A New Algorithm of Unit Commitment Based on On/Off Decision Criterion

HUI ZHENG and BEI GOU
University of Texas at Arlington
701 S. Nedderman Drive Arlington, TX 76019
UNITED STATES OF AMERICA

Abstract: - Based on the mathematical model presented in [1], this paper presents a new algorithm to decide the on/off schedule in Unit commitment by using Lagrangian Relaxation method. Theoretical analysis of the presented algorithm is provided. A simple search scheme, which avoids the forward searching and back tracing used by dynamic programming (DP) algorithm, is developed. Theoretical analysis and numerical test results show that the presented algorithm can be the substitute of the existing DP algorithm.

Key-Words: - Unit commitment, Lagrangian relaxation method, dynamic programming

1 Introduction

NOMENCLATURE

$F_i(P_i^t)$	Generator fuel cost function in a quadratic form $F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2$
N	Total number of generator units
$P_{i,\min}$	Minimum real power generation of unit i
$P_{i,\max}$	Maximum real power generation of unit i
P_i^t	Generation output power of unit i at hour t
P_{load}^t	Load demand at hour t
R^t	Spinning reserve at hour t
ST_i^t	startup cost of unit i at hour t
T	Total number of hours.
$T_{i,down}$	Minimum down time of unit i
$T_{i,off}$	Continuously off time of unit i
$T_{i,on}$	Continuously on time of unit i
$T_{i,up}$	Minimum uptime of unit i.
U_i^t	Status of unit (on=1, off=0)
λ^t, μ^t	Lagrangian multipliers at hour t

Unit commitment (UC) is an important application which schedules the generations in an economic way that the total system production cost over the scheduled time horizon is minimized under the constraints of the spinning reserve and operational constraints of generator units. Generally, UC problem is a nonlinear, mixed integer combinatorial optimization problem. Many approaches have been developed to try to solve UC problem.

Many methods such as priority list, Dynamic Programming (DP) [2], [3], [4], and Lagrangian Relaxation (LR) method[1], [5], [6], [7] have been developed to solve the combinatorial optimization problem. Recently, Artificial Neural Networks (ANN) [8],[9], Simulated Annealing (SA)[10],[11], fuzzy logic [12], [13] and Genetic Algorithms (GA) [14], [15] are also applied to solve this problem.

Up to now Lagrangian Relaxation (LR) method is still the most popular method for solving the unit commitment problem. The basic idea of LR is to relax system constraints in the objective function by using Lagrangian multipliers. The relaxed problem is then decomposed into N sub-problems for each unit. The dynamic programming process is used to search optimal commitment for single units. The Lagrangian multipliers are updated based on violations of systems constraints. For the UC problem, the primal function is always greater than or equal to the function which is defined as weak duality. The difference in value between the primal and dual function yields the duality gap which provides the measure of the sub-optimality of the solution. Most of the LR research has been concentrating on finding an appropriate technique for updating the Lagrangian multipliers, while minimizing the duality gap. Most of the studies update the Lagrangian multipliers using sub-gradient search algorithm.

In this paper, a new algorithm is developed to substitute the conventional dynamic programming (DP) for solving the dual problem. The new method avoids the searching forward and tracing back. It only uses an auxiliary function compared with the start cost as the On/Off decision criterion of the unit. The obtained numerical test results show that the proposed method is simple, efficient and has great potential in solving practical UC problems.

The organization of this paper is as follows. Section 2 briefly introduces the Lagrangian Relaxation method based on [1]. A new algorithm which has the potential to replace the existing dynamic programming is presented in Section 3. Section 4 presents simple numerical tests on the proposed algorithm. Conclusion is given in Section 5. The Appendix provides the proofs of the proposed theorems of the new algorithm.

2 Lagrangian Relaxation Method for Unit Commitment

UC problem is designed to minimize the production cost over the scheduled 24-hour time horizon under the constraints of generator operation and spinning reserves. The objective function to be minimized is [1]

$$F(P_i^t, U_i^t) = \sum_{t=1}^{T}\sum_{i=1}^{N}[F_i(P_i^t) + ST_i^t(1-U_i^{t-1})]U_i^t \quad (1)$$

Subject to

(1) power balance constraint

$$P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t = 0 \quad (2)$$

(2) spinning reserve constraint

$$P_{load}^t + R^t - \sum_{i=1}^{N} P_{i,\max} U_i^t \le 0 \quad (3)$$

(3) generation limit constraints

$$P_{i,\min}U_i^t \le P_i^t \le P_{i,\max}U_i^t \quad i = 1,2,\dots N \quad (4)$$

