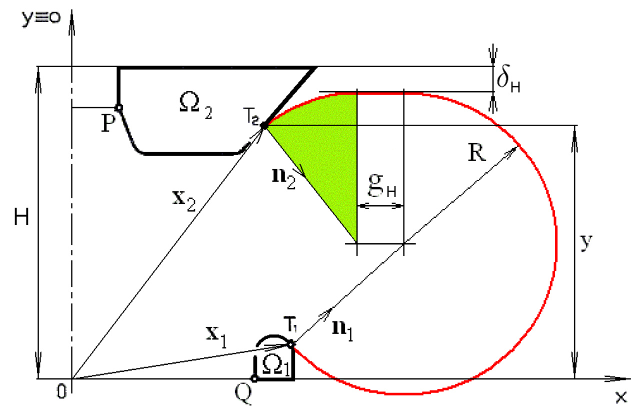

Figure 5 Model of the upper limitation of bellow; Ω_1-lower retainer; Ω_2-upper retainer; P, Q-terminal points of bellows; T_1, T_2-contact points; $\mathbf{x}_1$, $\mathbf{x}_2$-vectors of coordinates of contact points T; $\mathbf{n}_1$, $\mathbf{n}_2$-unit vectors of exterior normal of boundary of domain Ω in contact points T; R-radius of the circular arc of bellow; g_H-upper contact line

configuration is value R uniquely defined by variable v_2. Radius R is function of variable $v_2 - R(v_2)$, see figure 5

$$R(v_2) = \frac{(H - y_{T_2} - \delta_H)}{1 + \sin \varphi_2(v_2)} \quad . \quad (16)$$

Equation (14) can be transformed in the form, where variable g_H is function of variables v_1 and v_2.

$$g_H(v_1, v_2) = L - [v_1 + v_2 + R(v_2).(\varphi_2(v_2) - \varphi_1(v_1))] \quad . \quad (17)$$

Rank of the set of equations decreases by one degree. Two unknowns' v_1 and v_2 have to be found only. For numerical solution is Newton's method used.

2.4 Both sides limit

If the bellow is limited from upper side and lower side simultaneously, the solution is easier from mathematical point of view – even though the number of variables have increased, see figure 6.

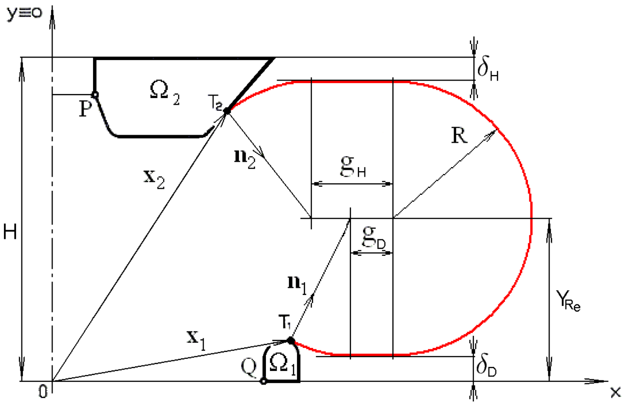

Figure 6 Model of the both sides limitation of bellow; Ω_1-lower retainer; Ω_2-upper retainer; P, Q-terminal points of bellows; T_1, T_2-contact points; $\mathbf{x}_1$, $\mathbf{x}_2$-vectors of coordinates of contact points T; $\mathbf{n}_1$, $\mathbf{n}_2$-unit vectors of exterior normal of boundary of domain Ω in contact points T; R-radius of the circular arc of bellow; g_D-lower contact line; g_H-upper contact line

Radius of bellow is uniquely defined

$$R = \frac{(H - \delta_D - \delta_H)}{2} \quad , \tag{18}$$

and center point of radius is given

$$Y_{R_e} = R + \delta_D \quad . \tag{19}$$

Knowing R and Y_{Re}, task can be split up to three independent tasks:

Find v_1 – outcome of equation

$$x_{1,y}(v_1) + Rn_{1,y}(v_1) - Y_{R_e} = 0 \quad . \tag{20}$$

Find v_2 – outcome of equation

$$x_{2,y}(v_2) + Rn_{2,y}(v_2) - Y_{R_e} = 0 \quad . \tag{21}$$

Equations (20) and (21) will be solved by generalized Newton's method.

