

# Genetic Algorithms and Neural Networks in Optimal Location of Piezoelectric Actuators and Identification of Mechanical Properties

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*Abstract:* - Composite materials are a very important class of engineering materials with great properties and applications in a variety of complex structures. The correct design of such structures requires adequate analysis and, in particular, adequate and accurate models for simulation with numerical tools. Piezoelectric materials can convert mechanical energy into electric energy and vice-versa and are being applied in laminated composite structures, working either as sensors (mechanical load applications) or actuators (electric potential applications), allowing its use in a wide range of engineering applications. Two important subject matters in the type of applications involving laminated composite structures and piezoelectric materials are the identification of material constants and the optimal location of actuators and sensors. In this work two numerical procedures involving genetic algorithms and neural networks are proposed for these problems. A neural networks based methodology for the identification of mechanical properties is presented. The identification process makes use of the information collected from piezoelectric sensors. Pairs of sensors are placed on the surfaces of a composite laminated plate and the potential differences are obtained among pairs when the plate is charged. Since genetic algorithms are very expensive when the objective function has high computational cost, we introduce the artificial neural networks to improve the efficiency of the genetic algorithm for the optimal location of piezoelectric actuators. Neural networks make a choice of the chromosomes for which it is worthwhile to calculate the objective function; for the other chromosomes neural networks attribute a value to the objective function. For both procedures, the results obtained are compared with those present in literature.

*Key-Words:* - Neural Networks, Genetic Algorithms, Identification, Optimization, Piezoelectrics

## 1 Introduction

Piezoelectric materials are being applied in laminated composite structures, working either as sensors or actuators, allowing its use in a wide range of engineering applications. Benjeddou, [1], presents a bibliographical revision concerning finite elements modeling of adaptive composite structures.

An important aspect in the type of applications involving laminated composite structures and piezoelectric materials is the identification of material constants. In fact, for the case of material composites, as they are the result of the mixture of two or more materials, the obtaining of its final properties becomes particularly complex once the parameters to define depend not just on the properties of the materials involved as, also, on the manufacturing process. The necessary parameters for the model can

be obtained partially through destructive experimental tests of the final material or based on empiric formulas and the maker specifications on the base materials. In any of the cases, the guarantee of the results is reduced since the process of manufacture of the material can cause a great dispersion in the properties. An alternative form of obtaining the involved physical and mechanical parameters is through the resolution of an inverse problem in which the properties are identified from the structural response.

A procedure to obtain the material constants of composite laminates based on genetic algorithms was proposed in [2]. Following this work, a neural network model was proposed in [3]. Bibliographical revision on this subject can be found in [4].

Another important aspect in the type of applications involving laminated composite structures and

piezoelectric materials is to improve their intended performance based on the optimal location of actuators and sensors, a typically discrete problem. A bibliographical revision on the subject of adaptive composite structures, including aspects related to the optimal location can be found in [5]. Genetic algorithms have shown an adequate technique for the optimization problem, particularly to solve these problems [6], but they demonstrated low computational efficiency and high computational cost, essentially related to the great number of times that the evaluation function was calculated.

In this work two numerical procedures involving genetic algorithms and neural networks are proposed for the described problems. A neural network based approach is proposed to identify the properties of composite laminated plates. The information collected from piezoelectric sensors placed on the surfaces of the composite laminated plate is used as the structural response parameter. For the case of optimal location of actuators, a methodology that intends to reduce the number of times that the evaluation function is calculated is proposed, by introducing some "intelligent" characteristics to the genetic algorithm through its combination with neural networks, but without removing the random characteristic of the genetic algorithm.

A higher order finite element formulation allowing the response of the laminated plates is developed in order to make the forward computation for both procedures. The results obtained with these methods are compared with those present in literature.

## 2 Finite Element Modeling

For a composite laminated plate with piezoelectric materials, the linear piezoelectric constitutive relations, coupling the elastic field and the electric field, can be expressed as [7]

$$\sigma = Q \varepsilon - \bar{e} E \quad ; \quad D = e^T \varepsilon + p E \quad (1)$$

where  $\sigma$  is the elastic stress vector,  $\varepsilon$  is the elastic strain vector,  $Q$  is the elastic constitutive matrix in the laminate coordinate system,  $\bar{e}$  is the piezoelectric stress coefficients matrix in the same coordinate system,  $D$  is the electric displacement vector and  $p$  represents the dielectric matrix.  $E$  is the electric field vector which is the negative gradient of the electric potential,  $E = -\nabla \phi$ , where  $\phi$  represents the electric voltage applied.