(4) minimum up and down time constraints

$$U_i^t = \begin{cases} 1, & if \quad T_{i,on} > T_{i,up} \\ 0, & if \quad T_{i,off} > T_{i,down} \\ 0 or 1, & otherwise \end{cases} \quad (5)$$

By temporarily ignoring the coupling constraints, duel optimization procedure transfers the original optimization problem into the following optimization problem (details are available in [1]):

$$\min_{P_i^t, U_i^t} L(P,U,\lambda,\mu) = \sum_{i=1}^{N}\min\sum_{t=1}^{T}\{[F_i(P_i^t) + ST_i^t(1-U_i^{t-1})]U_i^t - \lambda^t P_i^t U_i^t - \mu^t P_{i,\max}U_i^t\} \quad (6)$$

Subject to $U_i^t P_{i,\min} \le P_i^t \le U_i^t P_{i,\max}$ for t=1,…,T, and the constraints are given in (5).

3 A New Algorithm for Solving the Sub-problems

3.1 Dynamic problem

In the conventional Lagrangian relaxation method, dynamic programming is used to obtain the dual solution for each unit separately.

In Eq. (6), if $U_i^t = 0$, the value of objective function to be minimized is trivial (i.e., it equals zero); if $U_i^t = 1$, the startup cost term and the term $\mu^t P_{i,\max}$ can be ignored since they keep unchanged at this moment during the minimization of the objective function. The optimization problem, therefore, becomes the problem to decide P_i^t so that $\left[F_i(P_i^t) - \lambda^t P_i^t\right]$ is minimized.

The term $\left[F_i(P_i^t) - \lambda^t P_i^t\right]$ will be minimized by the optimality condition to find the dual power

$$\frac{d}{dP_i^t}\left[F_i(P_i^t) - \lambda^t P_i^t\right] = 0 \quad (7)$$

The dual power is then obtained

$$P_i^{t,opt} = \frac{\lambda^t - b_i}{2c_i} \quad (8)$$

There are three cases to check $P_i^{t,opt}$ against its limits [1]:

(1) If $P_i^{t,opt} < P_{i,\min}$ then $P_i^t = P_{i,\min}$

(2) If $P_{i,\min} \le P_i^{t,opt} \le P_{i,\max}$ then $P_i^t = P_i^{t,opt}$

(1) If $P_i^{t,opt} > P_{i,\max}$ then $P_i^t = P_{i,\max}$

3.2 A new Lagrangian algorithm

In our method, the new on/off decision criterion will be used to substitute the DP method.

We define the new variable

$$X_i^t = F_i(P_i^t) - \lambda^t P_i^t - \mu^t P_{i,\max} \quad (9)$$

Then the minimum of the Lagrangian function for each generating will be changed to

$$\min\sum_{t=1}^{T}[X_i^t + ST_i^t(1-U_i^{t-1})]U_i^t \quad i = 1,\dots,N \quad (10)$$

Omitting the generator subscript i, equation (10) becomes:

$$\min\sum_{t=1}^{T}[X^t + ST(1-U^{t-1})]U^t \quad (11)$$

Here we assume ST is constant. If ST=0, then the on/off decision is very easy to make: If X^t is negative ($X^t < 0$), $U^t = 1$; If X^t is positive ($X^t > 0$), $U^t = 0$; If $X^t = 0$, U^t could be either 1or 0.

But usually ST > 0, then changing state (especially from 0 to 1) will affect the result of Eq. (11). So we have to modify the above criterion. Our main idea is: $\forall t \in [0,1,...,T]$, if $X^t < -ST$, then $U^t = 1$; If $X^t > ST$, then $U^t = 0$. If $-ST < X^t < ST$, hen U^t will depends on X^{t+1}. If still can not be decided, it will depends on X^{t+2}, and so on…

In order to solve this problem, we define new variables:

$$S(M) \overset{\Delta}{=} \sum_{t=1}^{M} X^t \tag{12}$$

with S(0) =0.

$$S_{\max}(M) = \max\{S(0), S(1), S(2), \ldots, S(M)\} \tag{13}$$

$$S_{\min}(M) = \min\{S(0), S(1), S(2), \ldots, S(M)\} \tag{14}$$

$$i_{\max}(M) = t' \quad \text{when } S(t') = S_{\max}(M) \tag{15}$$

$$i_{\min}(M) = t'' \quad \text{when } S(t'') = S_{\min}(M) \tag{16}$$

$$D(M) \overset{\Delta}{=} S_{\max}(t) - S_{\min}(t) \tag{17}$$

As shown in Fig 1:

$S(10) = 1$; D(10)=7;

$S_{\max}(10) = 4$; $S_{\min}(10) = -3$;

$t_{\max}(10) = 8$; $t_{\min}(10) = 5$.