Find g_H and g_D – two equations are needed.
First is derived from (10) and (14)

$$[v_1 + v_2 + R.(\varphi_2(v_2) - \varphi_1(v_1)) + g_D + g_H] = 0 \quad . \tag{22}$$

Second is derived from (11) and (15)

$$x_{1,x}(v_1) + Rn_{1,x}(v_1) + g_D = x_{2,x}(v_2) + Rn_{2,x}(v_2) + g_H \quad . \tag{23}$$

Equations (22) and (23) can be solved analytically and variable g_D and g_H will be received directly.

4 Multi Bellows Problem Formulation

There are several approaches to multi-BAS issues. One of them is that a model for each type of spring is built (two-bellows, three-bellows spring), obviously free part of the below is always replaced by part of circle. This approach was tested but it proved to be too complicated. Each type of spring needs a different model designed and developed from the scratch.
As much more efficient way of modelling multi-BAS was found, when worked with multi-BAS as with several one-bellow air springs connect in into a series. This approach is limited by equations (6) and (7). They are solved in iterative process.
At the beginning, heights of the bellows are chosen so the equation (7) equals. Effective area of all bellows should fit in equation (6), in convergent criterion respectively

$$|Si - S(z)| < \varepsilon \qquad i = 1,2,\ldots,n \quad . \tag{24}$$

Good efficiency (speed and stability of convergence) is reached with bisection method.

5 Conclusion

Model described in this article works very fast and results are available in seconds. Model was finalized in format of CAD system and was equipped with a user-friendly interface. Designer, who is not familiar with modelling, can use it as a black-box tool.
Designer of BAS, using the system and its graphical outputs can improve BAS designs and adjust the characteristics of newly designed spring so they fulfill exactly customer expectations.
Designer of machines where BAS is used can, using this system, change shapes of BAS retainers so they fit in these machines.
There are areas for further development. The assumption of constant bellows length is not quite exact. Bellows from fabric-reinforced elastomer change the length in dependence on deformation and on air pressure too. This

