Using Recursive Least Squares Estimator For Modelling a Speech Signal

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Abstract: - In this paper we will present a detailed study of recursive least squares method, and then we choose an application of this algorithm to identify the parameters of the autoregressive model AR associated to the speech signal. The technique proposed here makes it possible to identify the parameters of the autoregressive moving average model ARMA with unbiased estimates, this method is based on the minimization of a quadratic criterion.

Key-Words: - Least squares, bias estimator, identification, model, simulation, speech signal.

1 Introduction

Noise estimation consists of estimating the distribution of the noise in the frequency domain. As the spectrum of the signal is obtained by using a Fourier transformation and according to the central limit theorem, we make the assumption that the noise magnitude is normally distributed in each frequency band of the spectral domain. If the noise is considered as stationary, in each frequency band its mean and variance can be considered as constant for the whole signal [4]. When the noise characteristics are varying slowly, the estimates of its parameters have to be update for each new frame of the signal.

Lynch et al. [1987] proposed an approach which defines speech and noise metrics bases on statistical assumptions about the characteristics of the speech and noise waveforms.

Savoji [1989] proposed an algorithm based on the combination of energy and zero-crossing rate. It is a modification of the algorithm proposed by Lynch et al., with the introduction of a three state model (speech, silence and transition).

In order to detect the speech parameters, we have proposed an identification approach which consists to determine the parameters of a mathematical model, when the structure is established according to a given criterion, the parameters of the model are obtained by minimization of the error prediction between the measured output signal and the signal estimated according to a criterion of optimality, we are interested more particularly to the method which is based on the whitening error of prediction.

Formally, the operation identification can be summarized by the figure below [2].

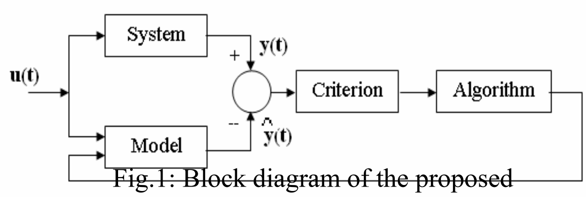

Fig.1: Block diagram of the proposed approach with identification by output error.

It is important to say that there is not single algorithm of parametric estimate for all the types of models of noise provide asymptotically unbiased parametric estimates. For each structure of noise, there are specific algorithms making it possible to obtain good results, there is first of all appropriate to specify the principal models [5].

In the paragraphs which follow we will present the basic algorithm of recursive least squares (RLS).

2 ARMA model identification

The modelling of autoregressive moving average model noted ARMA (n, m) is defined by the following equation in the differences [3]:

$$\sum_{i=0}^{n} a_i\, y\,(t-i) = \sum_{i=0}^{m} b_i\, u\,(t-i) + e(t) \tag{1}$$

With $a_0 = 1$, $b_0=0$
u(t): Input signal of system
y(t): Output signal of system
e(t): White noise.

Using Z transform, the equation (1) can be written as:

$$\sum_{i=0}^{n} a_i Y(Z) Z^{-i} = \sum_{i=0}^{m} b_i U(Z) Z^{-i} + E(Z) \tag{2}$$

Where:

$$A(Z) = \sum_{i=0}^{n} a_i Z^{-i} \ ; \quad B(Z) = \sum_{i=0}^{m} b_i Z^{-i}$$

Therefore, the equation (2) will be written in this form:

$$A(Z)\, Y(Z) = B(Z)\, U(Z) + E(Z) \tag{3}$$

The error of prediction can be written:

$$E(Z) = A(Z)\, Y(Z) - B(Z)\, U(Z) \tag{4}$$

From the equation (4), we can draw the following diagram:

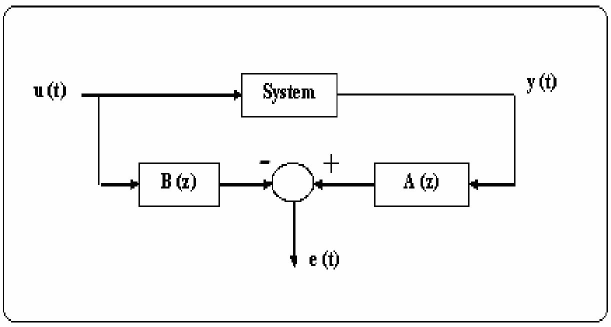

Fig.2: Structure realization of ARMA model

By developing the equation (1), we obtain:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \ldots - a_n y(t-n) + b_1 u(t-1) + \ldots + b_m u(t-m) + e(t) \tag{5}$$

3 Determination of parameters

We think in an intuitive way that if we increase the number of observations; the problem will be reduced to the resolution of a system of linear equations.