We assume that the solution of Eq. (10) is U_t^* t=1,…,T.

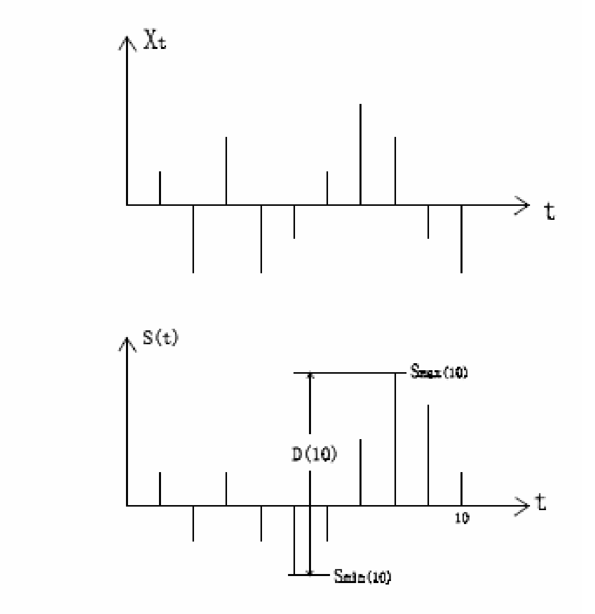

Fig. 1. A example of S(M).

Our new on/off decision criterion is (the proof is given in the Appendix I):

1. $\forall t \in 1,2,\ldots,M, D(M) < ST$

(1) If $U_0^* = U_{M+1}^*$, then

$$U_t^* = U_0^* \qquad t=1,2,\ldots,M \tag{18}$$

(2) If $U_0^* = 0$, $U_{M+1}^* = 1$ then

$$\begin{cases} U_t^* = U_0^*, & t = 1,2,\ldots t_{\max} \\ U_t^* = U_{M+1}^*, & t = t_{\max}+1,\ldots,M \end{cases} \tag{19}$$

(3) If $U_0^* = 1$, $U_{M+1}^* = 0$ then

$$\begin{cases} U_t^* = U_0^*, & t = 1,2,\ldots t_{\min} \\ U_t^* = U_{M+1}^*, & t = t_{\min}+1,\ldots,M \end{cases} \tag{20}$$

(4) If M=24, then

$$\begin{cases} U_t^* = 0, S(T) > 0 & t = 1,2,\ldots T, \\ U_t^* = 1, S(T) < 0 & t = 1,2,\ldots,T \end{cases} \tag{21}$$

2. $\forall t \in 1,2,\ldots,M, D(M-1) < ST$, D(M) > ST,

(1) If $X^M > 0$, then

$$U_t^* = 0 \qquad t = t_{\min},\ldots,M$$

especially if $X^t > ST$, then $U_t^* = 0$.

(2) If $X_M < 0$, then

$$U_t^* = 1 \qquad t = t_{\max},\ldots,M$$

especially if $X^t < ST$, then $U_t^* = 1$.

From the criterion 2, we can get the solution of interval from $t_{\min}$ (or $t_{\max}$) to M directly. The other interval's solution could be gotten from criterion 1.

For every generator, the new algorithm is given as follows:

Step 1: Set t = 0, $t' = t$;

Step 2: Set S(0) = 0;

Step 3: If t < T, t = t+1; Otherwise stop.

Step 4: Calculate S(t), $S_{\max}$, $S_{\min}$, $t_{\max}$, $t_{\min}$, D;

Step 5: If D < ST, Go to step 3.

Step 6: If D > ST,

(1) If $X^t > 0$,

$$U_k^* = U_{t'} \qquad k = t'+1,\ldots,t_{\min}$$

$$U_k^* = 0 \qquad k = t_{\min}+1,\ldots,t$$

(2) If $X^t < 0$,

$$U_k^* = U_{t'} \qquad k = t'+1,\ldots,t_{\max}$$

$$U_k^* = 1 \qquad k = t_{\max}+1,\ldots,t$$

Go to step 2

4 Numerical Results

In order to illustrate the effectiveness of the new method, the results of dynamic programming and our presented algorithm are compared. Two methods are used in solving the equation (10). The program was written in MATLAB on a PC/Pentium 400. We use a random vector X with 1,000,000 random variables and random number ST in the test. The results show that dynamic programming uses 40.7 seconds but our new method only need 27.9 seconds.