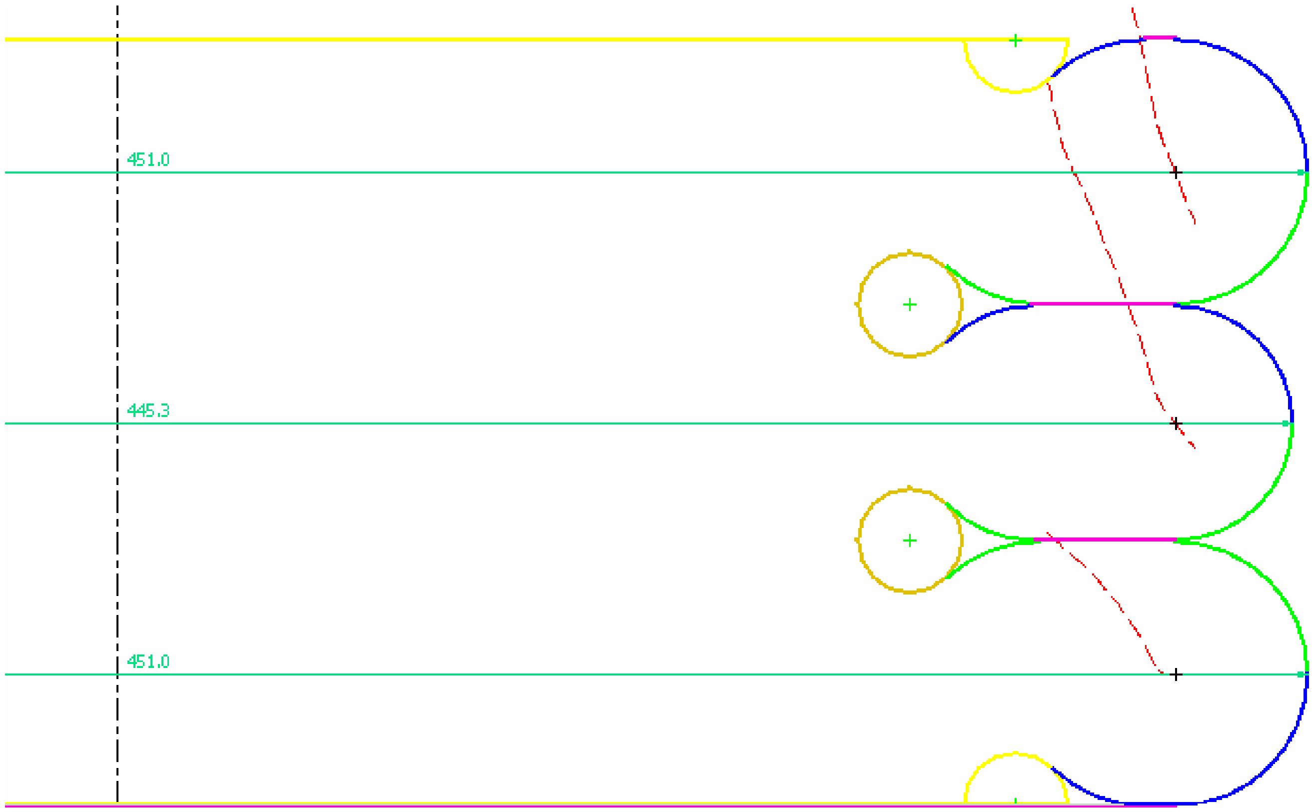

Figure 7 Consequent shape of multi bellows air spring

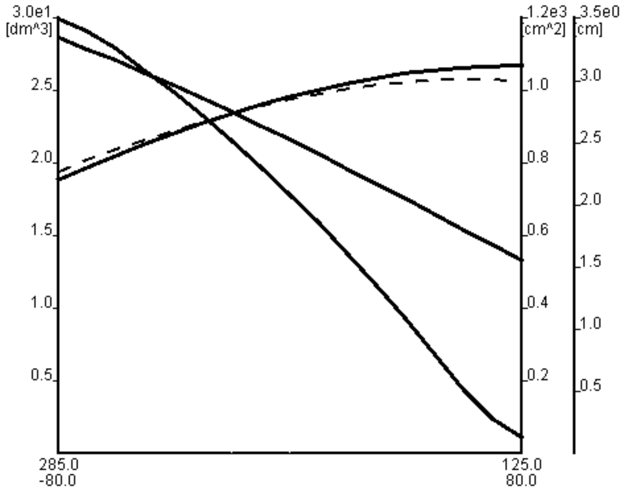

Figure 8 Graph of volume, effective area and index of effective area of bellows air spring

issue needs new formulation of problem involving the interaction in the fabric-reinforced elastomer out of which bellows are made. This model can provide quite new scope of solving tasks, for example radial loading of retainers (shift in axis x) or inclination of them.

It would be certainly very interesting to compare results of this research with a similar project abroad. Unfortunately, even after extensive search such a project was not found.

Acknowledge

This work was supported by Czech Ministry of Education under project code MSM 4674788501.

References:

[1] L.Prasil, V.Kracik, D.Frydrych: Shape Molelling of Air Bellows Springs, *Algoritmy 2005*, pp. 142–149, Podbanske, Slovak Republic, ISBN 80-227-2192-1

[2] D.Frydrych, L.Prasil, V.Kracik: Numerical models of bellows air springs for calculation of geometrical characteristics, *Hydraulika a Pneumatika 2004*, pp. 51–56, Svit, Slovak Republic, ISBN 80-968961-2-1

[3] O.Krejcir: Pneumatic vibro-isolation (czech original: Pneumaticka vibroizolace), Doctoral thesis, VSST Liberec, 1986