In the case of a disturbed system, we carry out N measurements of observations, we can write, according to the equation (1), a matrix form: [2]

$$y(t) = \theta^T \varphi(t) + e(t) \tag{6}$$

With
θ^T: parameters vector to be identified
$\varphi^T(t)$: data vector.

θ^T and $\varphi^T(t)$ two vectors can be defined as:

$$\theta^T = [a_1 \quad a_2 \quad \ldots \quad a_n \ , \ b_1 \quad b_2 \quad \ldots \quad b_m]$$
$$\varphi^T(t) = [-y(t-1) \ldots -y(t-n)\ , u(t-1) \ldots u(t-m)]$$

We define the error of prediction as the difference between the output system and the output of the model:

$$\varepsilon(t) = y(t) - \hat{y}(t) \tag{7}$$

$$\hat{y}(t) = \hat{\theta}^T (t-1) \varphi(t) \tag{8}$$

With $\hat{\theta}^T (t-1)$: represent the estimated parameters.

The method of least squares is based on the determination of the best parameters, i.e. those which will minimize a certain criterion of optimality. It represents the sum of the squares of the errors of predictions, and which is mentioned with the lower part [1], [2]:

$$J_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} [\varepsilon(t)]^2 \tag{9}$$

The minimization of the criterion $J_N(\theta)$ consists to find an optimum, i.e. to calculate the derivative:

$$\left[\frac{\delta J_N(\theta)}{\delta \theta}\right]_{\theta=\hat{\theta}(N)} = 0 \tag{10}$$

$$\frac{\delta J_N(\theta)}{\delta \theta} = -\frac{2}{N} \left\{ \sum_{t=1}^{N} \varphi(t) \left[y(t) - \theta^T \varphi(t) \right] \right\}_{\theta=\hat{\theta}(N)} \tag{11}$$

From equation (10) and (11), we deduce the optimal solution within the meaning of least squares as the following form:

$$\hat{\theta}(N) = [\sum_{t=1}^{N} \varphi(t)\, \varphi^T(t)]^{-1} . \sum_{t=1}^{N} \varphi(t) y(t) \tag{12}$$

We note that the matrix $\varphi(t)\varphi^T(t)$ is large, if the number of samples N is significant, and then the calculation of its reverse is not advised, for that we use the recursive least squares estimation.

4 Recursive least squares algorithm

For the recursive algorithm, we note:

$$R(t)=\sum_{k=1}^{t}\varphi(k)\varphi^T(k)=R(t-1)+\varphi(t)\varphi^T(t) \tag{13}$$

According to the equations (12) and (13), we have:

$$\hat{\theta}(t) = R^{-1}(t)\sum_{k=1}^{t}\varphi(k)y(k) \tag{14}$$

$$\hat{\theta}(t) = R^{-1}(t)\left[\sum_{k=1}^{t-1}\varphi(k)y(k)+\varphi(t)y(t)\right] \tag{15}$$

$$\hat{\theta}(t) = R^{-1}(t)\left[R(t-1)\hat{\theta}(t-1)+\varphi(t)y(t)\right] \tag{16}$$

$$\hat{\theta}(t) = R^{-1}(t)\left[R(t)\hat{\theta}(t-1)-\varphi(t)\varphi^T(t)\hat{\theta}(t-1)+\varphi(t)y(t)\right] \tag{17}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1)+R^{-1}(t)\varphi(t)\left[y(t)-\hat{\theta}^T(t-1)\varphi(t)\right] \tag{18}$$

According to this last equation (18), we notice that the solution of recursive least squares contains the term $R^{-1}(t)$ which requires a matrix inversion at every moment t.

Then, we use the lemma of matrix inversion which arises in the following form [2]:

$$[A+BCD]^{-1} = A^{-1}-A^{-1}B\left[DA^{-1}B+C^{-1}\right]^{-1}DA^{-1} \tag{19}$$

We put:

$$A = R(t-1),\ B=\varphi(t),\ C=1,\ D=\varphi^T(t)$$

However

$$R^{-1}(t)=\left[R(t-1)+\varphi(t)\varphi^T(t)\right]^{-1} \tag{20}$$

The application of the lemma of matrix inversion on the equation (20), gives:

$$R^{-1}(t)=R^{-1}(t-1)-\frac{R^{-1}(t-1)\varphi(t)\varphi^T(t)R^{-1}(t-1)}{1+\varphi^T(t)R^{-1}(t-1)\varphi(t)} \tag{21}$$

The introduction of the matrix of the profit of adaptation $P(t) = R^{-1}(t)$, allows writing the algorithm of recursive least squares RLS as the following form:

$$\hat{\theta}(t)=\hat{\theta}(t-1)+P(t)\varphi(t)\left(y(t)-\hat{\theta}^T(t-1)\varphi(t)\right) \tag{22}$$

$$P(t)=P(t-1)-\frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{1+\varphi^T(t)P(t-1)\varphi(t)} \tag{23}$$