The results show that the new method is faster than the dynamic programming, but identical optimization solution is obtained. As mentioned in 3.1, in solving the equation (11), dynamic programming needs 4N(T-1)+2 additions and 2N(T-1) comparisons, but our method only needs 2NT additions and 2NT comparisons. So the new method is more efficient than the dynamic programming. More numerical test results will be reported in further papers.

5 Conclusion

This paper presents a new approach to determine the hourly unit status instead of single-unit dynamic programming. The new On/Off decision criterion is depended on an auxiliary variable S which is the sum of X^t. The proposed method has been tested. Test results show that the proposed algorithm is able to obtain the identical results as dynamic programming but with less CPU time. The results imply that the proposed method is simple, efficient and has potential for solution of practical UC problem.
In the conventional Lagrangian relaxation method, dynamic programming is used to obtain the dual solution for each unit separately.

Appendix I
Proof of the new method

For the function $F = \sum_{t=1}^{T} [X^t + ST(1-U^{t-1})]U^t$, the solution U_t^* of minF have following proprieties:

Theorem 1.1: Assume that we have known U_0^*, U_{M+1}^*, and D(M) < ST, then:

(1) If $U_0^* = U_{M+1}^*$,

$$U_t^* = U_0^* \quad t = 1,2,\ldots,M \tag{22}$$

(2) If $U_0^* = 0, U_{M+1}^* = 1$,

$$\begin{cases} U_t^* = 0 & t = 1,2,\ldots,t_{\max} \\ U_t^* = 1 & t = t_{\max}+1,\ldots,M \end{cases} \tag{23}$$

(3) If $U_0^* = 1, U_{M+1}^* = 0$,

$$\begin{cases} U_t^* = 1 & t = 1,2,\ldots,t_{\min} \\ U_t^* = 0 & t = t_{\min}+1,\ldots,M \end{cases} \tag{24}$$

Proof: Let's first define the following variable:
n: the number of the state (0 or 1) changed.
F_{fixed}: the cost of the decided time i.e. $t \notin 1,2,\ldots,M$.
F_{n1}^{n2}: the total cost when the number of undecided state at 'off' is n1 and at 'on' is n2.
Then,

$$F_{n1}^{n2} = F_{fixed} + F_{undecided} \tag{25}$$

(i) Assuming $U_0^* = U_{M+1}^* = 1$, then

If n = 0,

$$F_0^1 = F_{fixed} + S(M) \tag{26}$$

$$F_1^0 = F_{fixed} + ST \tag{27}$$

Because S(M) ≤ D(M)<ST, therefore $F_0^1 < F_1^0$.

If n = 1,

$$\begin{aligned} F_1^1 &= F_{fixed} + S(M) + ST - S(off) \\ &\geq F_{fixed} + S(M) + ST - D(M) > F_0^1 \end{aligned} \tag{28}$$

If n = 2,

$$\begin{aligned} F_1^2 &= F_{fixed} + S(M) + ST - S(off) \\ &\geq F_{fixed} + S(M) + ST - D(M) > F_0^1 \end{aligned} \tag{29}$$

or

$$\begin{aligned} F_2^1 &= F_{fixed} + S(M) + 2ST - 2S(off) \\ &\geq F_{fixed} + S(M) + 2ST - 2D(M) > F_0^1 \end{aligned} \tag{30}$$

When n = 3,4,5,…, using the similar method we can also get that $F_n < F_0^1$.

So $U_1^* = U_2^* = \cdots = U_M^* = 1$.

(ii) Assuming $U_0^* = U_{M+1}^* = 0$, then

If n = 0,

$$F_0^1 = F_{fixed} + ST + S(M) \tag{31}$$

$$F_1^0 = F_{fixed} \tag{32}$$

Because $|\mathrm{S(M)}| \leq \mathrm{D(M)} < \mathrm{ST}$, therefore, $F_1^0 < F_0^1$

If n = 1,

$$\begin{aligned} F_1^1 &= F_{fixed} + ST + S(on) \\ &\geq F_{fixed} + ST - D(M) > F_1^0 \end{aligned} \tag{33}$$

If n = 2,

$$F_1^2 = F_{fixed} + 2ST - 2S(on)$$
$$\geq F_{fixed} + 2ST - 2D(M) > F_1^0 \quad (34)$$

or

$$F_2^1 = F_{fixed} + ST + S(on)$$
$$\geq F_{fixed} + ST - D(M) > F_1^0 \quad (35)$$

When n = 3,4,5,…, using the similar method we can also get that $F_n < F_1^0$.