In order to approximate the elasticity problem of a two-dimensional laminate, a higher order finite element formulation was developed, based in the

following displacement field

$$\begin{cases} u_1(x, y, z, t) = u(x, y, t) + z\phi_1(x, y, t) + z^3\theta_1(x, y, t) \\ u_2(x, y, z, t) = v(x, y, t) + z\phi_2(x, y, t) + z^3\theta_2(x, y, t) \\ u_3(x, y, z, t) = w(x, y, t) + z\psi_3(x, y, t) + z^2\theta_3(x, y, t) \end{cases} \quad (2)$$

where  $u_1, u_2, u_3$  are the displacement components at any point in the laminate space in the  $x, y, z$  directions, respectively,  $u, v, w$  are the displacements of a generic point on the reference surface,  $\phi_1, \phi_2$  are the rotations of a normal to the reference surface related to the  $y$  and the  $x$  axes, respectively, and  $\theta_1, \theta_2, \theta_3, \psi_3$  are the higher order terms.

Hamilton's principle is used to derive the equations for the finite element formulation and a nine node Lagrangian quadrilateral element is applied to the formulation. This model has nine degrees of freedom at each node for the elastic behaviour and one additional electric potential degree of freedom for each piezoelectric layer for the piezoelectric behaviour. The electrical potentials  $\phi$  are assumed to be constant for each piezoelectric layer within each element.

The system of equilibrium equations obtained for the laminated composite plate with embedded or surface bonded piezoelectric layers can be written as

$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} u \\ \phi \end{Bmatrix} = \begin{Bmatrix} F_u(t) \\ F_\phi(t) \end{Bmatrix} \quad (3)$$

where  $M_{uu}$  is the mass matrix,  $K_{uu}$  is the elastic stiffness matrix,  $K_{\phi\phi}$  is the dielectric 'stiffness' matrix and  $K_{u\phi} = K_{\phi u}^T$  are the coupling matrices between elastic mechanical and electrical effects.  $\{u \ \phi\}^T$  are the system of generalized displacements and voltages at sensors and  $\{F_u(t) \ F_\phi(t)\}^T$  are the mechanical loads and the applied electrical charges at time  $t$ , defining the force vector of the system. A more complete description can be obtained in [4, 5].

## 3 Genetic Algorithms and Neural Networks

### 3.1 Genetic Algorithms

Genetic algorithms are search and optimization techniques inspired by Darwin's theory of natural evolution [8]. Genetic algorithms start with a population of chromosomes (a digit sequence, in

general binaries), each one of them representing a solution in the search space. Each solution is evaluated using a fitness function which demonstrates the merit of the respective individual. Based on the fitness, a random selection is made, in which the best chromosomes have a higher chance of being selected. The successive application of genetic operators (crossover and mutation) to the selected individuals generates tendentially more fitting populations. When the algorithm attains a pre-established criterion, it stops.

The genetic algorithms have a high probability of tending to the global minimum; that is, to the best performance. The risk of the algorithm being stuck in a local minimum is relatively low if the search is made from a large enough random set of solutions and if the population diversity is assured during the process.

The basic requisites for the construction of a genetic algorithm are: 1) Codification technique of the population individuals in chromosomes, 2) Characteristics (dimension and generation) of the initial population, 3) Evaluation function that allows the measurement of the merit of each individual, 4) Selection method used to make sure that the best individuals have a higher probability of remaining in the population and/or of reproduction, 5) Genetic operators necessary for the attainment of new individuals and the probability with which they will operate, 6). Stop criteria. A more detailed description of these aspects can be found in [5].

### 3.2 Neural Networks

Some appealing features of neural networks are their ability to learn through examples, they do not require any prior knowledge and can approximate well any non-linear continuous function [9]. Among the several architectures used in practice, feedforward type neural networks, shown in Fig. 1, have been considered more suitable for the purposes of the signature analysis.

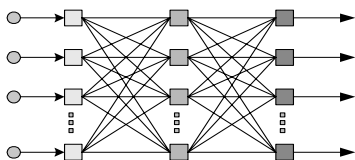


Fig. 1 – Feedforward Neural Network

A feedforward neural network consists of several layers, each one with some neurons, linked to each other by weights. The weights determine the nature and the strength of the connection and the number of nodes considered in the input and output layers depend on the specifications of the problem. The

number of hidden layers, the number of neurons in each hidden layer as well as the activation function type for each neuron is selected according to the experience and some convergence criterions.