5 Properties of least squares estimator

The estimator θ (t) is known as unbiased if this condition is checked:

$$E\left[\hat{\theta}(t)\right] = \theta_0$$

By replacing the equation (6) in (12), we obtain:

$$\hat{\theta}(N) = \left[\sum_{k=1}^{N}\varphi(k)\varphi^T(k)\right]^{-1}\sum_{k=1}^{N}\varphi(k)\left[\varphi^T(k)\theta_0+e(k)\right]$$

$$E\left[\hat{\theta}(N)\right]=\theta_0+E\left([\sum_{k=1}^{N}\varphi(k)\varphi^T(k)]^{-1}.\sum_{k=1}^{N}\varphi(k)e(k)\right)$$

Where E: represent the expectation.

The method of least squares provides an unbiased estimate if *e (k)* a centred random sequence (average null). Finally, from where, we write: $E[\theta(N)] = \theta_0$ and the estimator is unbiased.

$$\hat{\theta}(N) \rightarrow \theta_0 \quad when \quad N\rightarrow \infty$$

6 Example of simulation

Let us consider a stable physical system having the following transfer function:

$$H(z)=Z^{-1}\frac{1+2Z^{-1}}{1+0.3Z^{-1}+0.8Z^{-2}}$$

With $a_1 = 0.3$; $a_2 = 0.8$; $b_1 = 1$; $b_2 = 2$

The implementation of algorithm RLS on PC, by using the programming language MATLAB, gave us the following results:

N \ $\hat{a}_i,\hat{b}_i$	$\hat{a}_1$	$\hat{a}_2$	$\hat{b}_1$	$\hat{b}_2$	S.D
04*256	0.2995	0.7900	0.9252	1.9984	0.0356
10*256	0.3050	0.8055	1.0206	1.9686	0.0221
20*256	0.3088	0.8036	0.9776	2.0116	0.0148
30*256	0.2929	0.7940	0.9989	1.9987	0.0031
50*256	0.3016	0.8041	1.0036	2.0060	0.0018

Tab.1: Estimate parameters according to N.

σ^2 \ $\hat{a}_i,\hat{b}_i$	$\hat{a}_1$	$\hat{a}_2$	$\hat{b}_1$	$\hat{b}_2$	S.D
1.00	0.3088	0.8036	0.9776	2.0116	0.1148
0.64	0.2975	0.8017	1.0109	1.9945	0.0071
0.25	0.3031	0.7993	0.9885	2.0016	0.0066
0.04	0.2996	0.7999	1.0010	1.9978	0.0014
0.01	0.2999	0.79996	1.9990	1.9990	0.0012

Tab.2: Estimate parameters according to the variance

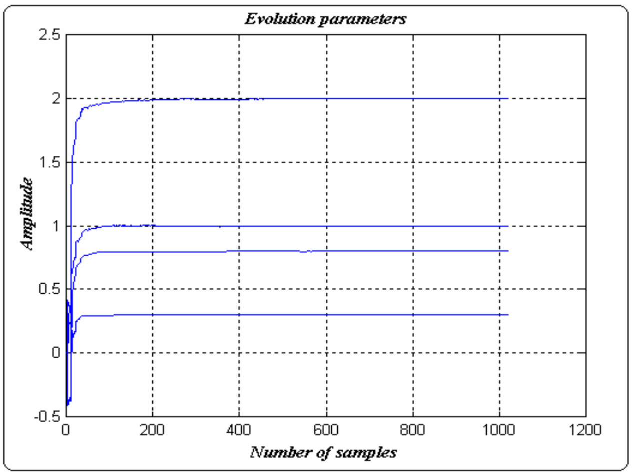

Fig.3: Evolution parameters

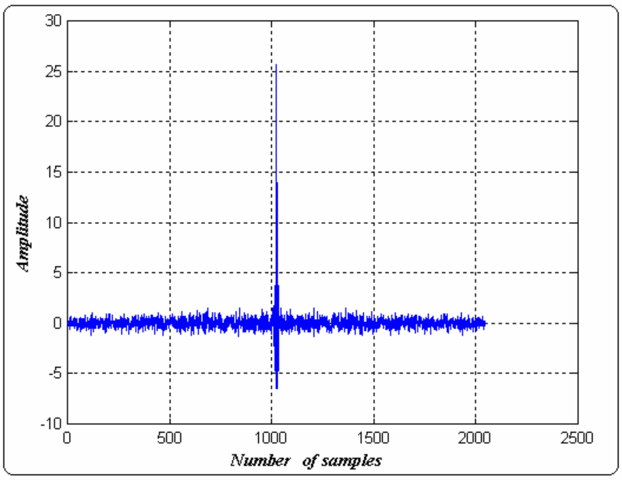

Fig.4: Correlation of error prediction

7 Application to the speech signal

Let us consider the output signal as a speech signal which is in the form of a data file which corresponds to the following sentence: (un *loup s'est jeté immédiatement sur la petite chèvre*), (See Fig.5 and Fig.6).