So $U_1^* = U_2^* = \cdots = U_M^* = 0$.

From equation (26) to (35), propriety (1) is proved.

(iii) Assuming $U_0^* = 0, U_{M+1}^* =$, then

Because $S_{\max} \geq 0$ and $S_{\min} \leq 0$, therefore

If n = 1,

$$\exists F_i^{\min} = F_{fixed} + ST + S(M) - S_{\max}(M) \quad (36)$$

when

$$U_t^* = 0 \quad t = 1,2,\ldots,t_{\max} \quad (37)$$
$$U_t^* = 1 \quad t = t_{\max} + 1,\ldots,M$$

If n = 0,

$$F_0^1 = F_{fixed} + ST + S(M) \geq F_1^{\min} \quad (38)$$
$$F_1^0 = F_{fixed} + ST \geq F_1^{\min} \quad (39)$$

If n = 2,

$$F_1^2 = F_{fixed} + 2ST + S(M) - S(off)$$
$$\geq F_{fixed} + 2ST + S(M) - D(M)$$
$$= F_{fixed} + 2ST + S(M) - [S_{\max}(M) - S_{\min}(M)]$$
$$= F_{fixed} + 2ST + S(M) - S_{\max}(M) + [ST + S_{\min}(M)]$$
$$\geq F_1^{\min} \quad (40)$$

or

$$F_2^1 = F_{fixed} + 2ST + S(on)$$
$$\geq F_{fixed} + 2ST - D(M)$$
$$\geq F_{fixed} + ST$$
$$\geq F_{fixed} + ST + S(M) - S_{\max}(M) = F_1^{\min} \quad (41)$$

When n = 3,4,5,…, using the similar method we can also get that $F_n > F_1^{\min}$. So propriety (2) is alos proved.

Propriety (3) could be proved in the same way.

Theorem 1.2: $\forall t \subset 1,2,\ldots,M$, Assume that D(M-1) < ST and D(M) > St, then:

(1) If $X^M > 0$,

$$U_t^* = 0 \quad t = t_{\min},\ldots,M$$

especially if $X^t > ST$ then $U^t = 0$.

(2) If $X^M < 0$,

$$U_t^* = 1 \quad t = t_{\max},\ldots,M$$

especially if $X^t < -ST$ then $U^t = 1$.

Proof: Because D(M-1) < ST and D(M) >ST, so $t_{\max} = M$.

Define n' as the number of the state (0 or 1) changed at internal [$t_{\min}$, M], then:

Assuming $U_{t_{\min}}^* = 0$ and $U_{M+1}^* = 0$,

If $n' = 0$,

$$F_1^0 = F'_{fixed}$$
$$F_0^1 = F'_{fixed} + ST + D(M) > F_1^0$$

If $n' = 1$,

$$F_1^1 = F'_{fixed} + ST + S(on) > F_1^0$$

If $n' = 2$,

$$F_1^2 = F'_{fixed} + 2ST + 2S(on) > F_1^0$$

or

$$F_2^1 = F'_{fixed} + ST + S(on) > F_1^0$$

When $n' = 3,4,5,\ldots$, using the similar method we can get that $F_n > F_1^0$.

In the other conditions, we can also get $F_n > F_1^0$. So propriety (1) is also proved.

It is easy to see if $X_t > ST$, then $U_t^* = 0$.

Using the same approach, propriety (2) can also be proved. Especially if $X_t < -ST$, then $U_t^* = 1$.

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Hui Zheng received the B.S.E. and M.S.E degrees in electrical engineering from Shanghai JiaoTong University in 1992 and 1997, respectively. Currently, he s a PhD student at the Energy System Research Center of the Department of Electrical Engineering, the University of Texas at Arlington (UTA), USA. His areas of interest are power system transient stabitliy analysis and unit commitment.

Bei Gou Received the B.S. degree in electrical engineering from North China University of Electric Power, China, in 1990, and M.S. degree from Shanghai JiaoTong University, China, 1993. From 1993 to 196, he taught at the department of electric power engineering in Shanghai JiaoTong University. He worked as a research assistant at Texas A&M University since 1997 and received his Ph.D. degree in 2000. He worked at ABB Energy Information Systems, Santa Clara CA for two years, and at ISO New England for one year as a senior analyst. He is currently an assistant professor at Energy Systems Research Center (ESRC) at the University of Texas at Arlington. His main interests are power system state estimation, power market operations, power quality, power system reliability and distributed generators.