The application of artificial neural network consists of two stages, namely training and testing. During the training stage an input-to-output mapping, using the available sample data, is presented to the network. The network evaluates its own output based on the presented input and compares this value with the target (presented) output. The actual output error is used to adjust the node weights so that the error can be reduced. The learning stage stops once a cross validation pre-set error threshold is reached and the node weights are frozen at this point. During the testing stage, data that have not been presented to the network in the learning stage are provided as input and the corresponding output is calculated using the fixed node weights.

In this work, the training of the neural network has been performed with a second order type algorithm, the Levenberg Marquardt [10].

## 4 Numerical Applications

### 4.1 Identification of Mechanical Properties

#### 4.1.1 Problem Definition

Consider a simply supported glass/epoxy laminated plate, made with 4 glass/epoxy equal thickness layers and with stacking sequence [0/90/90/0]. The plate dimensions are 200x200x2.5 mm.

The fiber glass (*f*) and the epoxy matrix (*m*) have the following mechanical properties:

$$\begin{aligned}
 E_f &= 85.0 \text{ GPa} ; G_f = 35.420 \text{ GPa} ; \nu_f = 0.2 \\
 E_m &= 3.4 \text{ GPa} ; G_m = 1.308 \text{ GPa} ; \nu_m = 0.3
 \end{aligned}
 \tag{4}$$

The laminae from the laminated plate can be manufactured varying the fiber volume fraction from 0.4 to 0.7. Then, considering the Halphin-Tsai equations [11], the corresponding properties of the laminae are presented in table 1. We assume  $E_2 = E_3$ ,  $G_{12} = G_{13} = G_{23}$  and  $\nu_{12} = \nu_{13} = \nu_{23}$ .

Fiber Volume	$V_f = 0.4$	$V_f = 0.7$
$E_1$ (GPa)	36.04	60.52
$E_2$ (GPa)	9.03	20.20
$G_{12}$ (GPa)	2.85	8.13
$\nu_{12}$	0.26	0.23

Tab. 1 – Space of Properties

A discretization with 36 equal elements is considered.

Six piezoelectric sensors are placed on the plate surfaces. A schematic representation of the plate with the piezoelectrics is illustrated in Fig. 1.

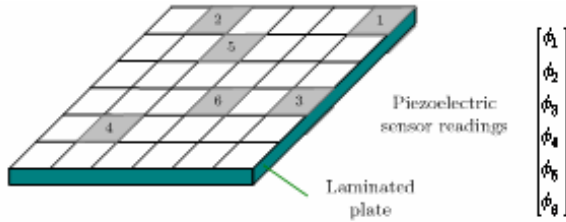


Fig. 1 – Plate with 6 Pairs of Sensors

The plate is loaded with 5 N applied in its center and the sensor readings are obtained with the described finite element model. The application considers a PVDF piezoelectric, with  $E_1 = E_2 = E_3 = 2 \text{ GPa}$ ,  $G_{12} = G_{13} = G_{23} = 1 \text{ GPa}$ ,  $\nu_{12} = \nu_{13} = \nu_{23} = 0.29$ , as elastic properties and the following piezoelectric properties:  $\bar{e}_{311} = \bar{e}_{322} = 0.046 \text{ C/m}^2$  and  $\bar{p}_{33} = 1.062^{-09} \text{ F/m}$ .

The main objective is to determine the mechanical properties  $E_1, E_2, G_{12}$  and  $\nu_{12}$  of a target plate, designated as “experimental”.

**4.1.2 Neural Network Model**

The idea with this model is to use neural networks to interpret the relationship among the potential differences in the sensors and the mechanical properties associated to the laminate plate. A neural network with 4 layers and 6 neurons in the input layer, corresponding to the 6 potential differences in the sensors, and 4 neurons in the output layer, corresponding to the 4 properties, were defined. For the intern layers, 2 neurons were defined. This neural network is designed Net 6-2-2-4.

The training data are created as follows. In a first step, 16 sets of properties are defined according to a uniform distribution of fiber volume between 0.4 and 0.7. After, 30 different fiber volumes are randomly created between the same space, allowing 30 other sets of properties. For each set, the potential differences in the sensors are obtained using the finite element model. Then, 46 training patterns were created. From the 46 training patterns, 6 are randomly chosen to be used as cross validation.

With the Halphin-Tsai equations, the 6 sets of properties which were different from each other, as well as from the training ones, were created to be used as testing data. In order to simulate manufacture conditions, a perturbation is added to the testing values of properties according to

$$\bar{A} = A \left( 1 + \frac{\beta}{100} randn \right) \tag{5}$$

where  $A$  is the data without perturbation,  $\bar{A}$  is the data with perturbation,  $\beta$  is a parameter that indicates the level of perturbation considered (0.5 in this step) and  $randn$  is a random number with variance and standard deviation 1.