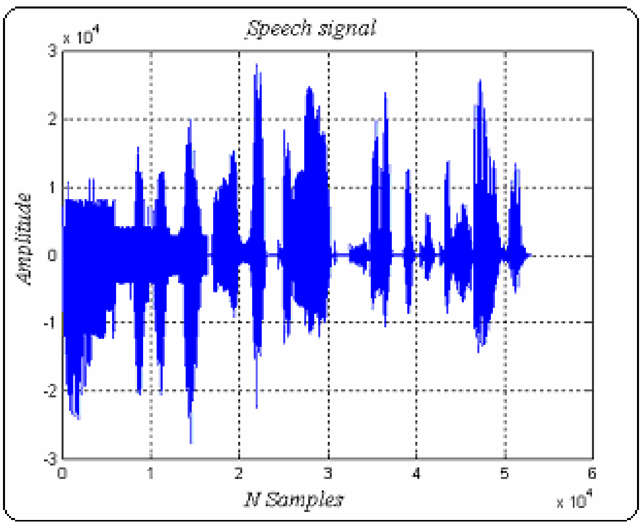

Fig.5: Representation of speech signal.

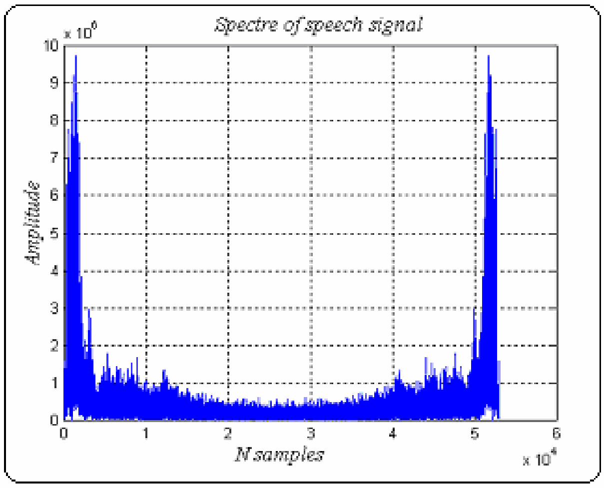

Fig.6: Spectrum representation of speech signal.

The model which corresponds to the signal of speech having the auto regressive structure AR of the form: [6].

$$y(t) = \sum_{i=1}^{n} a_i y(t-i) + v(t)$$

With

n: represent the order of the model

v (t): white noise

The use of the algorithm of recursive least squares RLS to identify the parameters of model AR, gives the following results:

Speech parameters	*RLS algorithm*	*Matlab function*
a_0	1.0000	1.0000
a_1	-0.7822	-0.7824
a_2	-0.1141	-0.1139
a_3	0.0235	0.0234
a_4	-0.0622	-0.0620
a_5	0.0005	0.0008
a_6	0.0097	0.0094
a_7	0.0040	0.0040
a_8	-0.0293	-0.0293
a_9	0.0132	0.0127
a_{10}	0.0658	0.0661

Tab.3: Estimate parameters of speech signal

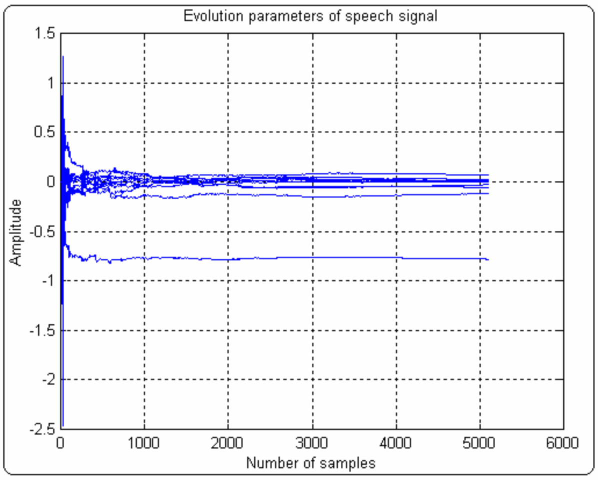

Fig.7: Evolution parameters of speech signal

8 Conclusion

The method of recursive least square which was presented in detail in this paper gives unbiased estimates for ARMA models.

In addition, this method does not use any information in a priori on the noise of measurement, and we showed that if the noise is not with null average value, the estimate of the parameters is biased. Generally, we did not use the law of statistical distributions of the noise and we supposed it not even known. For this reason, if we want to improve a quality of the estimator, we suppose the statistics are known in a priori, from where, we use the maximum likelihood method (RML).

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