Upon obtaining the testing data of “experimental” properties, their corresponding values of potential differences across the piezoelectric sensors are obtained using the finite element model. Finally, in order to simulate piezoelectric sensor reading conditions another perturbation is added, using the same expression (10) and considering  $\beta = 0.25$ .

The hyperbolic tangent function is used as transfer function for all neurons and the data are normalized between  $[-1,+1]$ .

**4.1.3 Obtained Results**

The response obtained from the neural network is presented in table 2. Additionally, this table shows the main relative errors (%) for each case.

Properties		Mat. 1	Mat. 2	Mat. 3
$E_1$ [x10 <sup>9</sup> ]	Exp.	38.018	43.296	47.139
	Net	37.632	43.141	46.922
	Error	1.02	0.36	0.46
$E_2$ [x10 <sup>9</sup> ]	Exp.	9.546	11.262	12.680
	Net	9.464	11.187	12.596
	Error	0.86	0.67	0.66
$G_{12}$ [x10 <sup>9</sup> ]	Exp.	2.999	3.491	3.975
	Net	2.985	3.483	3.951
	Error	0.45	0.21	0.59
$\nu_{12}$	Exp.	0.2580	0.2513	0.2471
	Net	0.2581	0.2513	0.2467
	Error	0.04	0.00	0.19

Properties		Mat. 4	Mat. 5	Mat. 6
$E_1$ [x10 <sup>9</sup> ]	Exp.	48.577	56.343	59.150
	Net	48.437	56.262	59.189
	Error	0.29	0.14	0.07
$E_2$ [x10 <sup>9</sup> ]	Exp.	13.272	17.292	19.098
	Net	13.224	17.245	19.200
	Error	0.36	0.27	0.53
$G_{12}$ [x10 <sup>9</sup> ]	Exp.	4.189	6.075	7.261
	Net	4.182	6.067	7.346
	Error	0.17	0.13	1.17
$\nu_{12}$	Exp.	0.2452	0.2353	0.2322
	Net	0.2448	0.2352	0.2316
	Error	0.18	0.05	0.23

Tab. 2 – Properties Obtained (Net 6-2-2-4)

As is shown in table 2, the neural network model identifies all the properties with mean relative errors lower than 1.2 %.

## 4.2 Optimization of Piezo Actuators Location

### 4.2.1 Problem Definition

Consider a rectangular composite plate (320x80x3 mm) with stacking sequence [0/-45/+45/0], clamped on the shortest side and free on the other sides, made of 4 epoxy laminae reinforced with S glass fiber, as shown in Figure 2.

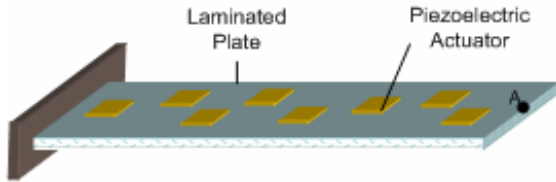


Fig. 2 – Plate with 8 Pairs of Actuators

The material properties of the epoxy laminae reinforced with S glass fiber are the following:  $E_1 = 55$  GPa,  $E_2 = E_3 = 16$  GPa,  $G_{12} = G_{23} = G_{13} = 7.6$  GPa,  $\nu_{12} = \nu_{13} = \nu_{23} = 0.28$ . The mechanical and electric properties of the actuators - PC5H Morgan Matroc, are the following:  $E_1 = E_2 = 60.24$  GPa,  $E_3 = 49.02$  GPa,  $G_{12} = G_{13} = G_{23} = 23$  GPa,  $\nu_{12} = \nu_{23} = \nu_{13} = 0.31$ ,  $d_{31} = d_{32} = -306 \times 10^{-12}$  C/N,  $d_{33} = -800 \times 10^{-12}$  C/N e  $p_{33} = 5.04 \times 10^{-8}$  F/m. The electric potentials applied on the 8 pairs of piezoelectric actuators are +200 V for the actuators bonded to the upper surface of the plate and -200 V for the actuators bonded to the lower surface.

A 4x14 elements plate discretization is used in which the dimension of the elements is similar to the dimension of the piezoelectric actuators, and the finite element formulation is used.

The aim is to find the best location for 8 pairs of piezoelectric actuators, PC5H with 20mm x 20mm dimensions, bonded to the lower and upper surfaces of the plate so as to maximize the transversal displacement in point A.

### 4.2.2 Neural Network/Genetic Algorithms Model

In this model a genetic algorithm is combined with a neural network. For the genetic algorithm, the choice of the binary codification was considered following the description: if the element includes the pair of actuators, the respective gene assumes the value of 1 (one); otherwise it is 0 (zero). Since one intends to determine 8 locations (the number of piezoelectric pairs) among 56 possible ones (the number of elements), the generation of the initial population is forced to produce only admissible chromosomes, that is to say, limited to 8 genes different from zero. The

crossover operation is modified to check how many piezoelectric elements exist in the resulting chromosomes and to assure its admissibility by taking or adding, in a random fashion, genes equal to the unit. The crossover thus developed assures the population diversity but, in spite of that, mutation operation is introduced, conditioned to the fact that the chromosomes produced are equally admissible.

The parameters and genetic operators which present better performance in this case are: Dimension of Population = 60, Probability of Crossover = 65%, Probability of Mutation = 2.5% and Selection Method = Tournament.

The idea of the model AG/NNET consists of using the data generated by the genetic algorithm (the chromosomes) and the corresponding values of the evaluation function to training a neuronal network that can introduce characteristics of intelligence to the genetic algorithm. This model is represented in figure 3.

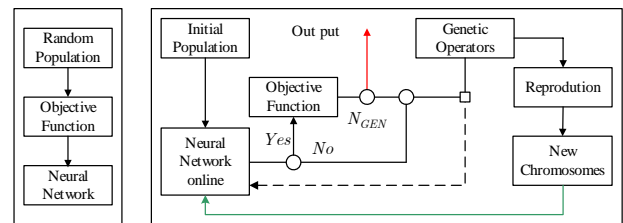


Fig. 3 – Outline of AG/NNET Model

Initially a set of chromosomes is randomly generated to the initial population. Each chromosome is evaluated using an evaluation function which supplies the fitness or merit of the respective individual. The pairs “chromosome-fitness” are used as data to train a neural network. After training, this net is incorporated in the conventional genetic algorithm. When a new chromosome is presented to the neural network, if the chromosome is considered “good”, it is selected to calculate the objective function; on the other hand, if the chromosome is considered “bad”, the fitness of the chromosome is the output of the network.

By definition of the problem, the chromosome is composed of a sequence of 56 genes, so, the input layer takes 56 neurons, one for each gene. To the output layer, once we intend to obtain a classification for the chromosome, a single neuron was considered. In the two internal layers, four neurons are used. Thus, the net is designated by Net 56-4-4-1.

The training of the network is made with cross validation. Additionally, a limit to the number of training epochs is imposed. It should be noted that, with the particular characteristics of this problem, no

testing evaluation is made to the network. In each generation of the genetic algorithm, the neural network is retrained, adding the new evaluated chromosomes to the old ones.

### 4.2.3 Obtained Results

Each problem was executed six times in a Pentium IV 3.0 GHz with 1024 Mb of memory RAM. The results obtained with the proposed model are compared with [6] (conventional genetic algorithm). The resolution of the two models was made with the same hardware support. The optimal location is the same as [6] and is shown in figure 4.

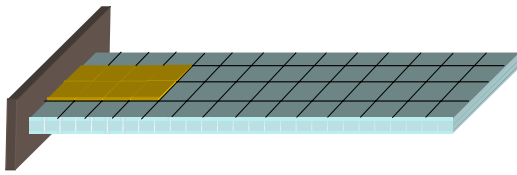


Fig. 4 – Optimal Location

In table 3 the results obtained are shown, namely the medium values of: the number of generations of the genetic algorithm, the number of fitness function calculations and the time need to obtain the solution.

Model	AG [6]	Presented AG + NNET
Nº of Generations	7	7
Nº of Fitness Functions Evaluated	258	42
CPU Time (s)	2143	346

Tab. 3 – Obtained Results

## 5 Conclusions

A numerical methodology to determine the material constants of composite laminates based on neural networks is developed. The sensor readings at 6 defined elements of the discretion plate are used as input for inverse identification. The identification of material constants of a glass-epoxy plate has been carried out with satisfactory results. The obtained results are encouraging and demonstrate the effectiveness of the proposed technique to the characterization of material constants of composite structures.

The numerical methodology for the optimal location of piezoelectric actuators involving the use of genetic algorithms and artificial neural networks allows improving computational efficiency, reducing in a significant way the computational cost as a consequence of the smallest number of calculations

of the fitness function. The obtained results confirm that the introduction of artificial neural networks in the evaluation of the chromosomes reduces drastically the time of calculation, without loss of the quality for the obtained solution.

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